One-quark connected contributions to baryon masses

With C* boundary conditions

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Summary

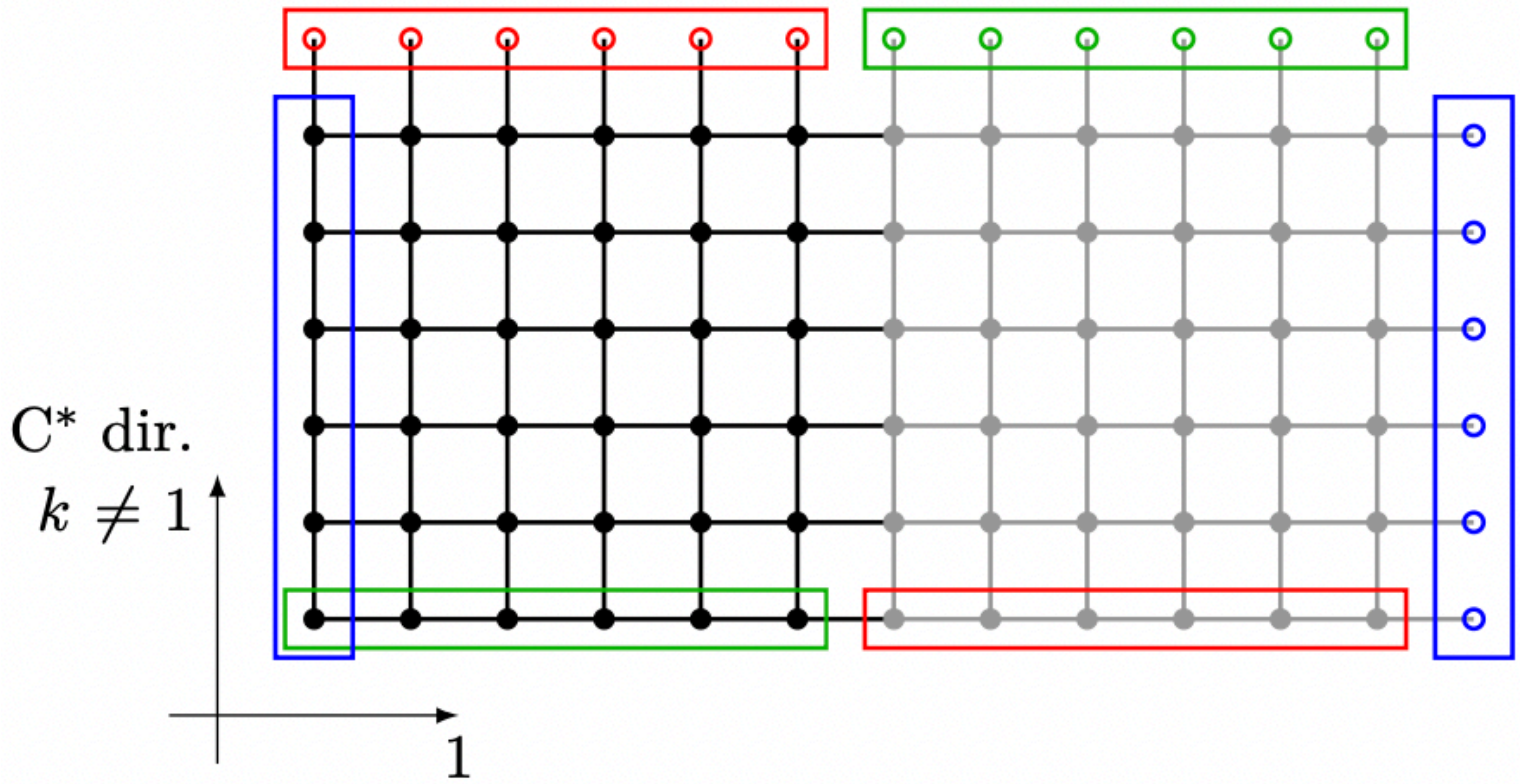
- Quark propagators with C* boundary conditions
- Baryon octet and decuplet, where to find one-quark connected contributions
- Three-quark connected contributions: current state of the measurements
- One-quark connected contributions:
    - Strategy of computation
    - Upper bound
- Outlook

Baryons with C* boundary conditions

Due to C* boundary conditions baryonic two-point correlation functions have additional contributions.

They can be understood looking at quark propagators in the orbifold construction

Baryons with C* boundary conditions



[1]: I. Campos et al. “openQ*D code: a versatile tool for QCD+QED simulations”. In: The European Physical Journal C (Mar. 2020).

Baryons with C* boundary conditions

Quark propagators in the orbifold construction have additional contributions:
(x, y belonging to the physical lattice):

$$\langle q_a^A(x)\ \bar{q}_b^B(y)\rangle = D^{-1}(x,y)_{ab}^{AB} \tag{1}$$

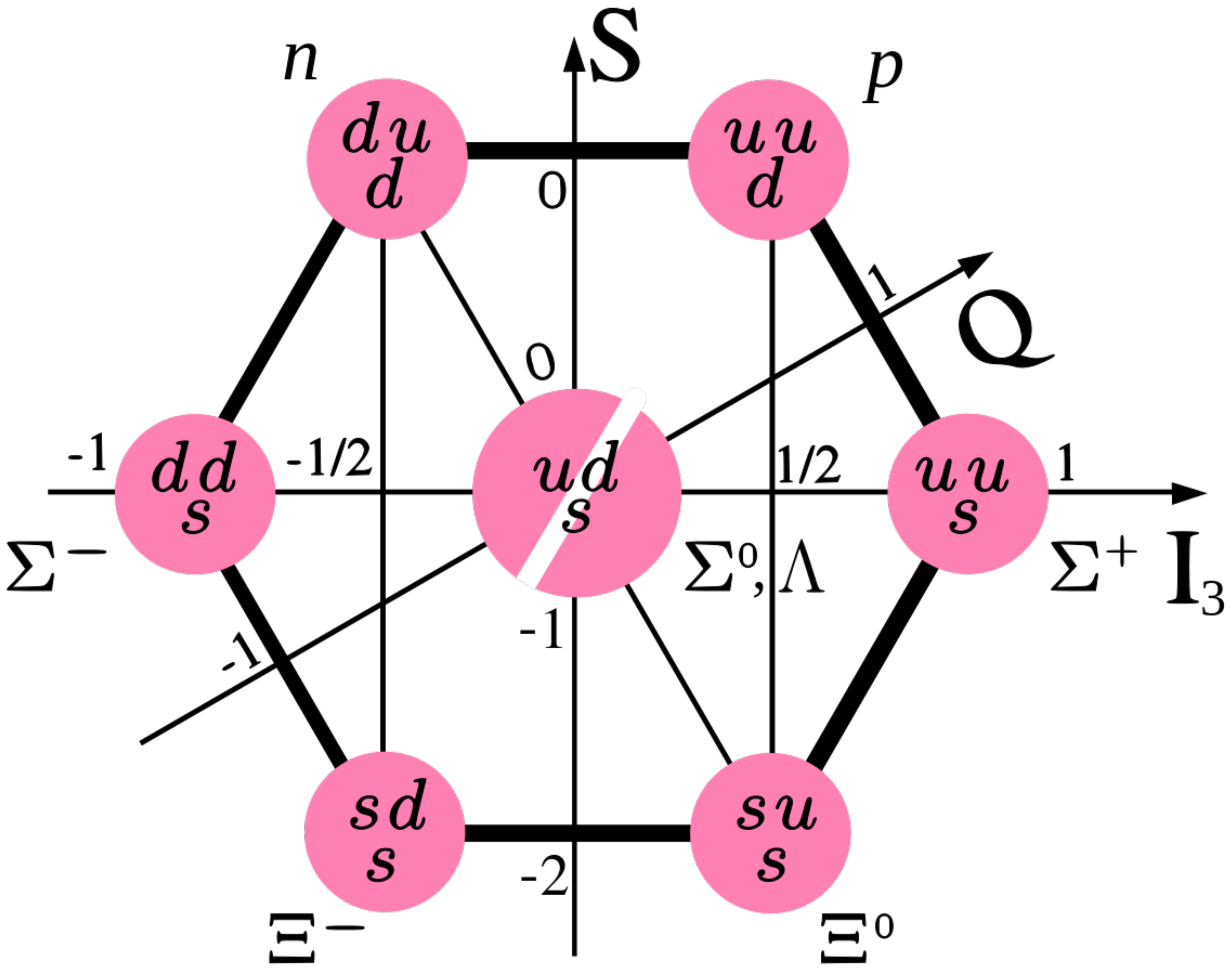
$$\langle q_a^A(x) q_b^{B,T}(y)\rangle = -D^{-1}(x, y + L\hat{1})_{ad}^{AB} C_{db} \tag{2}$$

$$\langle \bar{q}_a^{A,T}(x)\bar{q}_b^B(y)\rangle = C_{ad} D^{-1}(x + L\hat{1}, y)_{db}^{AB} \tag{3}$$

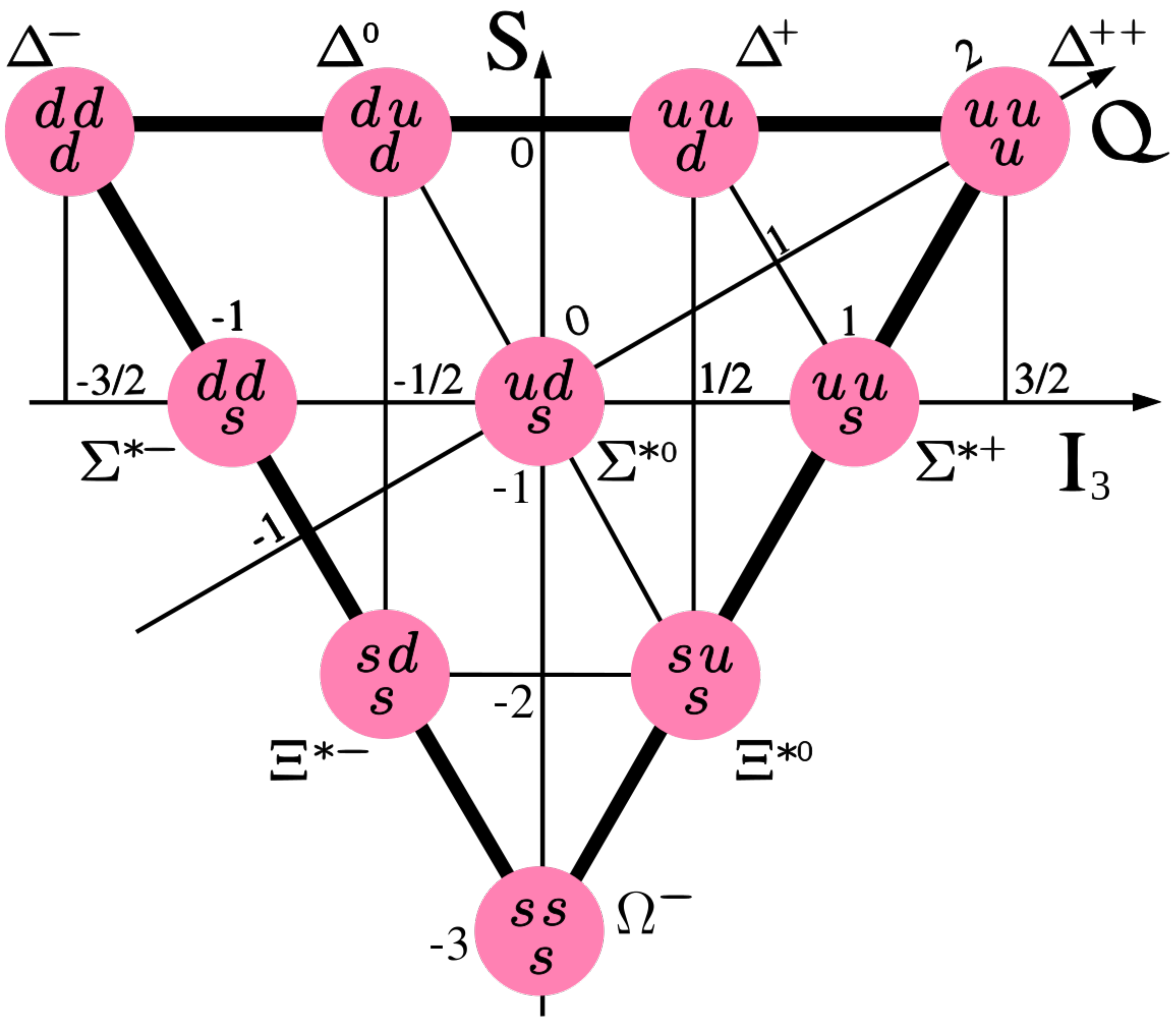
These additional contributions give additional contributions to baryon correlation functions

Baryons: octet and decuplet

Baryon octet



Baryon decuplet



Baryon correlators

Octet

- Interpolating operators:

$$v_c(x) = \sum_{\substack{abc\\ABC}} \epsilon_{ABC}\Gamma_{ab}[\chi_a^A(x)\eta_b^B(x)\chi_c^C(x)] \quad (1)$$

$$\overline{v}_c(x) = \sum_{\substack{abc\\ABC}} \epsilon_{ABC}\Gamma_{ba}[\overline{\chi}_c^C(x)\overline{\eta}_b^B(x)\overline{\chi}_a^{A,T}(x)] \quad (2)$$

- Two-point correlation function:

$$C(x_0) = \sum_{\mathbf{x}} \sum_{cc'} P^+_{cc'}\, v^c(x)\overline{v}^{c'}(0) \quad (3)$$

Where:

$$\Gamma = C\gamma^5 \qquad P^+ = \frac{Id + \gamma^0}{2} \quad (4)$$

Baryon correlators: decuplet vertices

Interpolating operators:

$$v^{m;d}(x) = \sum_{\substack{abc \\ ABC}} W^{d;m}_{abc;ABC} \psi^C_c(x) \psi^A_a(x) \psi^B_b(x) \quad (1)$$

$$\overline{v}^{m;d}(x) = \sum_{\substack{abc \\ ABC}} \overline{W}^{d;m}_{abc;ABC} \overline{\psi}^B_b(x) \overline{\psi}^A_a(x) \overline{\psi}^C_c(x) \quad (2)$$

$$W^{d;m}_{abc;ABC} = \epsilon^{ABC}[P^{ml}_{dc}\Gamma^l{}_{ab} + P^{ml}_{db}\Gamma^l{}_{ac} + P^{ml}_{da}\Gamma^l{}_{cb}] \quad (3)$$

$$\overline{W}^{d;m}_{abc;ABC} = \epsilon^{ABC}[P^{ml}_{cd}\Gamma^l{}_{ab} + P^{ml}_{bd}\Gamma^l{}_{ac} + P^{ml}_{ad}\Gamma^l{}_{cb}] \quad (4)$$

$$P^{ml} = [\delta^{ml} Id_{4\times4} - \frac{1}{3}\gamma^m\gamma^l] \qquad \Gamma^l = C\gamma^l \quad (5)$$

$A\,B\,C$ are colour indices, $a\,b\,c\,d$ are a Dirac indices and $m\,l$ are space indices

Baryon correlators: decuplet vertices

Two point correlation function:

$$C(x_0) = \sum_{\mathbf{x}} \sum_{\substack{dd' \\ m}} P^{+}_{dd'} \, v^{m;d}(x)\bar{v}^{m;d'}(0) \qquad (1)$$

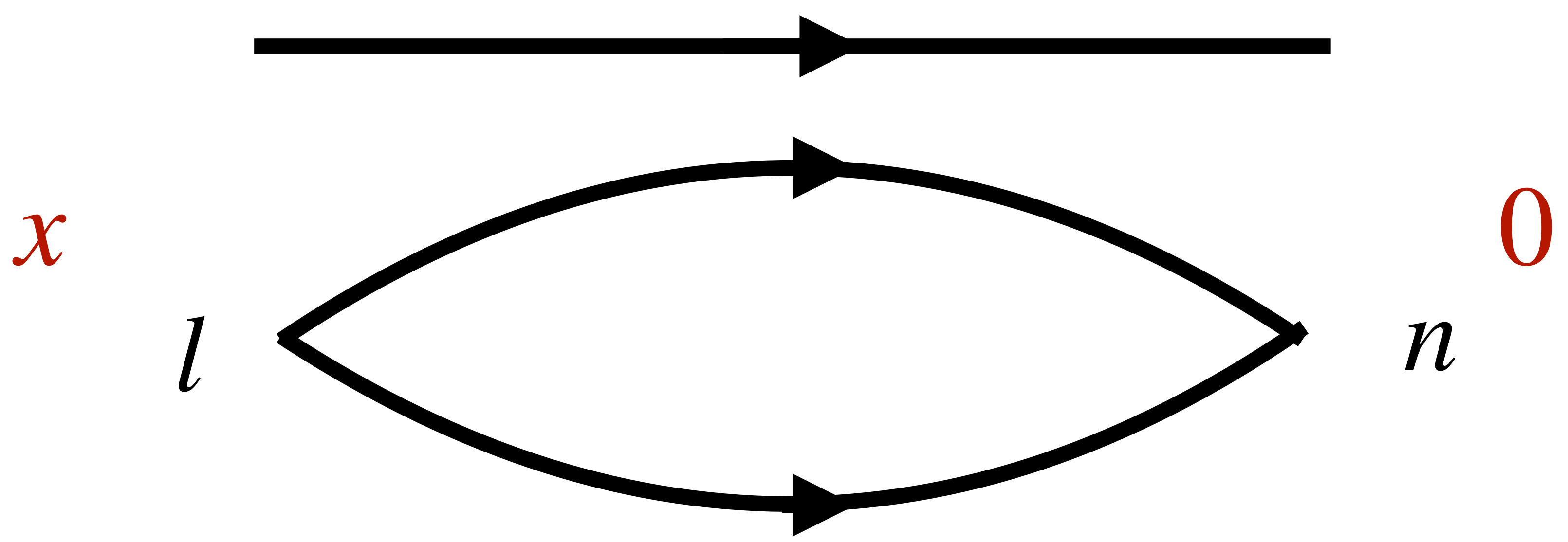
$$= \sum_{\mathbf{x}} \sum_{\substack{d'd \\ m}} \sum_{\substack{abc \\ a'b'c'}} \sum_{\substack{ABC \\ A'B'C'}} \overline{W}^{d';m}_{a'b'c';A'B'C'} P^{+}_{dd'} W^{d;m}_{abc;ABC} \psi^{C}_{c}(x)\,\psi^{A}_{a}(x)\,\psi^{B}_{b}(x)\,\overline{\psi}^{B'}_{b'}(0)\,\overline{\psi}^{A'}_{a'}(0)\,\overline{\psi}^{C'}_{c'}(0) \qquad (2)$$

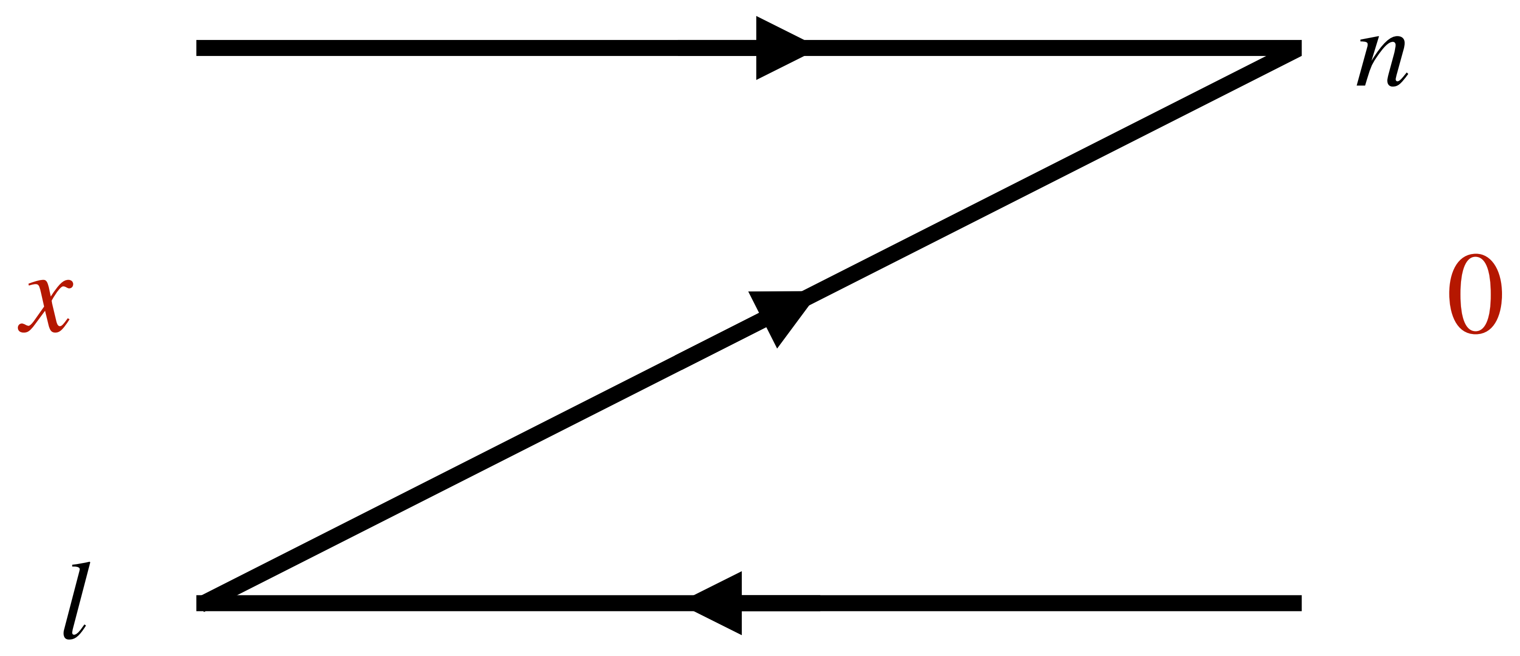
$A\,B\,C$ are colour indices, $a\,b\,c\,d$ are a Dirac indices and $m\,l$ are space indices

Baryon correlators: decuplet vertices

Three-quark connected

$$C(x_0) = \sum_{\mathbf{x}} \sum_{\substack{d'd \\ m}} \sum_{\substack{abc \\ a'b'c'}} \sum_{\substack{ABC \\ A'B'C'}} \overline{W}^{d';m}_{a'b'c';A'B'C'} P^{+}_{dd'} W^{d;m}_{abc;ABC} \overbrace{\psi^{A}_{a}(x)\overline{\psi}^{A'}_{a'}}(0)\, \overbrace{\psi^{B}_{b}(x)\overline{\psi}^{B'}_{b'}}(0)\, \overbrace{\psi^{C}_{c}(x)\overline{\psi}^{C'}_{c'}}(0) \qquad (1)$$





$A\ B\ C$ are colour indices, $a\ b\ c\ d$ are a Dirac indices and $m\ l$ are space indices

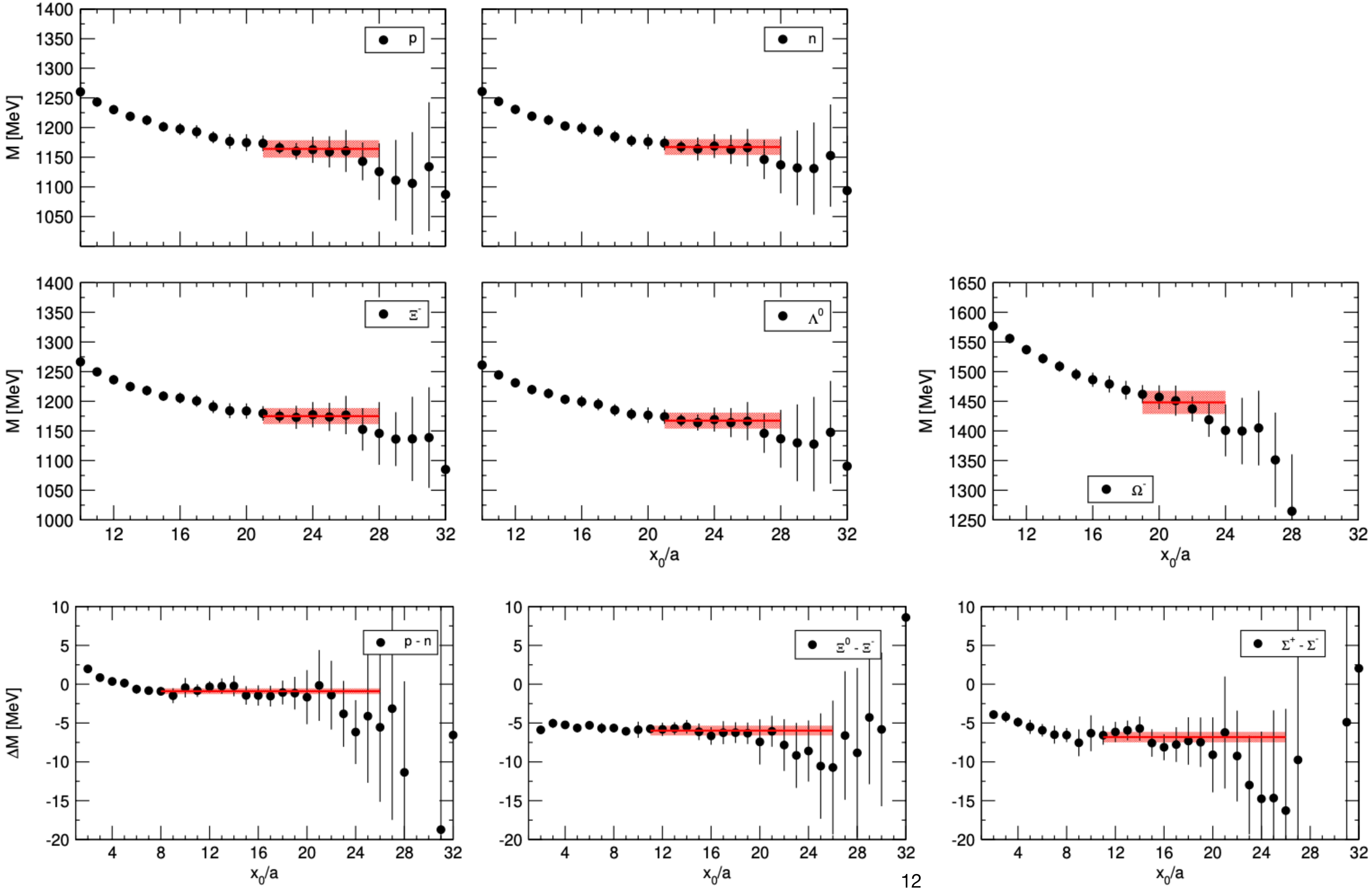
Baryon correlators: decuplet vertices

Three-quark connected

Ensemble	N. configurations	N. point sources/12
A500a50b324	1993	4
A360a50b324+RW2	2001	4
A380a07b324	2000	8
A380a07b324+RW1	2000	8
A450a07b324	2000	8

Baryon correlators: decuplet vertices

Three-quark connected

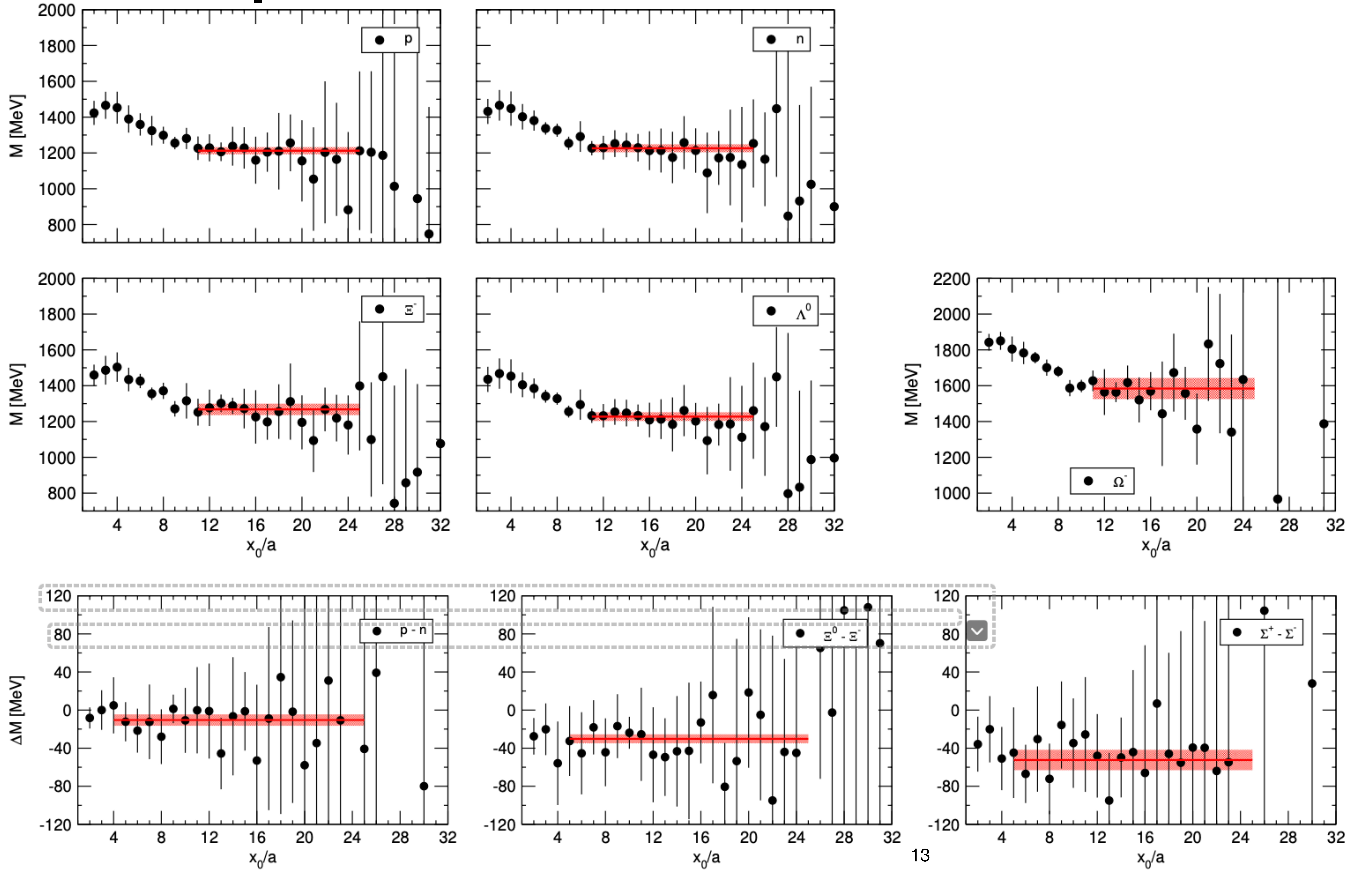


Baryon effective masses for the ensemble A380a07b324+RW1, with the selected plateaux and the fits to a constant.

[2]: L. Bushnaq et al. "First results on QCD+QED with C* boundary conditions". In Journal of high energy physics (Mar. 2023)

Baryon correlators: decuplet vertices

Three-quark connected



Baryon effective masses for the ensemble A360a50b324+RW2, with the selected plateaux and the fits to a constant.

[2]: L. Bushnaq et al. "First results on QCD+QED with C* boundary conditions". In Journal of high energy physics (Mar. 2023)

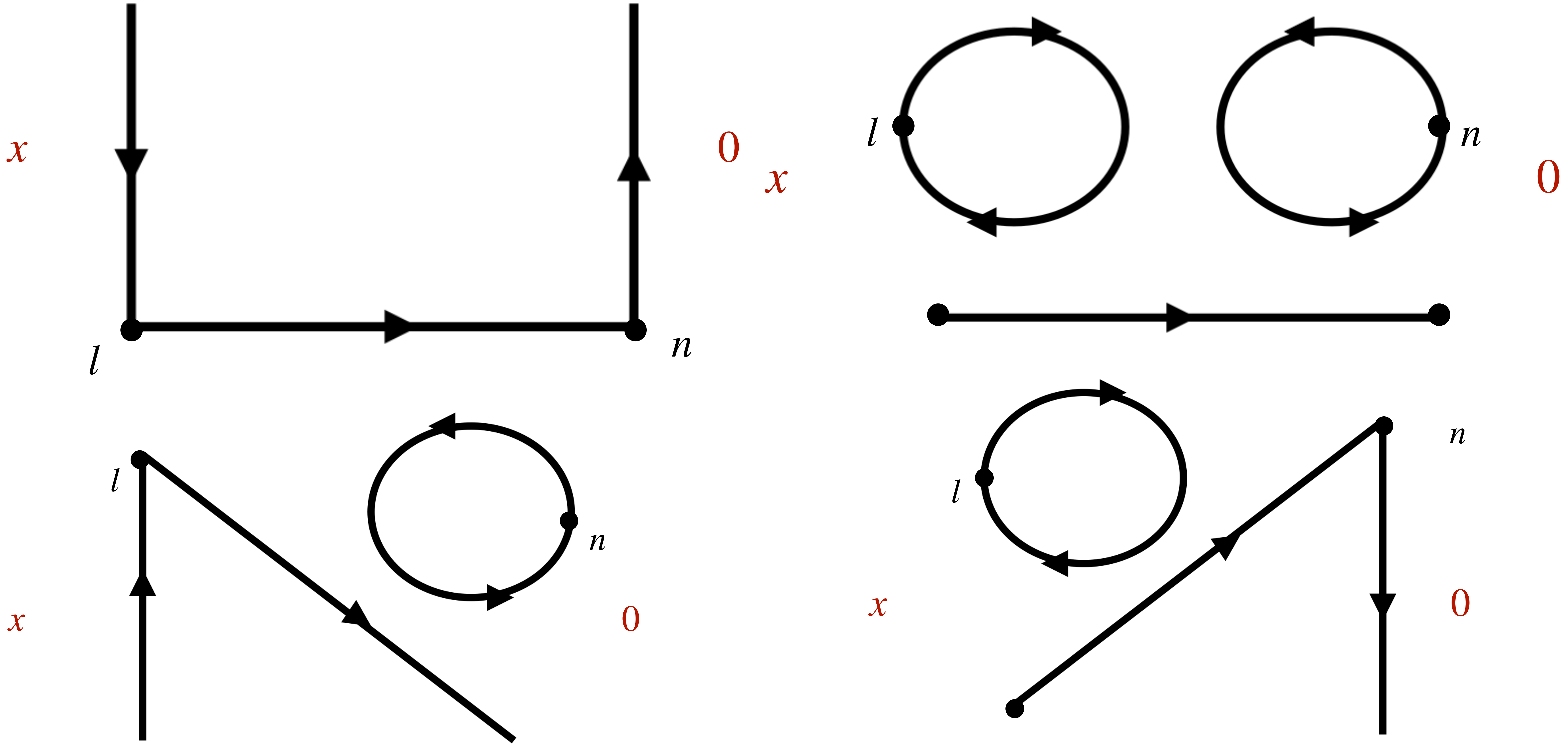
Baryon correlators: decuplet vertices

One-quark connected

$$C(x_0) = -\sum_{\mathsf{x}} \sum_{\substack{d'd \\ m}} \sum_{\substack{\alpha bc \\ \alpha' b' c'}} \sum_{\substack{ABC \\ A'B'C'}} \overline{W}^{d';m}_{a'b'c';A'B'C'} P^{+}_{dd'} W^{d;m}_{abc;ABC} \overline{\psi}^{A'}_{a'}(0) \overline{\psi}^{B'}_{b'}(0)\, \psi^{C}_{c}(x) \overline{\psi}^{C'}_{c'}(0)\, \psi^{B}_{b}(x) \psi^{A}_{a}(x) \qquad (1)$$

Baryon correlators: decuplet vertices

One-quark connected



Baryon correlators: decuplet vertices

One-quark connected

Defining the tensor: $$T^{A'B'C',ABC}_{\alpha'b'c',\alpha bc} = \sum_{\substack{d'd \\ m}} \overline{W}^{d';m}_{a'b'c';A'B'C'} P^{+}_{dd'} W^{d;m}_{abc;ABC} C_{a'\alpha'} C_{\alpha a} \qquad (1)$$

We can rewrite the two point correlation function as

$$C(x_0) = -\sum_{\mathbf{x}} \sum_{\substack{\alpha bc \\ \alpha'b'c'}} \sum_{\substack{ABC \\ A'B'C'}} T^{A'B'C',ABC}_{\alpha'b'c',\alpha bc} D^{-1}(L\hat{1},0)^{A'B'}_{\alpha'b'} D^{-1}(x,0)^{CC'}_{cc'} D^{-1}(x, x+L\hat{1})^{BA}_{b\alpha} \qquad (2)$$

L is the extension of the physical lattice in the first C* direction

One-quark connected contributions

Strategy of computation

We can rewrite $C(x_0)$ using one point source located at $(0,\bar{0})$ for each colour and Dirac index:

$$\eta(z)^{(A\alpha)}_{V\nu} = \delta_{VA}\delta_{\nu\alpha}\delta_{0,z} \qquad (1)$$

Instead of the two inverses of the Dirac operator with the second point in 0 we can then place the two spinors ψ resulting from the inversions

$$C(x_0) = -\sum_{\mathbf{x}} \sum_{\substack{\alpha bc \\ \alpha' b' c'}} \sum_{\substack{ABC \\ A'B'C'}} T^{A'B'C',ABC}_{\alpha' b' c',\alpha bc}\, \psi(L\hat{1})^{(B'b')}_{A'\alpha'}\, \psi(x)^{(C'c')}_{Cc}\, D^{-1}(x, x+L\hat{1})^{BA}_{b\alpha} \qquad (2)$$

One-quark connected contributions

Strategy of computation

For the last inversion we can use stochastic sources $\chi^{(n)}$ and in particular the relation:

$$\frac{1}{N_s}\sum_n \chi(x)_a^{(n)\dagger A}\chi(y)_b^{(n)B} = \delta_{AB}\delta_{ab}\delta_{xy} \qquad (1)$$

To get:

$$D^{-1}(x; x+L\hat{1})_{\alpha b}^{AB} = \frac{1}{N_s}\sum_n [D^{-1}\chi^{(n)}]_\alpha^A(x) \;\; \chi^{\dagger(n)}(x+L\hat{1})_b^B \qquad (2)$$

One-quark connected contributions

Strategy of computation

Finally we have:

$$C(x_0) = -\sum_{\mathrm{x}} \sum_{\substack{abc\\a'b'c'}} \sum_{\substack{ABC\\A'B'C'}} T^{A'B'C',ABC}_{a'b'c',abc}\, \psi(L\hat{1})^{(B'b')}_{A'a'}\, \psi(x)^{(C'c')}_{Cc}\, \frac{1}{N_s} \sum_n \left[D^{-1}\chi^{(n)}\right](x)^A_a \;\; \chi^{\dagger(n)}(x + L\hat{1})^B_b \tag{1}$$

One-quark connected contributions

Upper bound

Using the following definition of the norm of a matrix: $$||M|| = \sqrt{Tr[M^\dagger M]} \quad (1)$$

That for the Dirac operator translates to the Π two-point correlation function

$$||D^{-1}(x,y)||^2 = Tr\left[D^{-1}(x,y)^\dagger D^{-1}(x,y)\right] = Tr\left[\gamma^5 D^{-1}(y,x)\gamma^5 D^{-1}(x,y)\right] \quad (2)$$

It is observed that not only on average but also separately on each configuration it is translationally invariant and has the behaviour:

$$C(x,y) \propto e^{-M_\pi |y-x|} \quad (3)$$

One-quark connected contributions

Upper bound

We can use translational invariance:

$$||D^{-1}(0,L\hat{1})|| = ||D^{-1}(x, x+L\hat{1})|| \qquad (1)$$

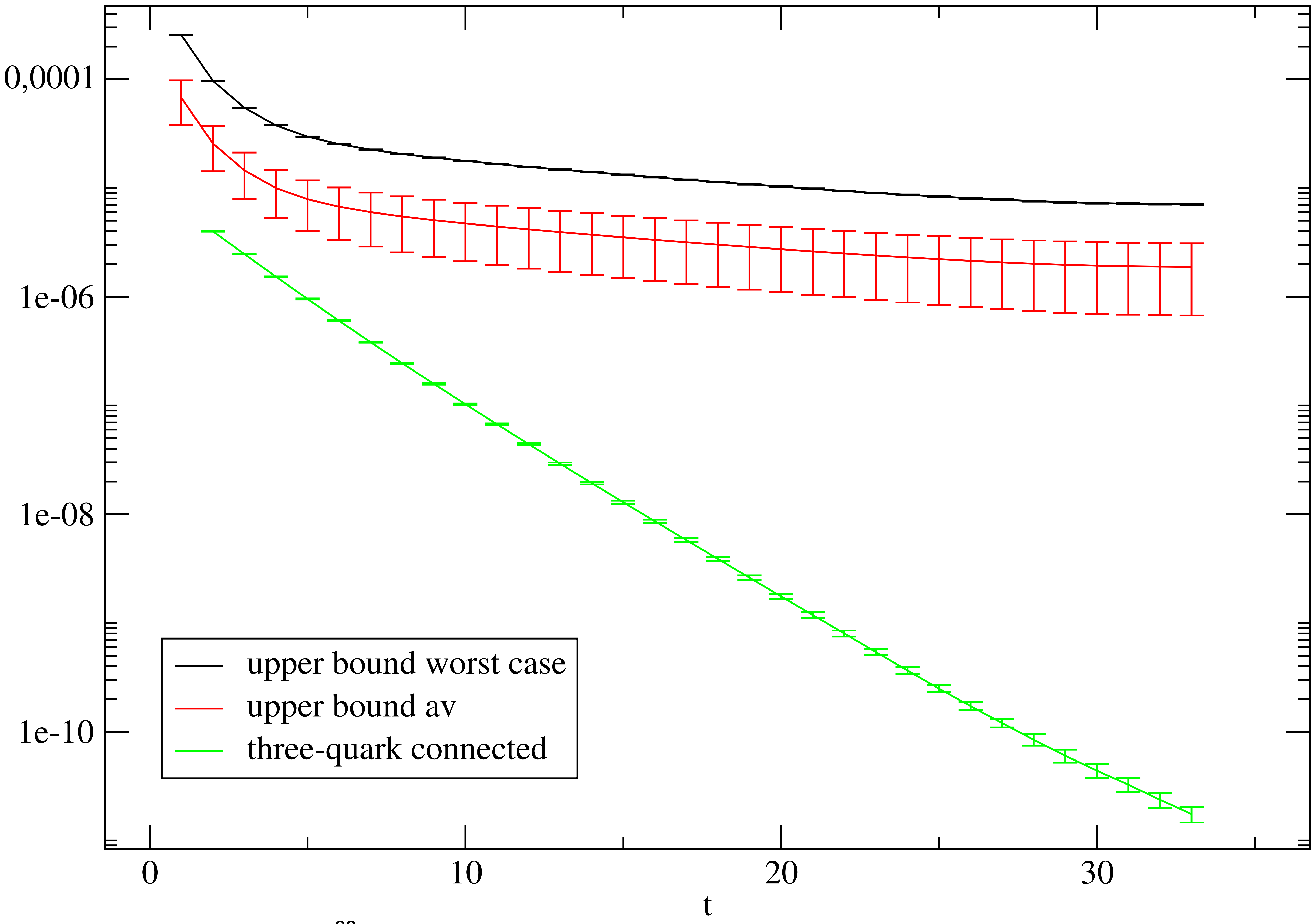
Then factorize the suppressing factors and obtain the upper bound.

$$|C(x_0)| \leq ||D^{-1}(0,L\hat{1})||^2 \sum_{\mathrm{x}} ||D^{-1}(x,0)|| \sqrt{\sum_{AB;\alpha b} \sum_{B'A';\alpha' b'} \sum_{CC';cc'} |T^{A'B'C',ABC}_{\alpha' b' c',\alpha bc}|^2} \qquad (2)$$

We can compare it with the measurements of the three-quark connected contributions

One-quark connected contributions

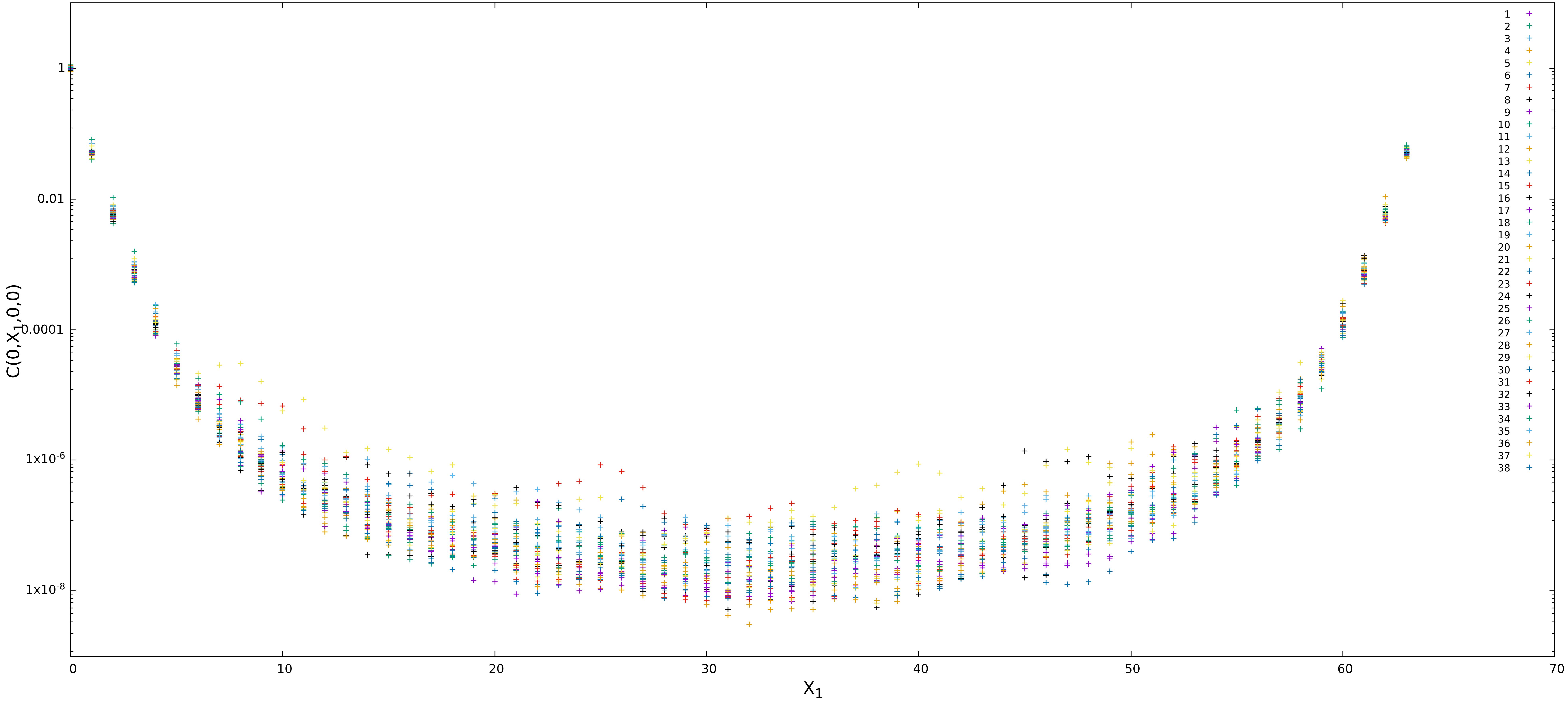
Upper bound of the one-quark connected contribution to the Ω^- correlator in comparison with the result for the three-quark connected piece for the ensemble A380a07b324



Outlook

- Other ways to compute upper bound
- Computation of the one-quark connected contributions
- Increase the statistics on three-quark connected contributions.

Pion correlator with C* boundary conditions



Two point correlation function of the down/strange quark (degenerate) on the ensemble A380a07b324 for 38 configurations (taken every 50 updates) as a function of the first spatial coordinate

One-quark connected contributions

Upper bound

Upper bound of the one-quark connected contribution to the Ω^- correlator for the coefficient equal to 1 in comparison with the result for the three-quark connected piece for the ensemble A380a07b324