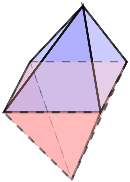
Combinatorics in the Sky (with Diamonds)



Paolo Benincasa

Max-Planck-Institut für Physik &

Instituto Galego de Física de Altas Enerxías

23 February 2024 – Galaxies meet QCD Workshop

Based on works in collaboration with:

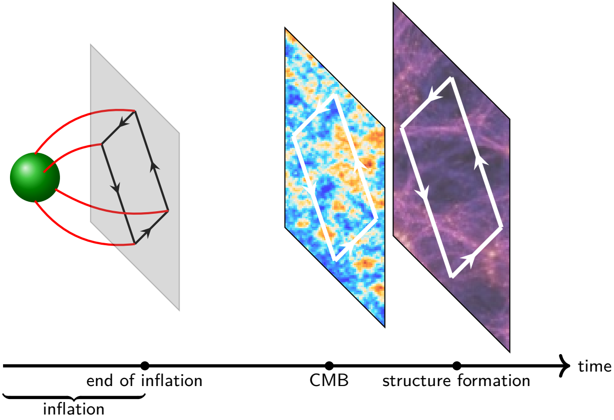
S. Albayrak, N. Arkani-Hamed, G. Dian,

C. Duaso Pueyo, A. McLeod, A. Pari, M. Parisi,

A. Postnikov, W. Torres Bobadilla, F. Vazão, C. Vergu

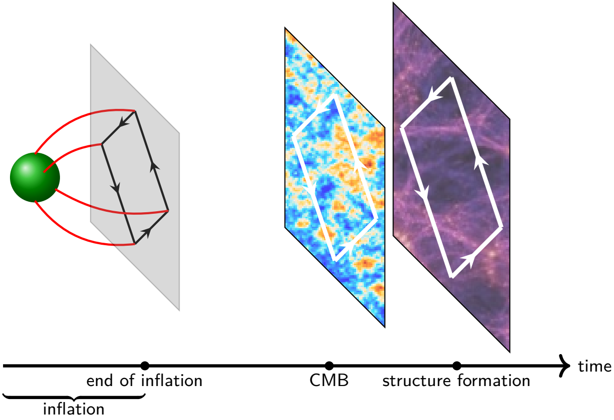
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Observables & Fundamental Principles



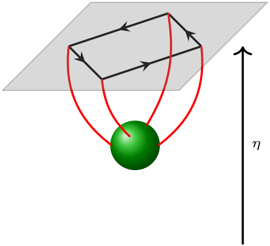
$\langle \Phi\Phi \cdots \rangle$ $\quad$ $\langle \delta T \delta T \cdots \rangle$ $\quad$ $\langle \delta\rho_g \delta\rho_g \cdots \rangle$

Observables & Fundamental Principles



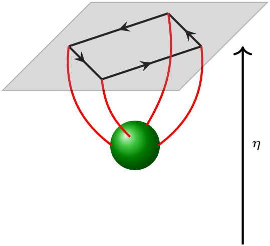
$\langle \Phi\Phi\cdots\rangle$ $\langle \delta T \delta T \cdots\rangle$ $\langle \delta\rho_g \delta\rho_g \cdots\rangle$

Observables & Fundamental Principles



$$\langle \Phi(\vec{p}_1) \cdots \Phi(\vec{p}_n) \rangle = \int \mathcal{D}\Phi \, \mathfrak{P}[\Phi] \Phi(\vec{p}_1) \cdots \Phi(\vec{p}_n)$$

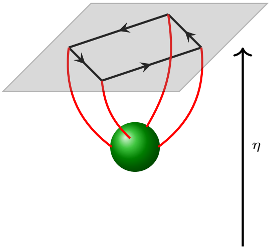
Observables & Fundamental Principles



$$\langle \Phi(\vec{p}_1)\cdots\Phi(\vec{p}_n)\rangle = \int \mathcal{D}\Phi\, \mathfrak{P}[\Phi]\, \Phi(\vec{p}_1)\cdots\Phi(\vec{p}_n)$$

Probability distribution

Observables & Fundamental Principles

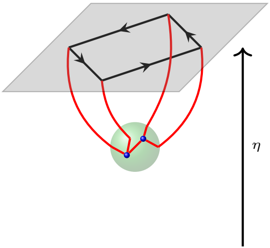


$$\langle \Phi(\vec{p}_1) \cdots \Phi(\vec{p}_n) \rangle = \frac{\int \mathcal{D}\Phi \, \Psi^\dagger[\Phi] \Phi(\vec{p}_1) \cdots \Phi(\vec{p}_n) \Psi[\Phi]}{\int \mathcal{D}\Phi \, |\Psi[\Phi]|^2}$$

$$\Psi[\Phi] := \langle \Phi | \hat{\mathcal{T}} \exp \left\{ -i \int_{-\infty}^{0} d\eta \, H(\eta) \right\} |0\rangle$$

Wavefunction of the universe
(transition amplitude from $|0\rangle$ to $\langle \Phi|$)

Observables & Fundamental Principles



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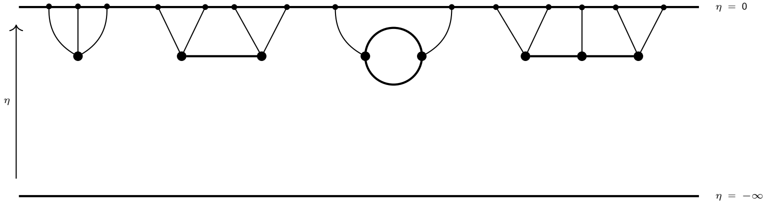
Wavefunction of the universe
(transition amplitude from $|0\rangle$ to $\langle\Phi|$)

Perturbation theory



Observables & Fundamental Principles

1. Can we put constraints on which state can propagate during inflation in a completely model independent way?

2. What is the imprint of the inflationary physics in the analytic structure of the relevant observables?

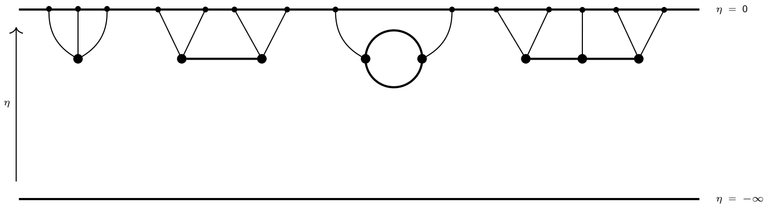
3. What are the rules governing physical processes at energies as large as $H|_{\text{infl}} \sim 10^{14}\ GeV$?

Why Combinatorics?

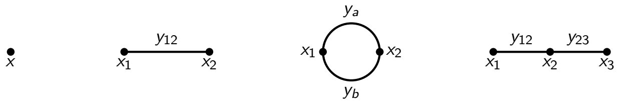
Deeper understanding of the physics encoded
into cosmological observables

Novel rules which can allow to go beyond the regime
in which the combinatorial description has been formulated

Cosmological Integrals



Cosmological Integrals



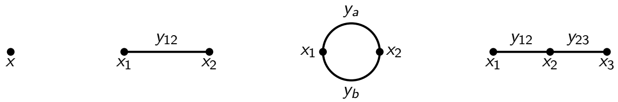
$$\mathcal{I}_{\mathcal{G}} = \prod_{s\in\mathcal{V}} \left[\int_{X_s}^{+\infty} dx_s\, \tilde{\lambda}(x_s - X_s)\right] \int_{\Gamma} \prod_{e\in\mathcal{E}^{(L)}} \left[\frac{dy_e}{y_e}\, y_e^{\beta_e}\right] \mu_d(y)\, \frac{\mathfrak{n}_{\delta}(x,y)}{\prod_{\mathfrak{g}\subseteq\mathcal{G}} q_{\mathfrak{g}}(x,y)}$$

Cosmology

Loop integration

Universal integrand

Cosmological Integrals



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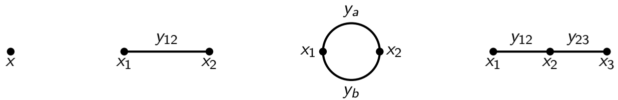
Cosmology → Power-law FRW $\tilde{\lambda}(x_s - X_s) \sim (x_s - X_s)^{\alpha-1}$

Loop integration

Universal integrand

Cosmological Integrals



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Cosmology

↓

Power-law FRW

$\tilde{\lambda}(x_s - X_s) \sim (x_s - X_s)^{\alpha-1}$

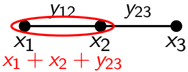
Loop integration

Universal integrand

External kinematics: $X_s := \sum_{j\in s} |\vec{p}^{(j)}|$, $y_e := \left|\sum_{j\in s_e} \vec{p}^{(j)}\right|$ $(e \in \mathcal{E}\setminus\{\mathcal{E}^{(L)}\})$

Loop momenta: $y_{e_1} := |\vec{l}|$, $y_{e_2} := |\vec{l} + \vec{p}^{(2)}|$, $\ldots$ $(e \in \mathcal{E}^{(L)})$

$$q_{\mathfrak{g}}(x,y) := \sum_{s\in\mathcal{V}_{\mathfrak{g}}} x_s + \sum_{e\in\mathcal{E}_{\mathfrak{g}}^{\text{ext}}} y_e$$



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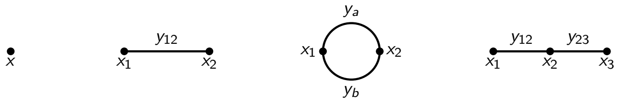
Cosmological Integrals



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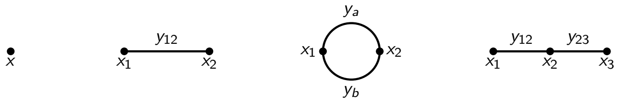
Cosmology

Loop integration

Universal integrand

(weighted) cosmological polytope

Cosmological Integrals



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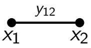
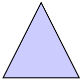
Cosmology

Loop integration

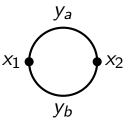
Universal integrand



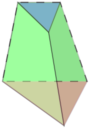
$$\omega\left(\mathcal{Y}, \mathcal{P}_{\mathcal{G}}\right) = \frac{\mathfrak{n}_\delta(x,y)}{\prod_{\mathfrak{g}\subseteq\mathcal{G}} q_{\mathfrak{g}}(x,y)} \, \frac{\prod_{s\in\mathcal{V}} dx_s \prod_{e\in\mathcal{E}} dy_e}{\text{Vol}\{GL(1)\}}$$



(weighted) cosmological polytope

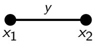


(Weighted) cosmological polytopes capture the singularity structure of $\mathcal{I}_{\mathcal{G}}$



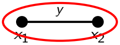
From Graphs to Polytopes

A flavour of cosmological polytopes



From Graphs to Polytopes

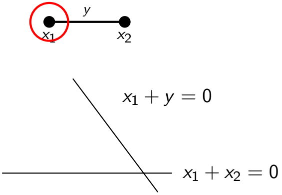
A flavour of cosmological polytopes



$$x_1 + x_2 = 0$$

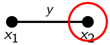
From Graphs to Polytopes

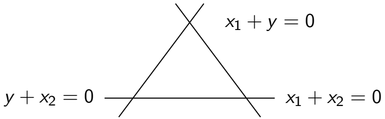
A flavour of cosmological polytopes



From Graphs to Polytopes

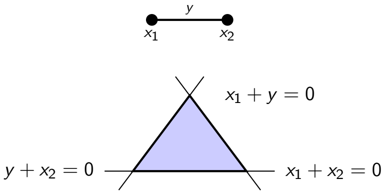
A flavour of cosmological polytopes





From Graphs to Polytopes

A flavour of cosmological polytopes

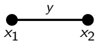


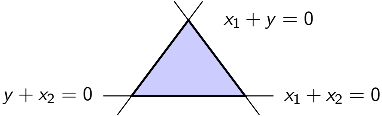
The singularities form a bounded region to which a function Ω is naturally associated

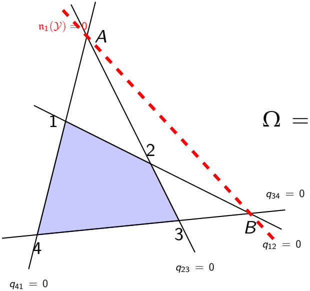
$$\Omega = \frac{1}{(x_1 + x_2)(x_1 + y)(y + x_2)} \equiv \frac{\mathfrak{n}_\delta}{q_{\mathcal{G}}\, q_{\mathfrak{g}_1}\, q_{\mathfrak{g}_2}}$$

From Graphs to Polytopes

A flavour of cosmological polytopes







$$\Omega = \frac{\mathfrak{n}_1}{q_{12}q_{23}q_{34}q_{41}} = \frac{1}{q_{12}q_{34}}\left[\frac{1}{q_{23}} + \frac{1}{q_{41}}\right]$$

Triangulation of the polytope

≡

Representation for the integrand

(Weighted) Cosmological Polytopes & $\mathcal{I}_{\mathcal{G}}$: A dictionary

Cosmological Polytope $\mathcal{P}_{\mathcal{G}}$	Cosmological Integral $\mathcal{I}_{\mathcal{G}}$
Canonical form ω	Integrand of $\mathcal{I}_{\mathcal{G}}$
Triangulations	Representations for the intgrand
Boundaries (Faces)	Residues of the integrands
Canonical form preserving transformations	Symmetries of the integrand
Paths along contiguous vertices	Symbols for $\mathcal{I}_{\mathcal{G}}$

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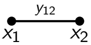
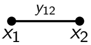
Towards a combinatorial RG: The IR/UV structure of $\mathcal{I}_{\mathcal{G}}$

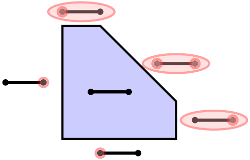


$$= \int_{\mathbb{R}_+} \prod_{j=1}^{2} \left[\frac{dx_j}{x_j} \, x_j^{\alpha} \right] \frac{1}{(x_1 + x_2 + \mathcal{X}_{\mathcal{G}})(x_1 + \mathcal{X}_{\mathfrak{g}_1})(x_2 + \mathcal{X}_{2_2})}$$

Towards a combinatorial RG: The IR/UV structure of $\mathcal{I}_{\mathcal{G}}$

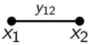


$$= \int_{\mathbb{R}_+} \prod_{j=1}^{2} \left[\frac{dx_j}{x_j} \, x_j^{\alpha} \right] \frac{1}{(x_1^1 x_2^0 + x_1^0 x_2^1 + \mathcal{X}_{\mathcal{G}} x_1^0 x_2^0)(x_1^1 x_2^0 + \mathcal{X}_{\mathfrak{g}_1} x_1^0 x_2^0)(x_1^0 x_2^1 + \mathcal{X}_{\mathfrak{g}_2} x_1^0 x_2^0)}$$



The integral converges for values of α that identifies points inside the Newton polytope

Towards a combinatorial RG: The IR/UV structure of $\mathcal{I}_{\mathcal{G}}$



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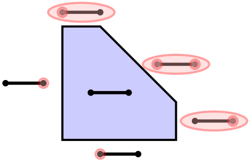

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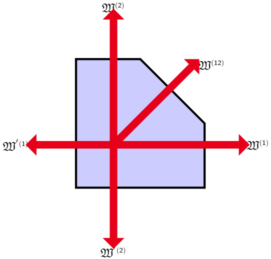

$$\mathfrak{W}^{(j_1 \ldots j_{n_s^{(\mathfrak{g})}})} = \begin{pmatrix} \lambda^{(j_1 \ldots j_{n_s^{(\mathfrak{g})}})} \\ \mathfrak{e}_{(j_1 \ldots j_{n_s^{(\mathfrak{g})}})} \end{pmatrix}$$

The integral diverges in the direction $\mathfrak{e}$ if the related λ is ≥ 0

$$\lambda^{(j_1 \ldots j_{n_s^{(\mathfrak{g})}})} = n_s^{(\mathfrak{g})} \alpha - (\text{tubings})$$

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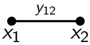
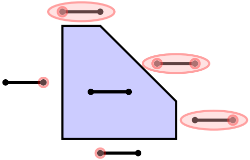
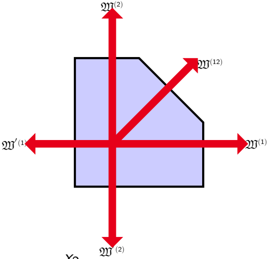
Towards a combinatorial RG: The IR/UV structure of $\mathcal{I}_\mathcal{G}$



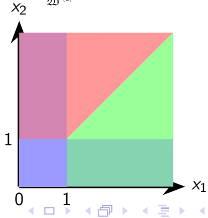
$$= \int_{\mathbb{R}_+} \prod_{j=1}^{2} \left[\frac{dx_j}{x_j} x_j^{\alpha} \right] \frac{1}{(x_1^1 x_2^0 + x_1^0 x_2^1 + \mathcal{X}_\mathcal{G} x_1^0 x_2^0)(x_1^1 x_2^0 + \mathcal{X}_{\mathfrak{g}_1} x_1^0 x_2^0)(x_1^0 x_2^1 + \mathcal{X}_{\mathfrak{g}_2} x_1^0 x_2^0)}$$





E.g.: if $\lambda^{(12)} \longrightarrow 0$: sector decomposition

$$\mathcal{I}^{\text{div}}_{\Delta_{j,12}} = \int_0^1 \frac{d\zeta_j}{\zeta_j} \frac{(\zeta_j)^{-\lambda^{(j)}}}{1+\zeta_j} \times \int_0^1 \frac{d\zeta_{12}}{\zeta_{12}} (\zeta_{12})^{-\lambda^{(12)}}$$



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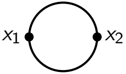
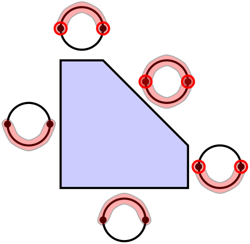
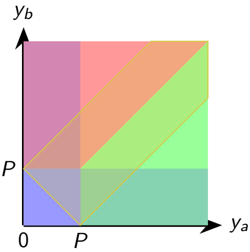
Towards a combinatorial RG: The IR/UV structure of $\mathcal{I}_\mathcal{G}$



$$= \int_{\mathbb{R}_+} \prod_{j=1}^{2} \left[\frac{dx_j}{x_j}\, x_j^{\alpha}\right] \int_{\Gamma} \prod_{e\in\mathcal{E}} \left[\frac{dy_e}{y_e}\, y_e^{\beta}\right] \mu(y) \times$$

$$\times \frac{2(x_1+x_2+y_a+y_b+\mathcal{X}_{\mathcal{G}})}{(x_1+x_2+\mathcal{X}_{\mathcal{G}})(x_1+x_2+y_a+\mathcal{X}_{\mathcal{G}})(x_1+x_2+y_b+\mathcal{X}_{\mathcal{G}})(x_1+y_a+y_b+\mathcal{X}_1)(x_2+y_a+y_b+\mathcal{X}_2)}$$





Conclusion

First clues on constraints on cosmological processes:
perturbative unitarity, flat-space limit,
factorisations, higher-codimensions singularities

General framework to have a direct formulation
with IR safe observables

Combinatorics allows to recast physical questions into
easier mathematical questions
Something to say for LSS?