Wave propagation in unbounded quasiperiodic media

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Abstract

This is a joint work with Pierre Amenoagbadji and Patrick Joly, two members of my group POEMS. It is devoted to the numerical solution of the Helmholtz equation in 1D unbounded quasiperiodic media. By this we mean that the coefficients appearing in the model are quasiperiodic functions of the 1D space variable, namely the trace along a line of a periodic function of n variables. When the coefficients are periodic (which is a special case), several methods have been proposed to characterize and compute the solution. However, when the coefficients are only quasi-periodic and not periodic, the above methods cannot be applied directly.

We use an original method, which we call the lifting method and which has been presented in several papers on homogenization theory. The original problem can thus be lifted to an nD "augmented" problem with periodic coefficients, and the 1D solution is the trace along this line of the nD solution. The advantage is that the periodicity of the augmented problem allows one to use the ideas that have been proposed for solving periodic Helmholtz equations in periodic waveguides. However, since the augmented equation is a degenerate elliptic equation (more precisely the principal part), the corresponding tools have to be adapted and new difficulties arise both in the analysis and in the design of the resulting numerical method.

I will present our results for the Helmholtz equation with dissipation (where the solution decays at infinity) and then for the equation without dissipation (where the solution can propagate to infinity), analyzing the latter case using a limiting absorption principle.