Universität Zürich[UZH]

Topologically localized phases

Bastien Lapierre

In collaboration with
Titus Neupert, Piet Brouwer, and Luka Trifunovic

Phys. Rev. Lett. ***129*** *(25), 256401 (2022), and work in progress*

This talk...

This talk...

1) Anderson localization 

This talk...

1) Anderson localization

2) Topology

This talk...

1) Anderson localization

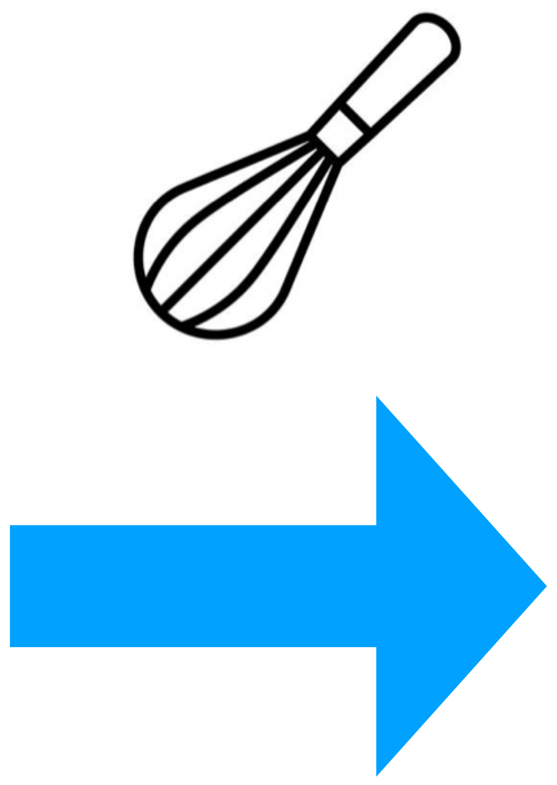
2) Topology

Topological insulator

This talk...

1) Anderson localization

2) Topology



Topologically Localized Insulator (TLI)

B.L, T. Neupert, L. Trifunovic,
Phys. Rev. Lett. ***129*** *(25), 256401 (2022)*,
"Topologically localized insulators"

Tenfold way topology

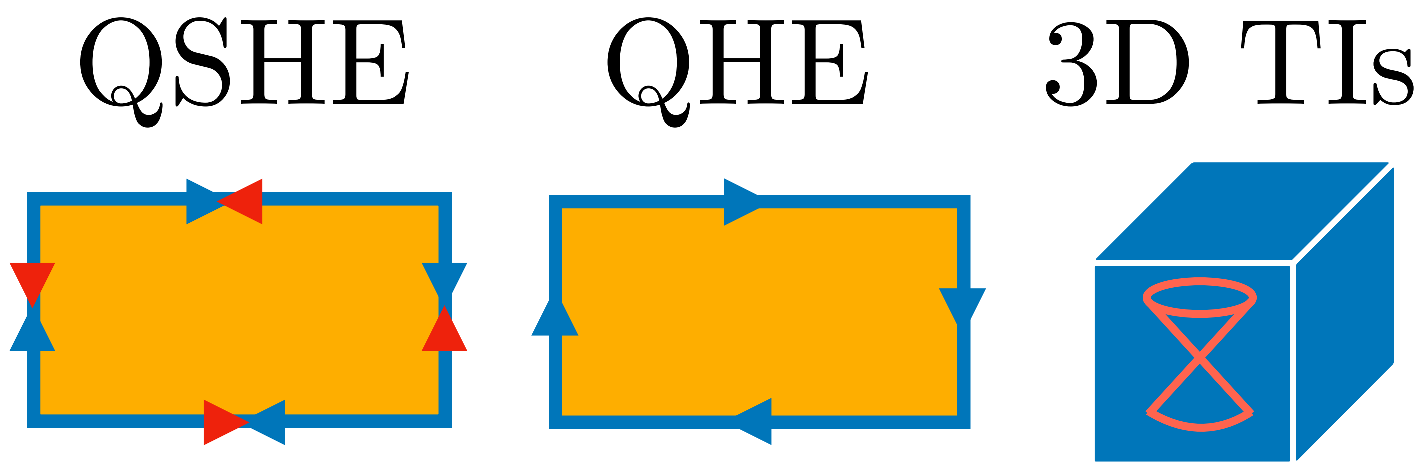
AZ	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

[A. P. Schnyder et al., Phys. Rev. B **78**, 195125]

Tenfold way topology

AZ	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

[A. P. Schnyder et al., Phys. Rev. B **78**, 195125]



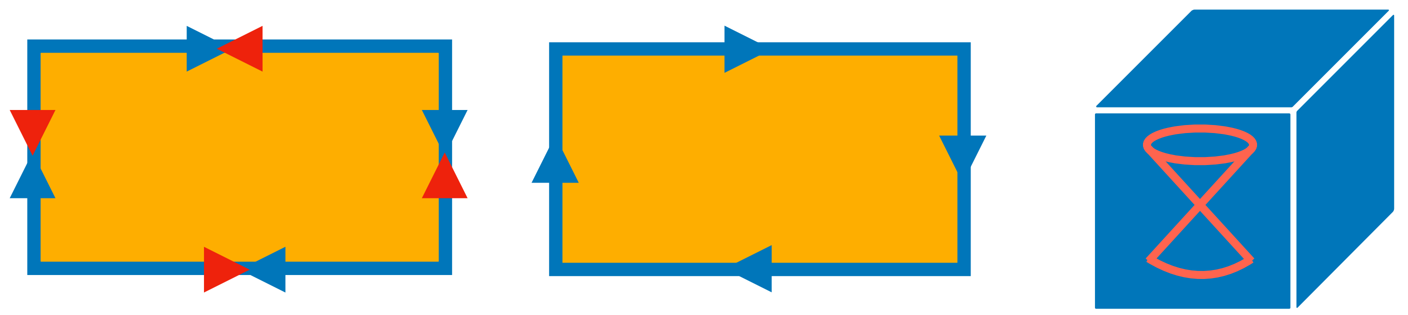
Immune to full bulk localization

Tenfold way topology

AZ	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

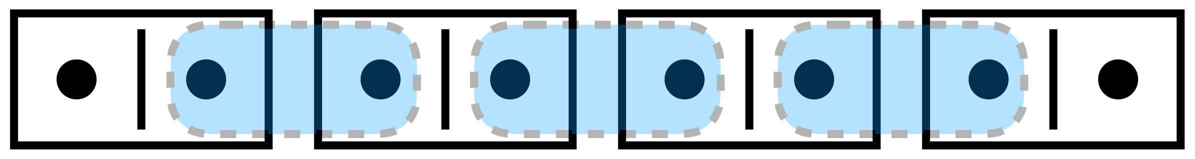
[A. P. Schnyder et al., Phys. Rev. B **78**, 195125]

QSHE QHE 3D TIs



Immune to full bulk localization

1d Kitaev chain



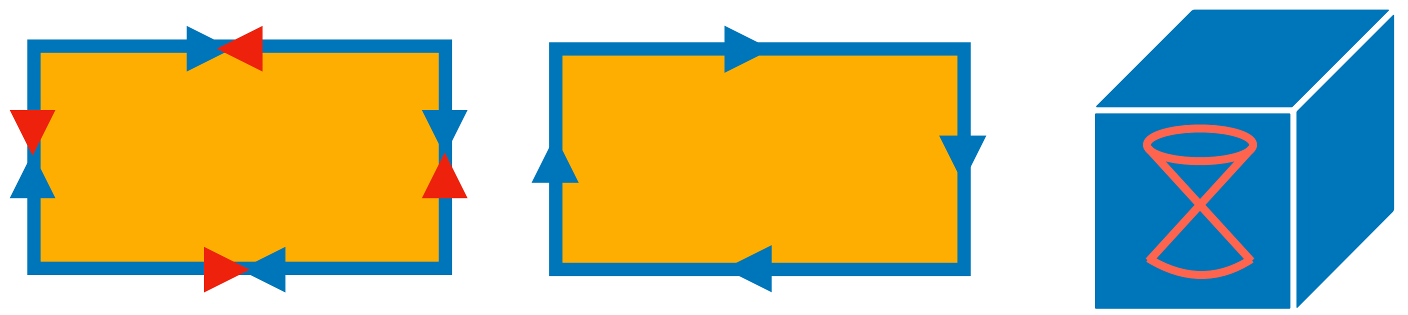
Fully localizable bulk

Tenfold way topology

AZ	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

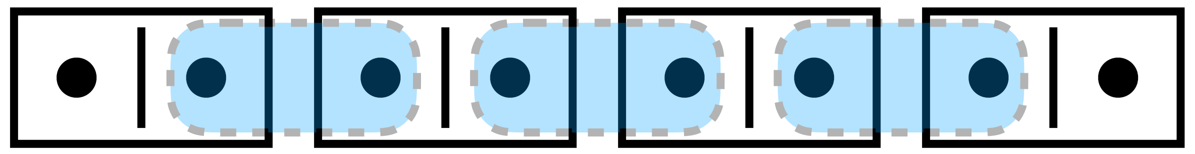
[A. P. Schnyder et al., Phys. Rev. B **78**, 195125]

QSHE QHE 3D TIs



Immune to full bulk localization

1d Kitaev chain



Fully localizable bulk

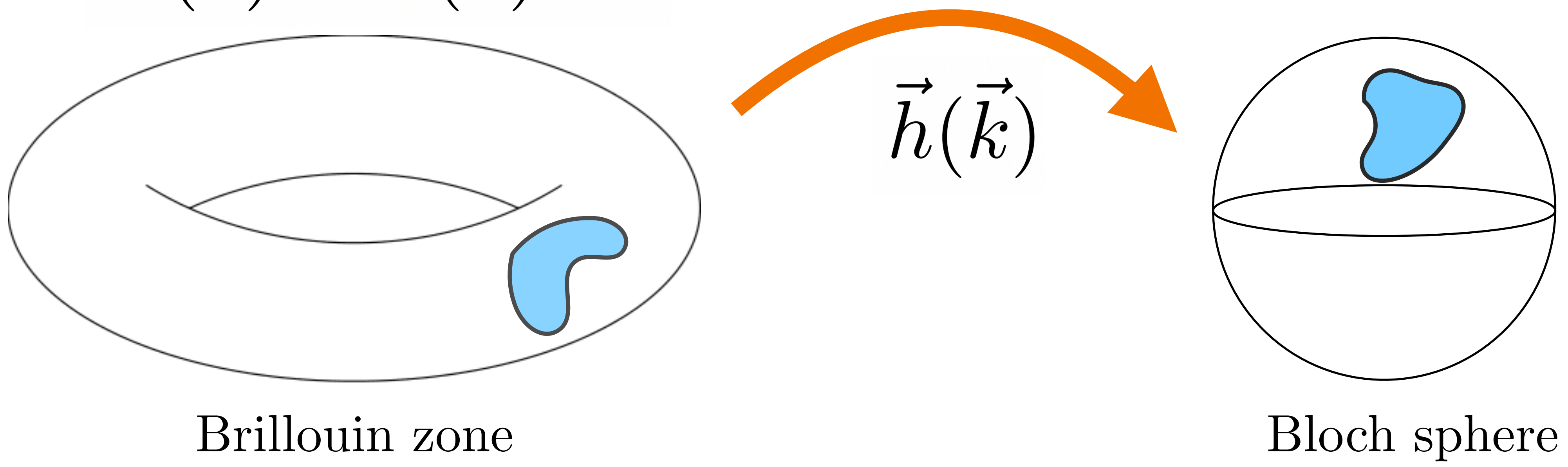
Can we classify topological phases with fully localized bulk?

Can we find new fully localized phases?

Chern insulators

2D Topological insulators in class A

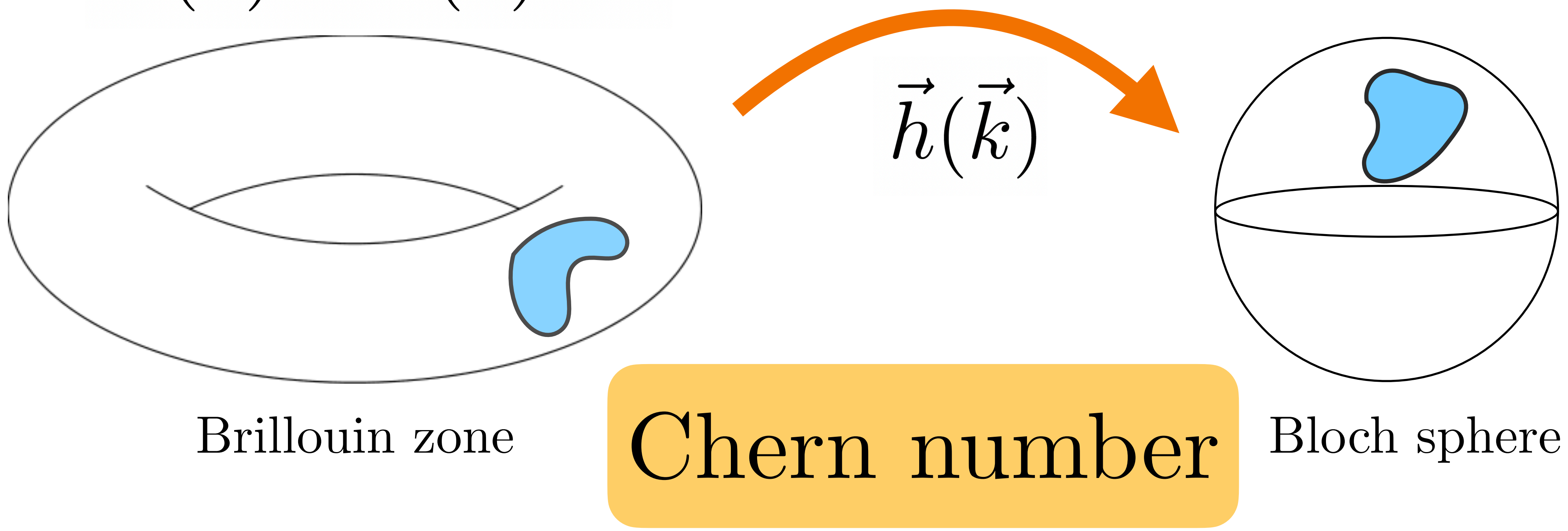
$$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$$



Chern insulators

2D Topological insulators in class A

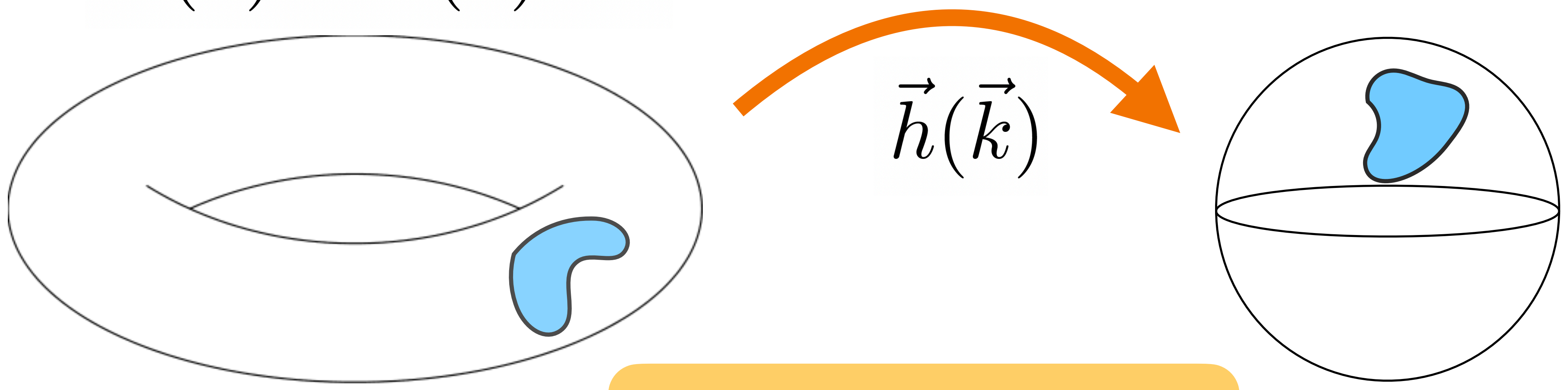
$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$



Chern insulators

2D Topological insulators in class A

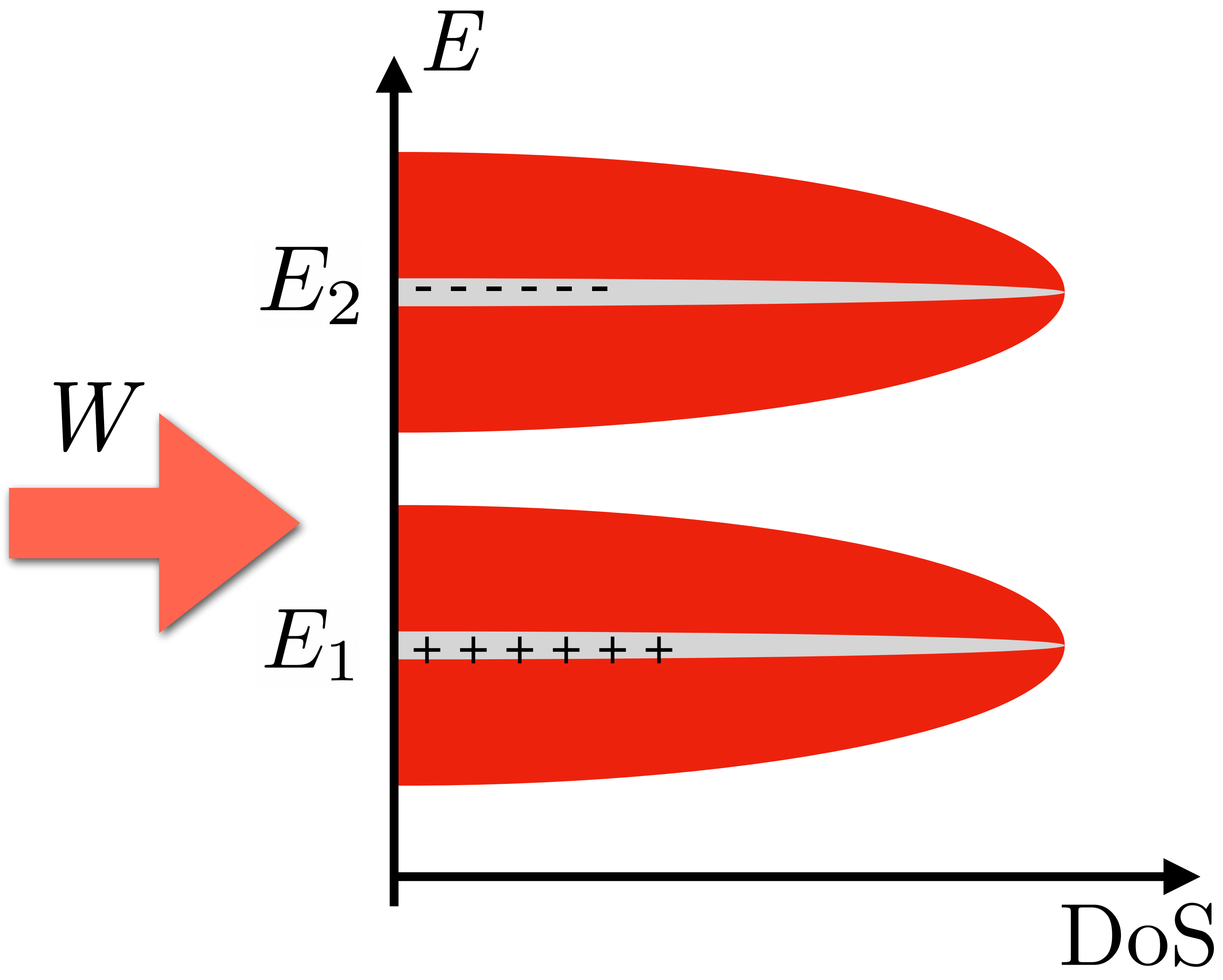
$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$



Brillouin zone

Chern number

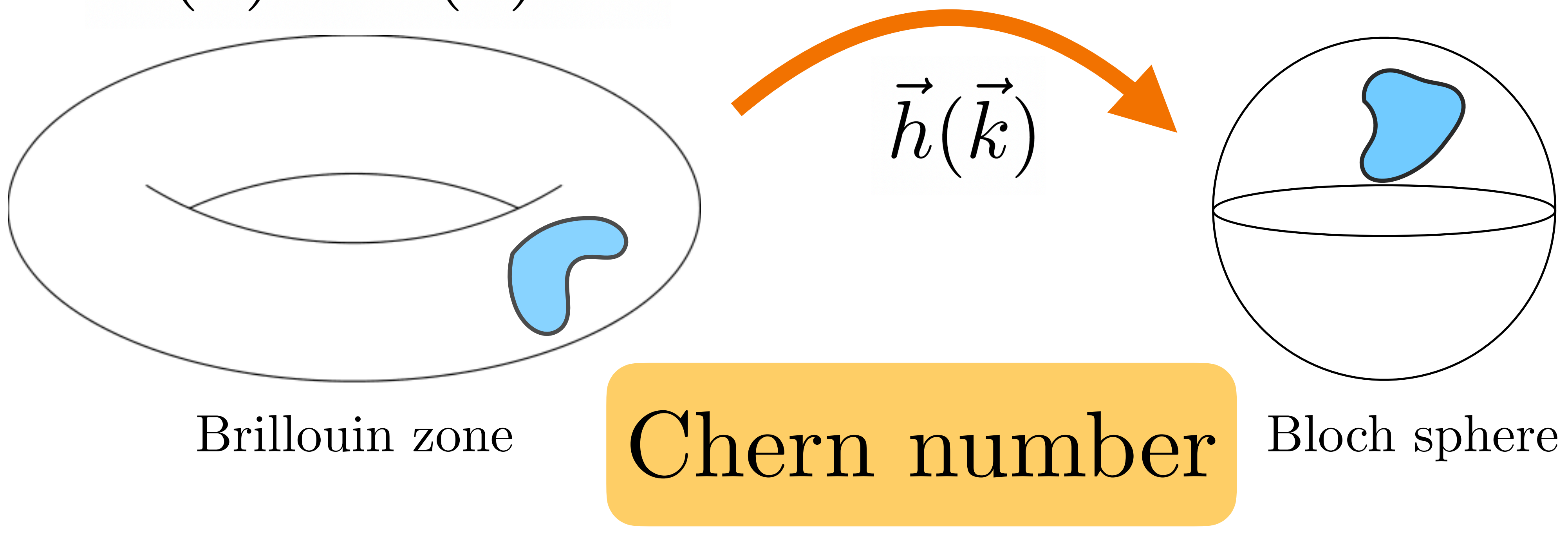
Bloch sphere

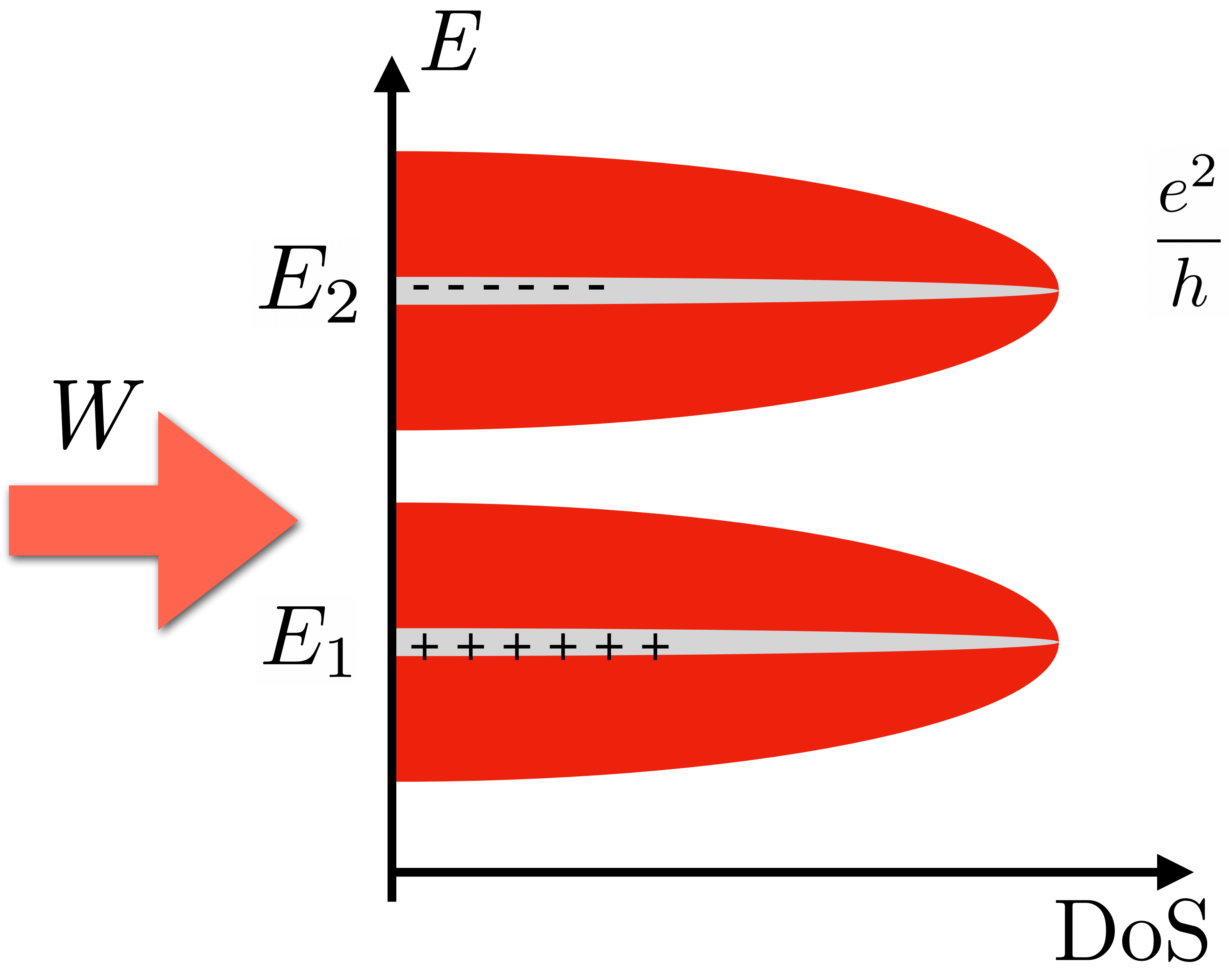


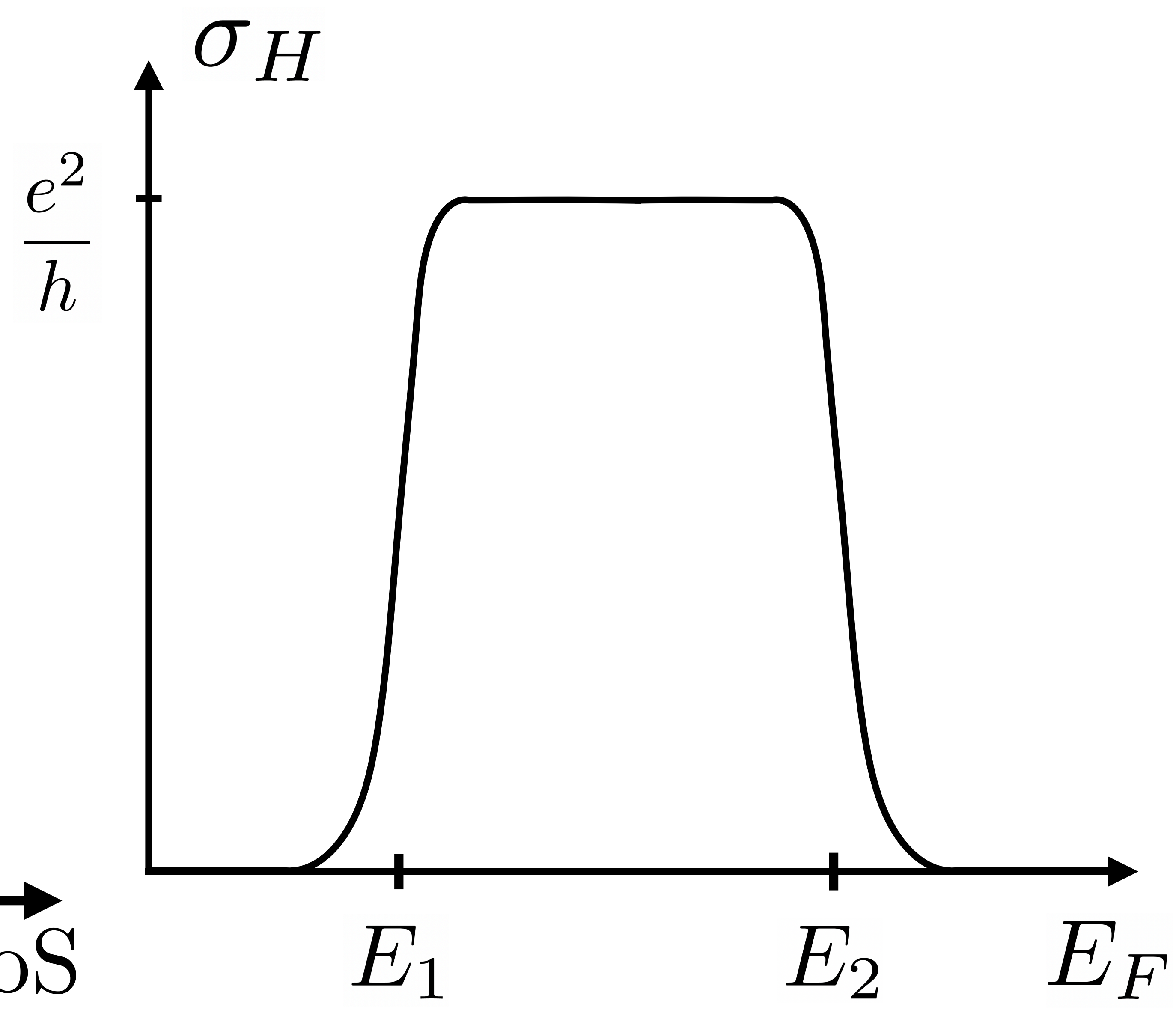
Chern insulators

2D Topological insulators in class A

$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$



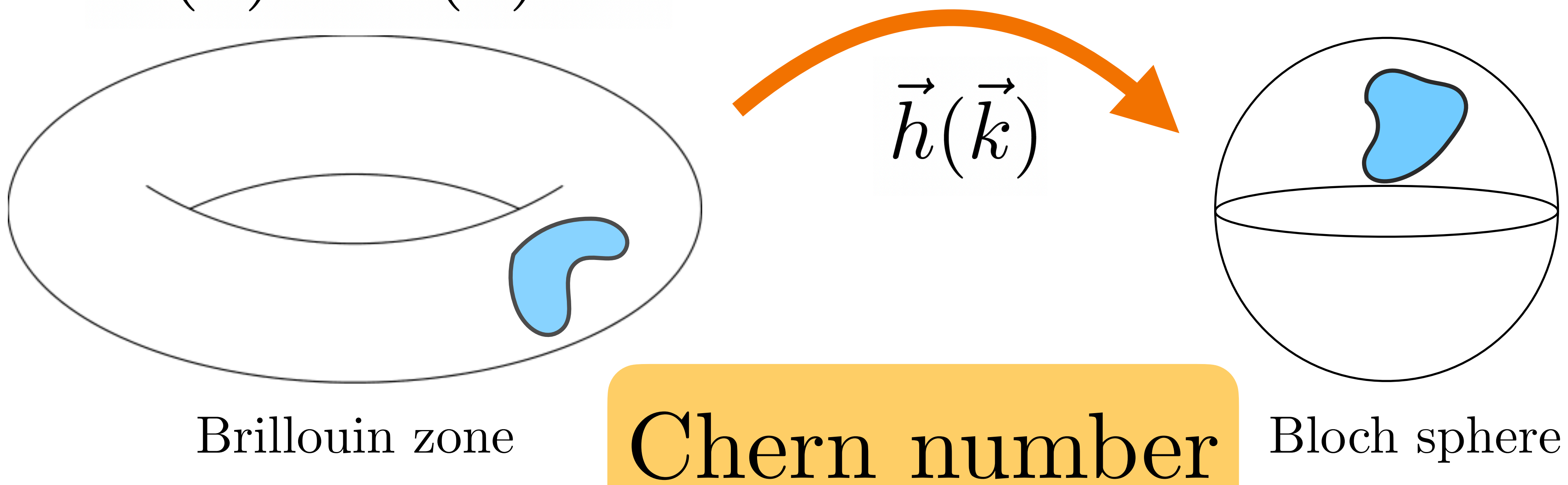




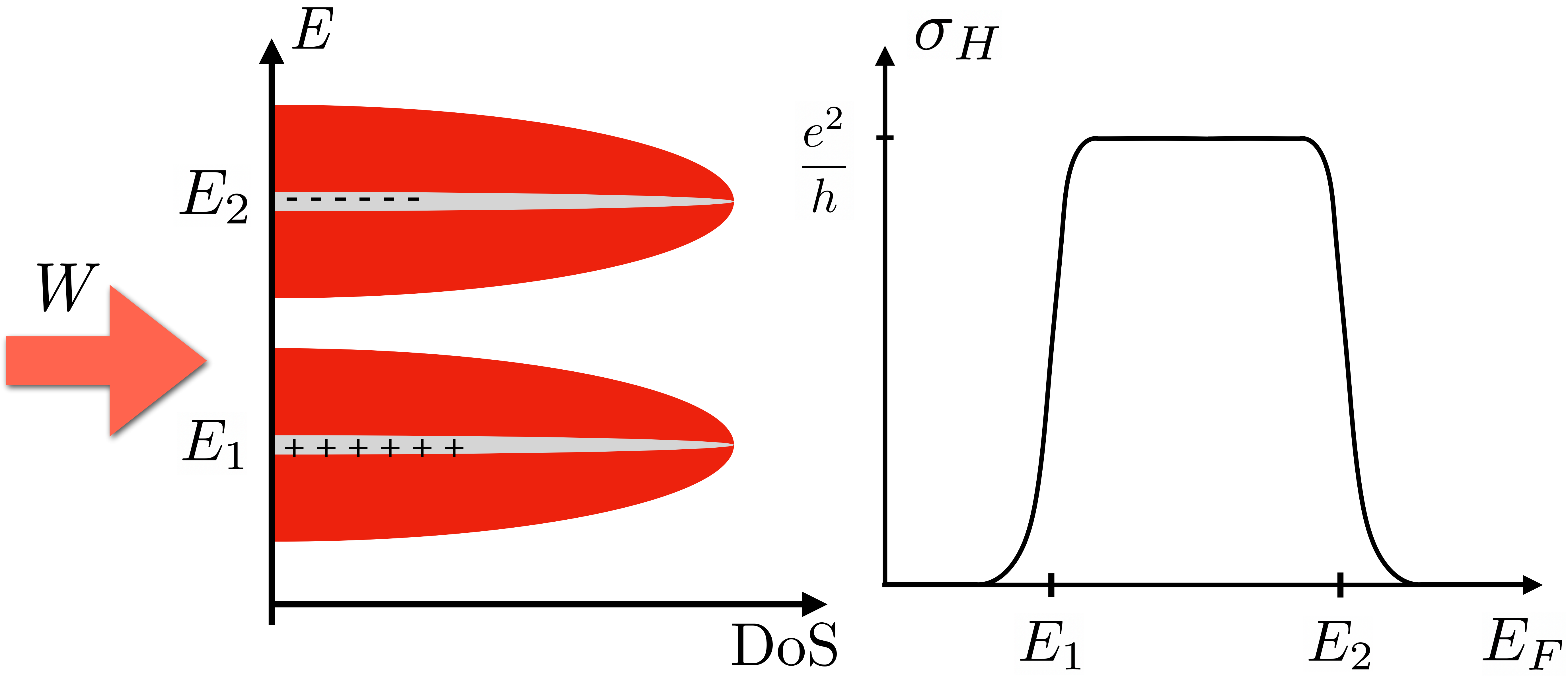
Chern insulators

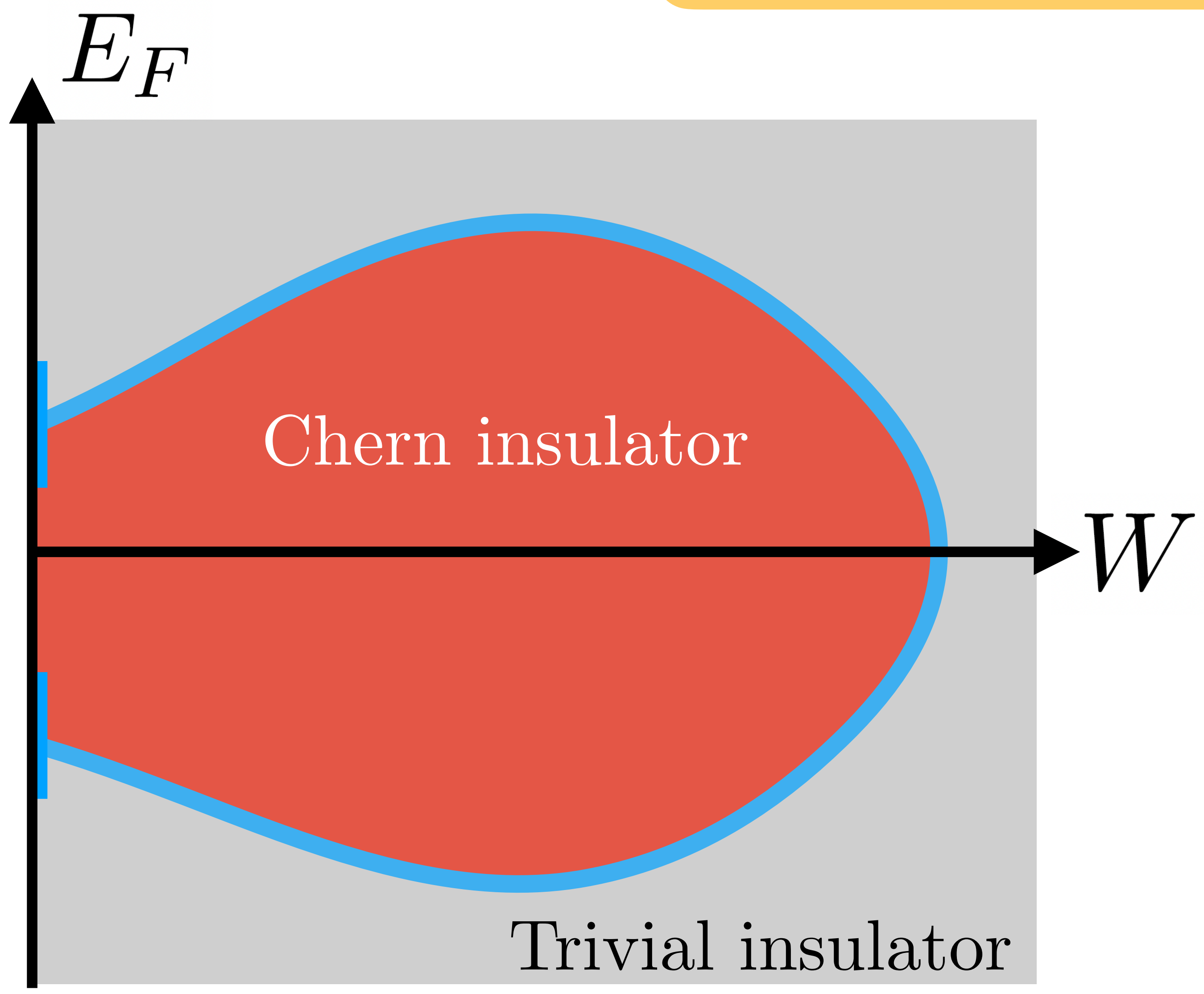
2D Topological insulators in class A

$$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$$



Chern number



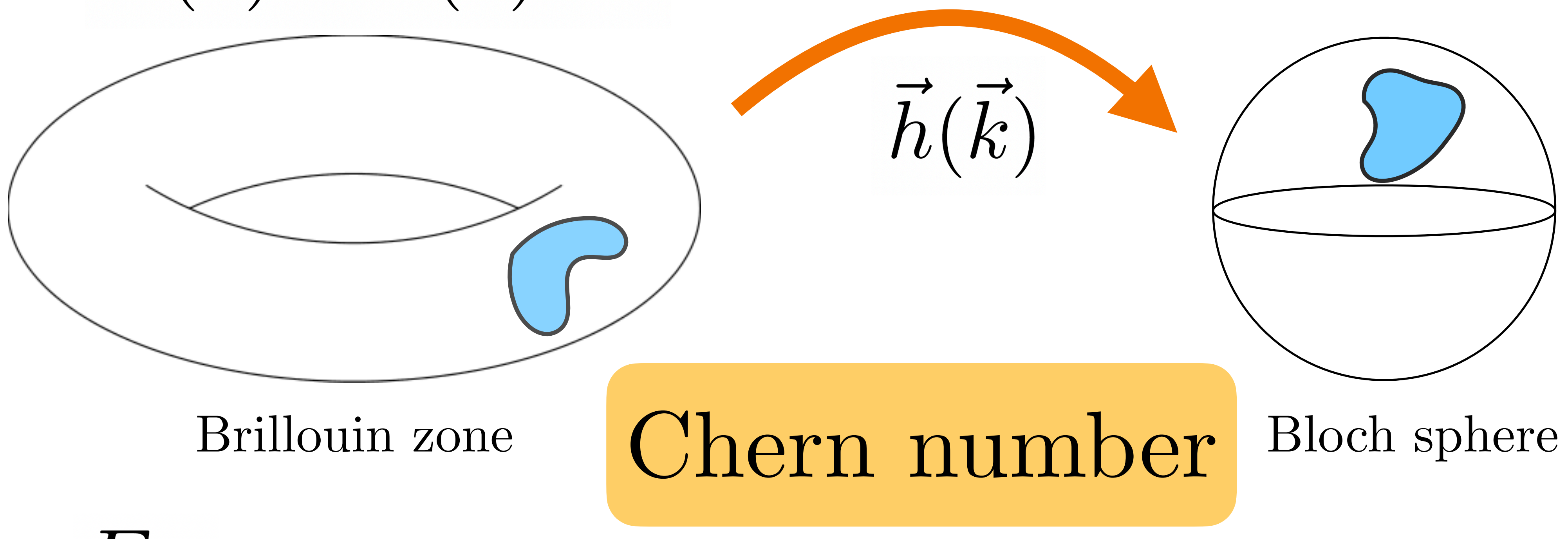


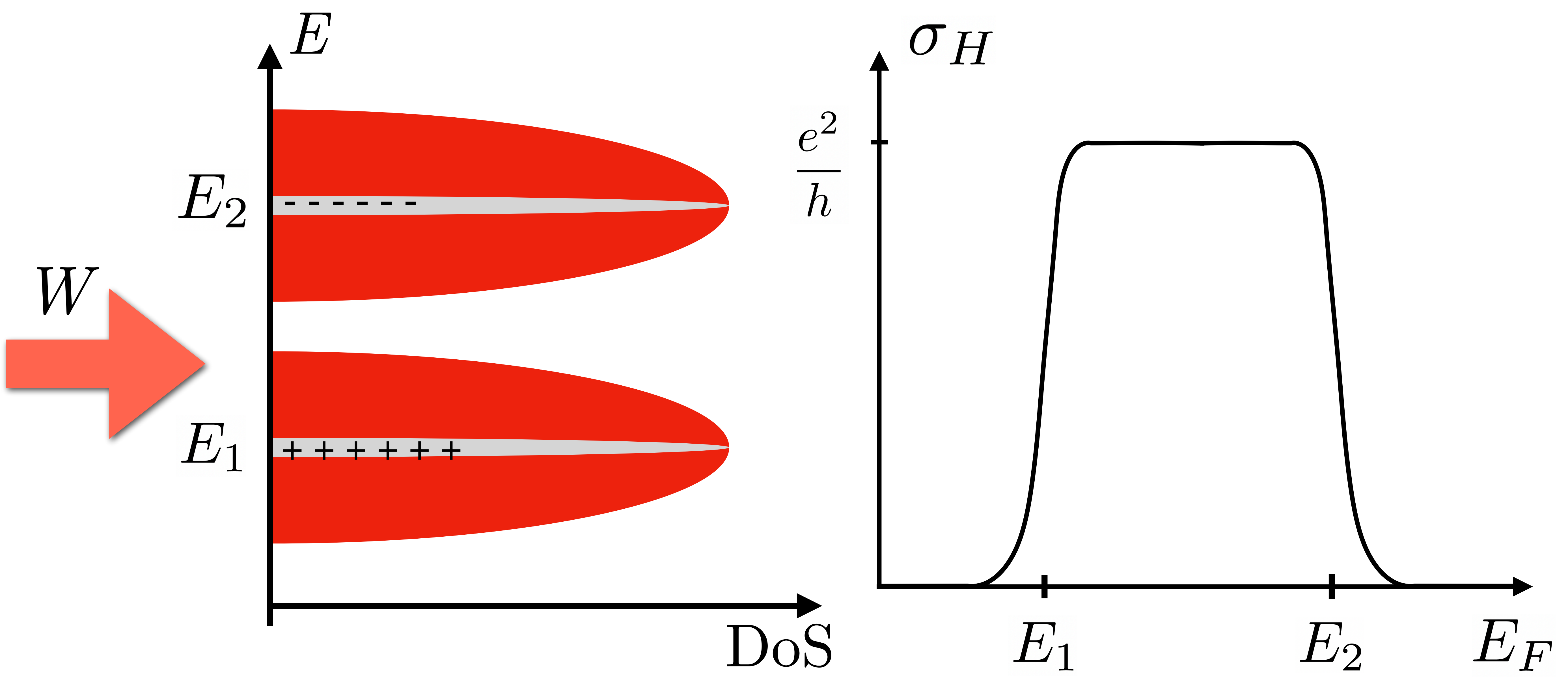
[E. Prodan, J. Phys. A: Math. Theor. 44, 113001 (2011)]

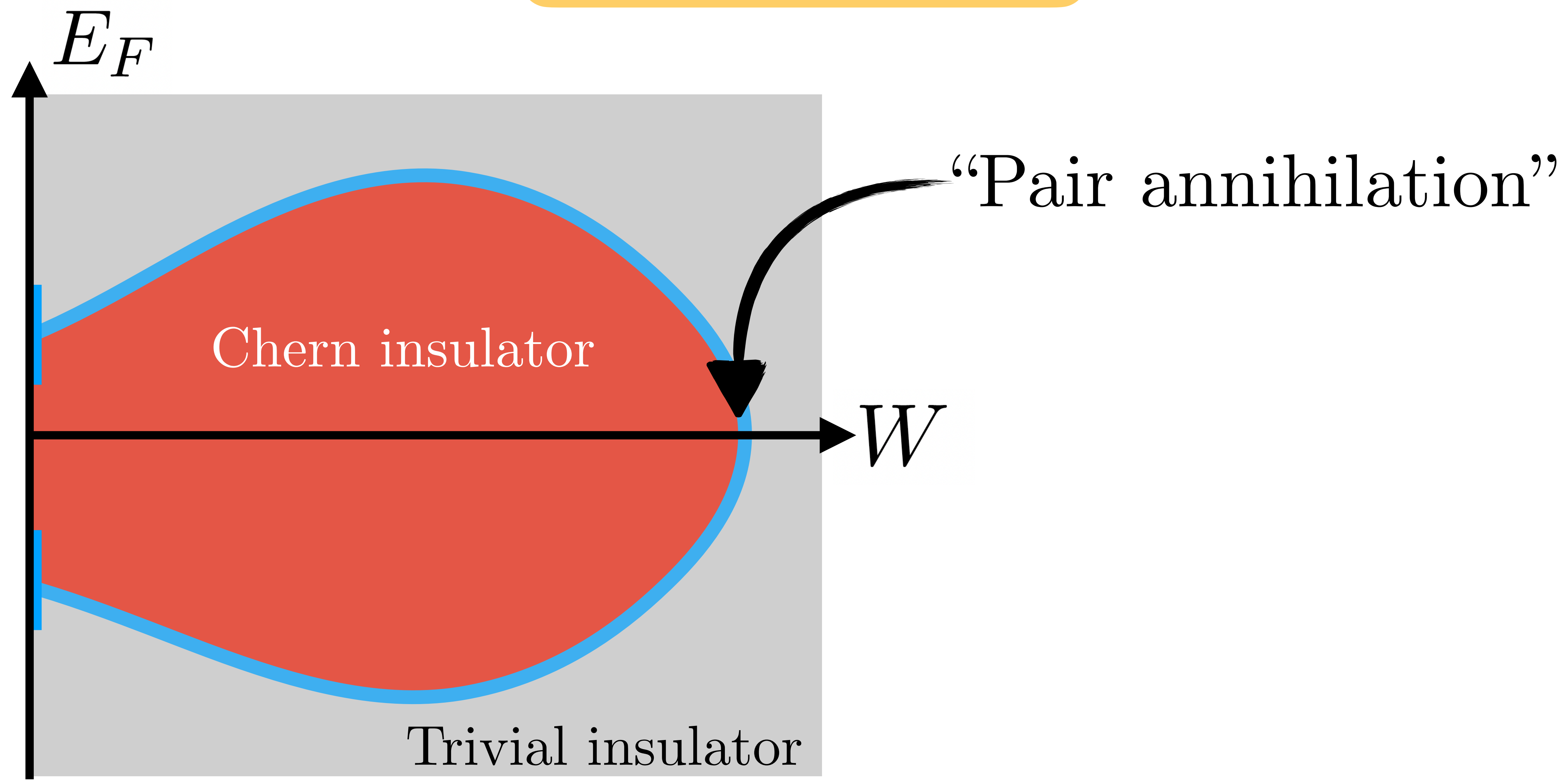
Chern insulators

2D Topological insulators in class A

$$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$$





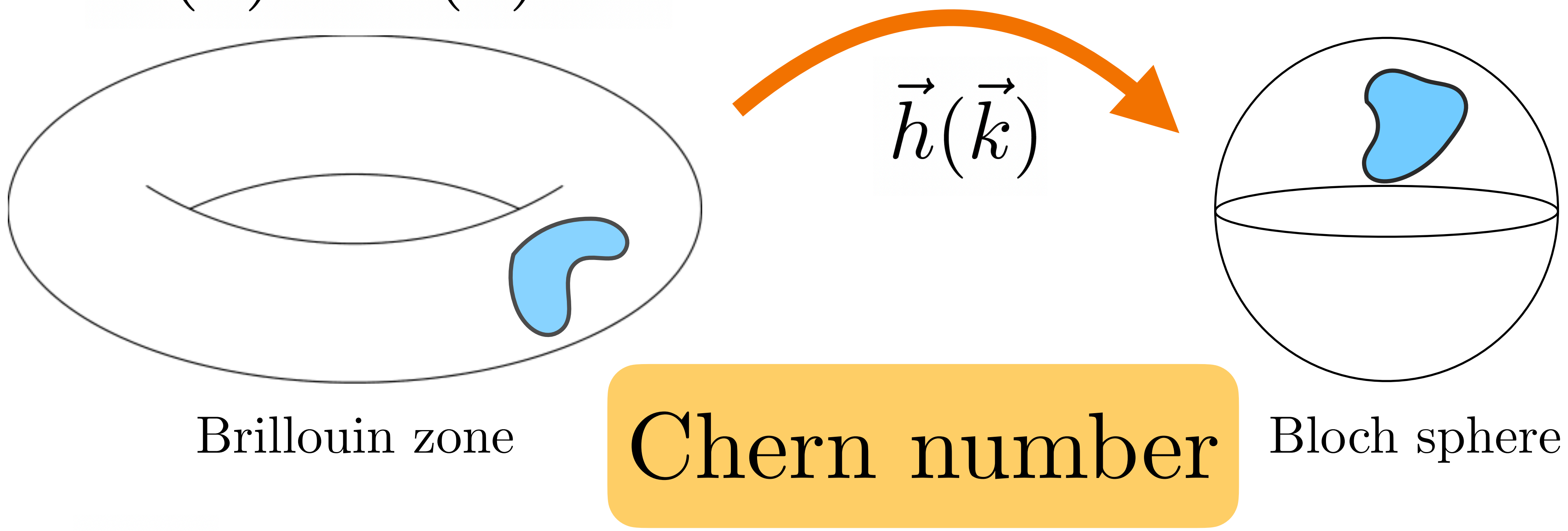


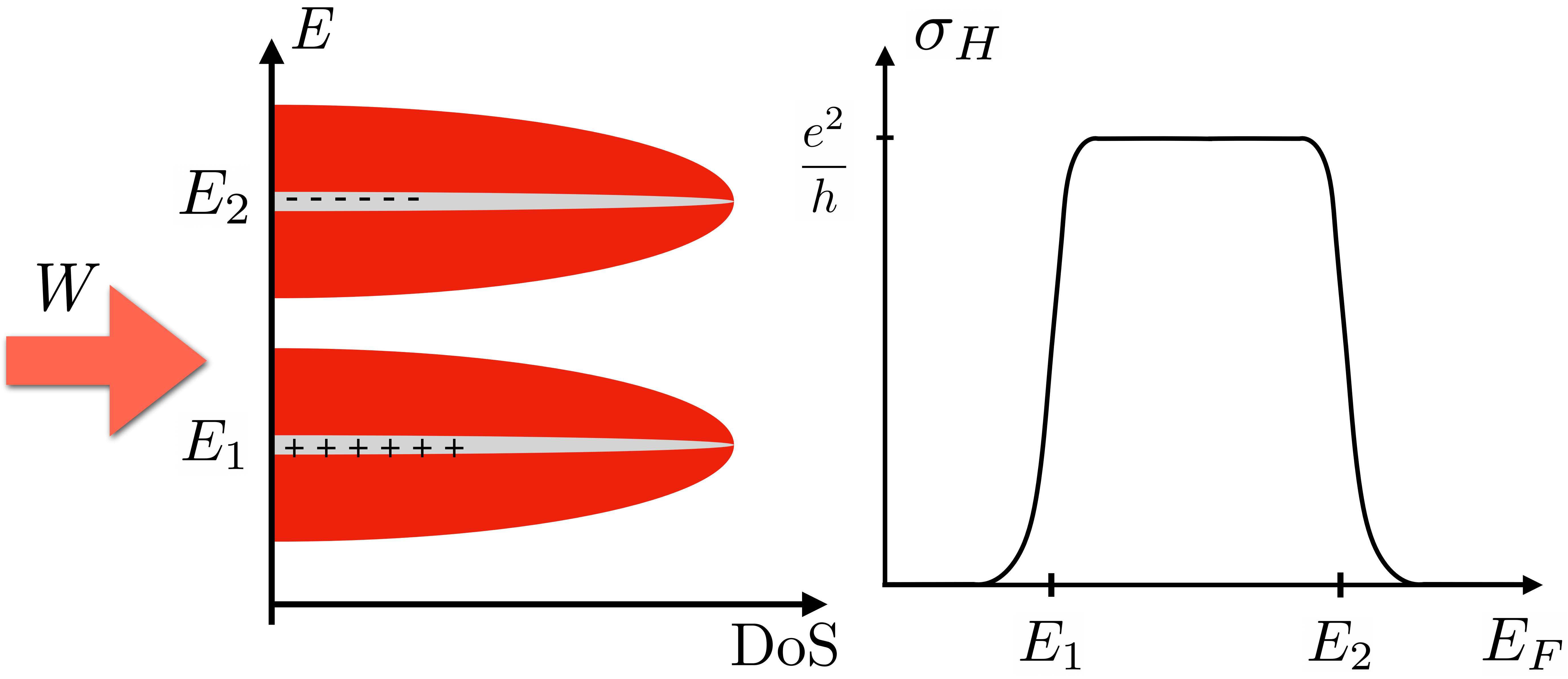
[E. Prodan, J. Phys. A: Math. Theor. 44, 113001 (2011)]

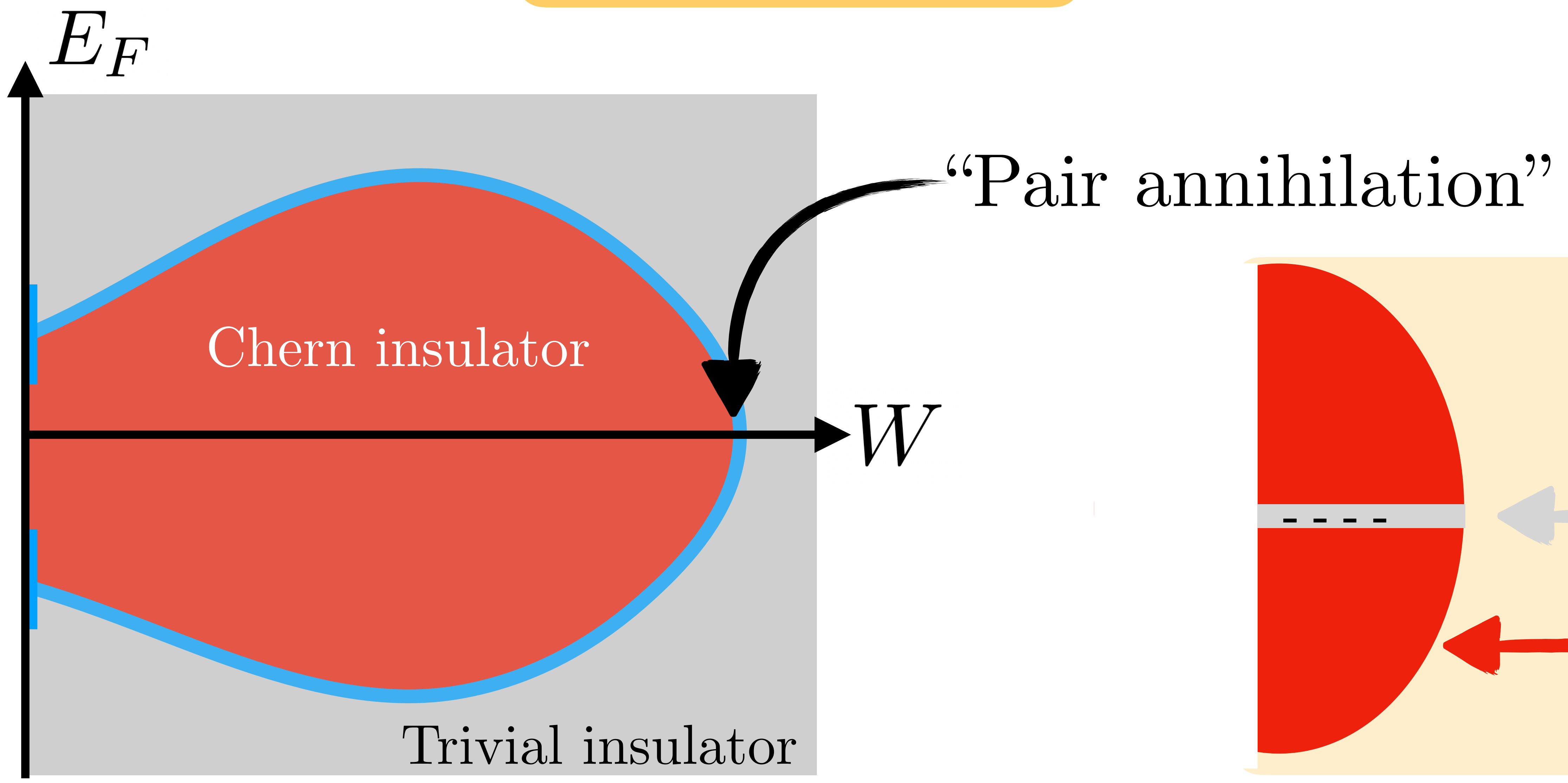
Chern insulators

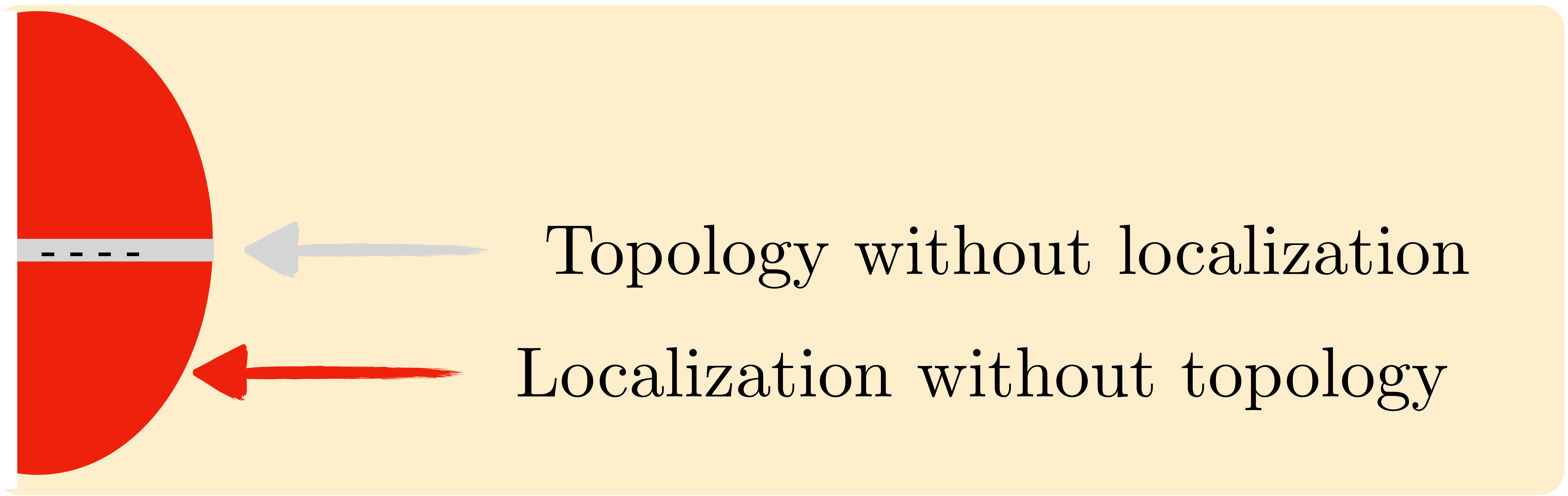
2D Topological insulators in class A

$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\sigma}$





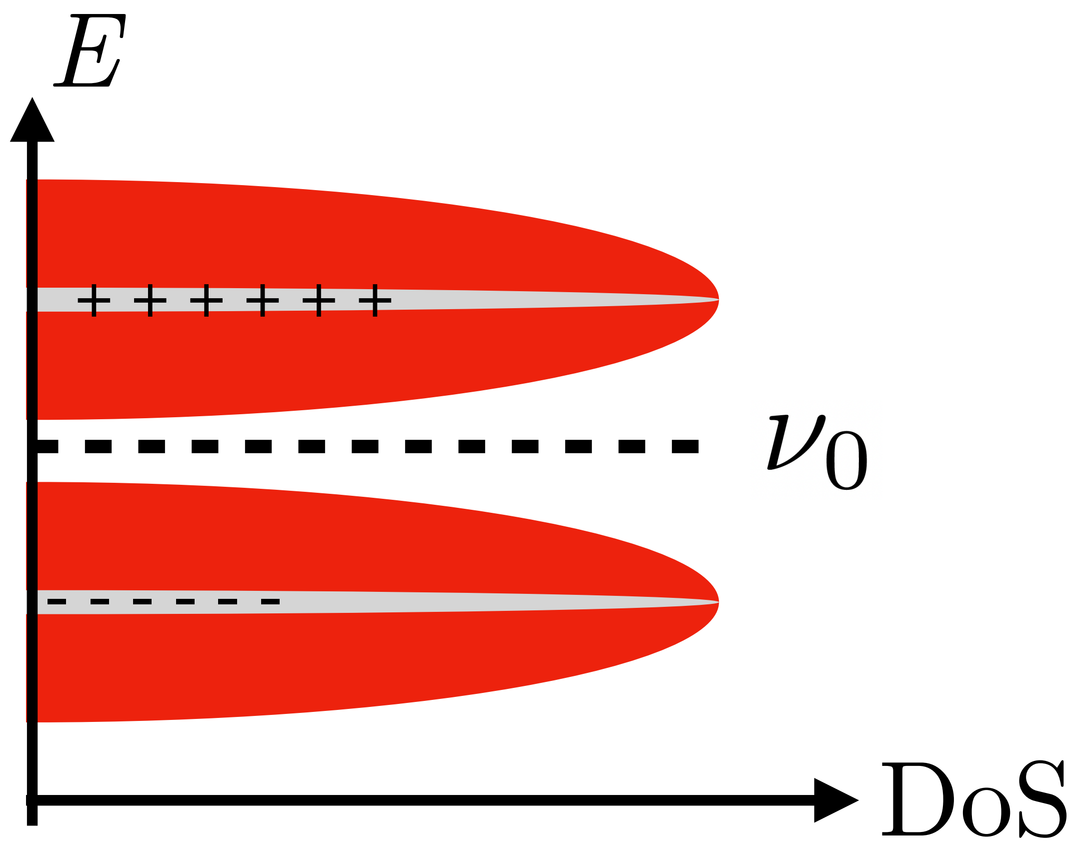




[E. Prodan, J. Phys. A: Math. Theor. 44, 113001 (2011)]

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



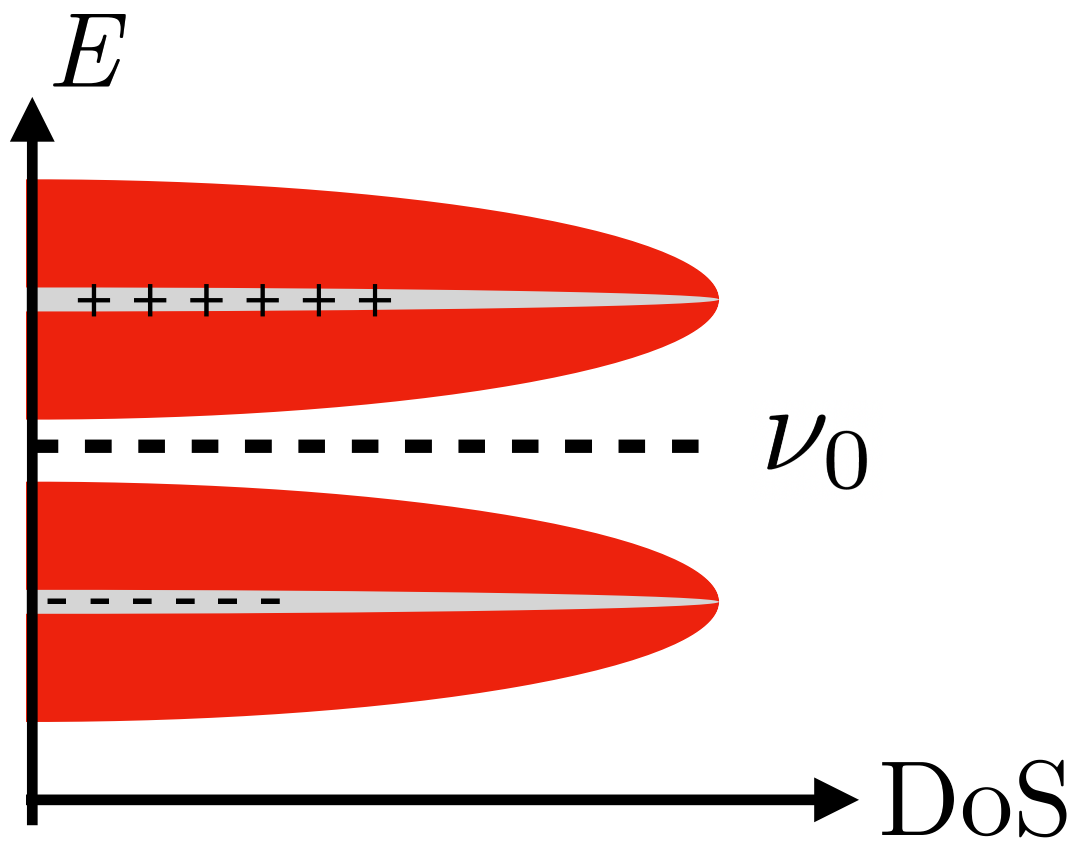
IQHE, Chern insulators, QSHE, 3D TIs...

Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



IQHE, Chern insulators, QSHE, 3D TIs...

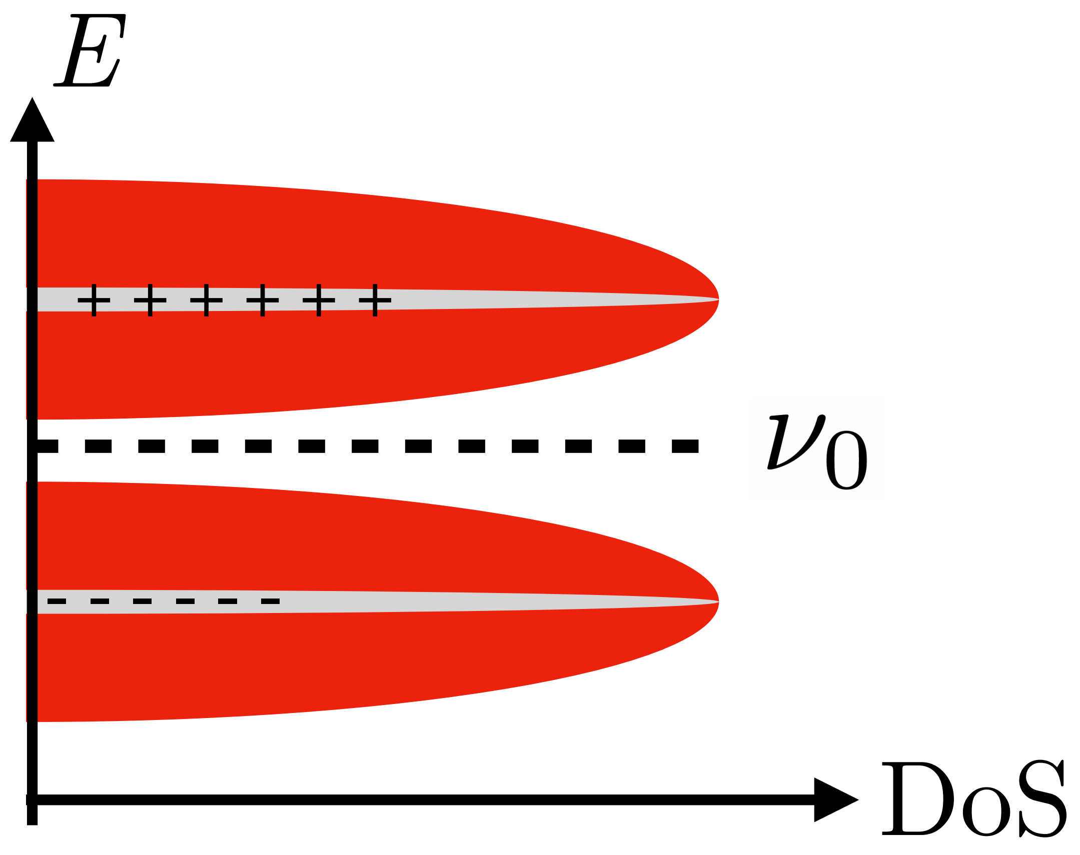
Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Phase transition: $\sigma_{xx} \neq 0$ (For fixed filling ν_0)

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



IQHE, Chern insulators, QSHE, 3D TIs...

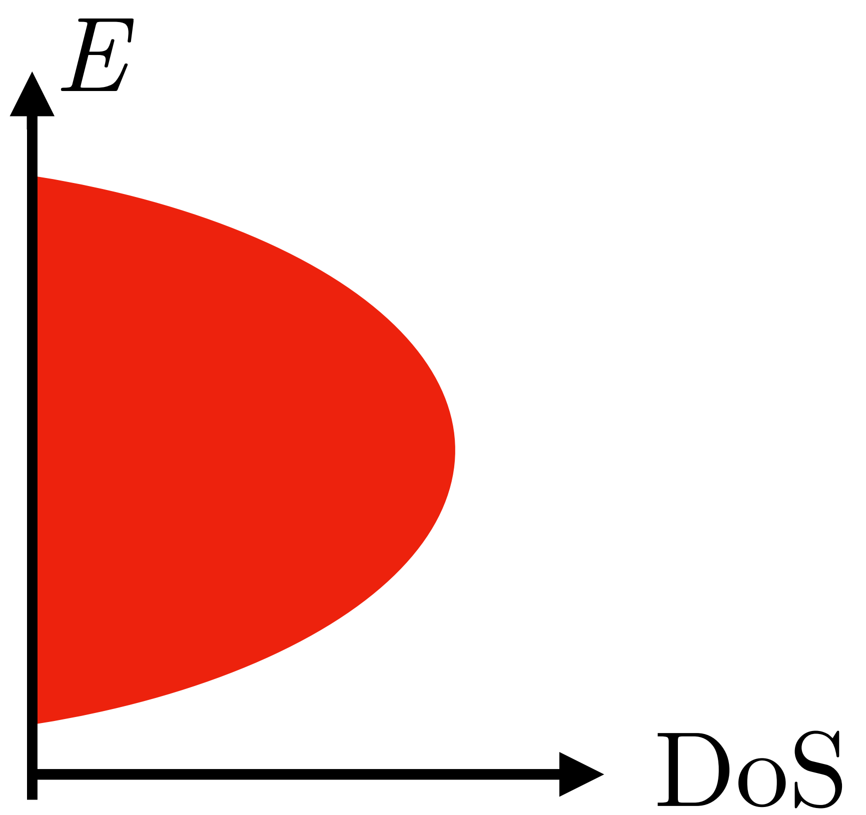
Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Phase transition: $\sigma_{xx} \neq 0$ (For fixed filling ν_0)

Topologically Localized Insulators (TLIs)

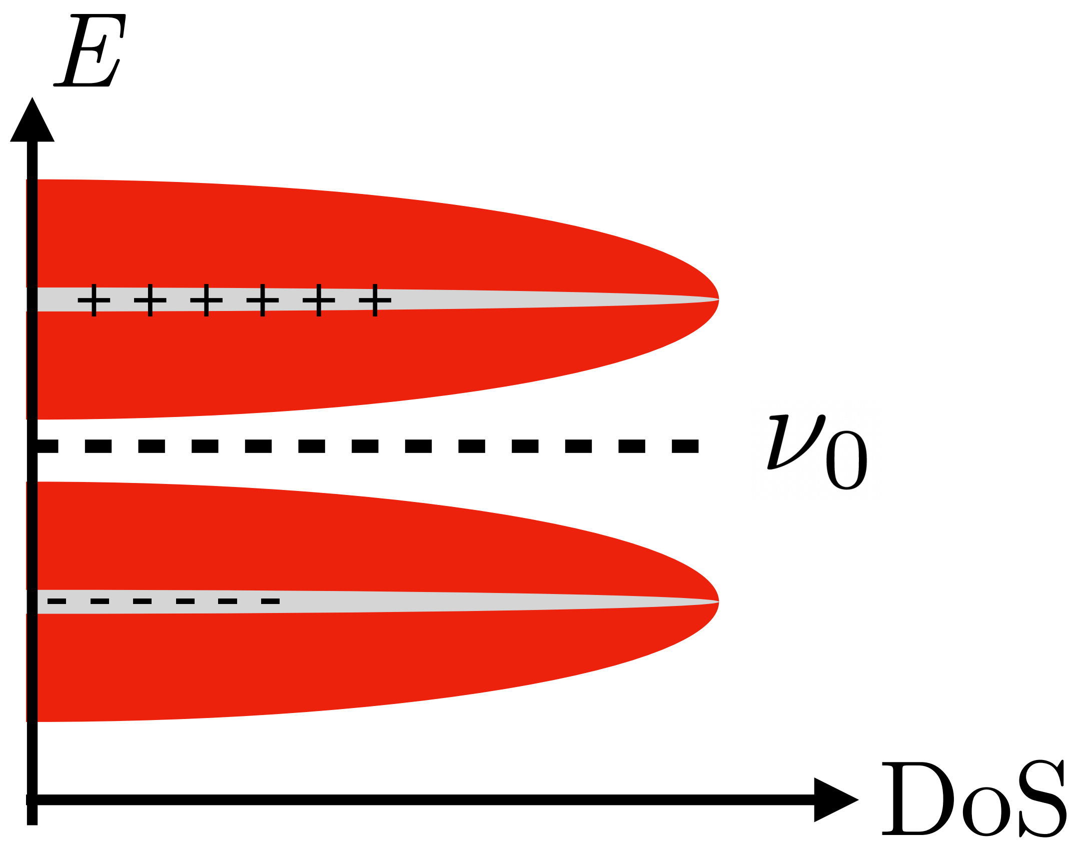
class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?



Full Anderson localization & non-trivial topology

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



IQHE, Chern insulators, QSHE, 3D TIs...

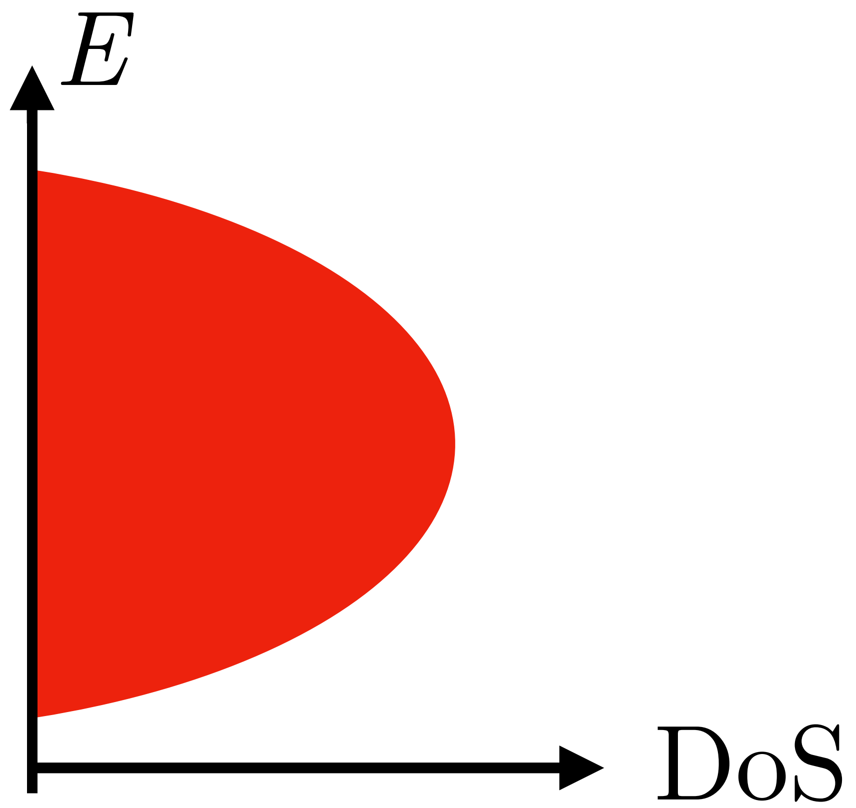
Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Phase transition: $\sigma_{xx} \neq 0$ (For fixed filling ν_0)

Topologically Localized Insulators (TLIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?



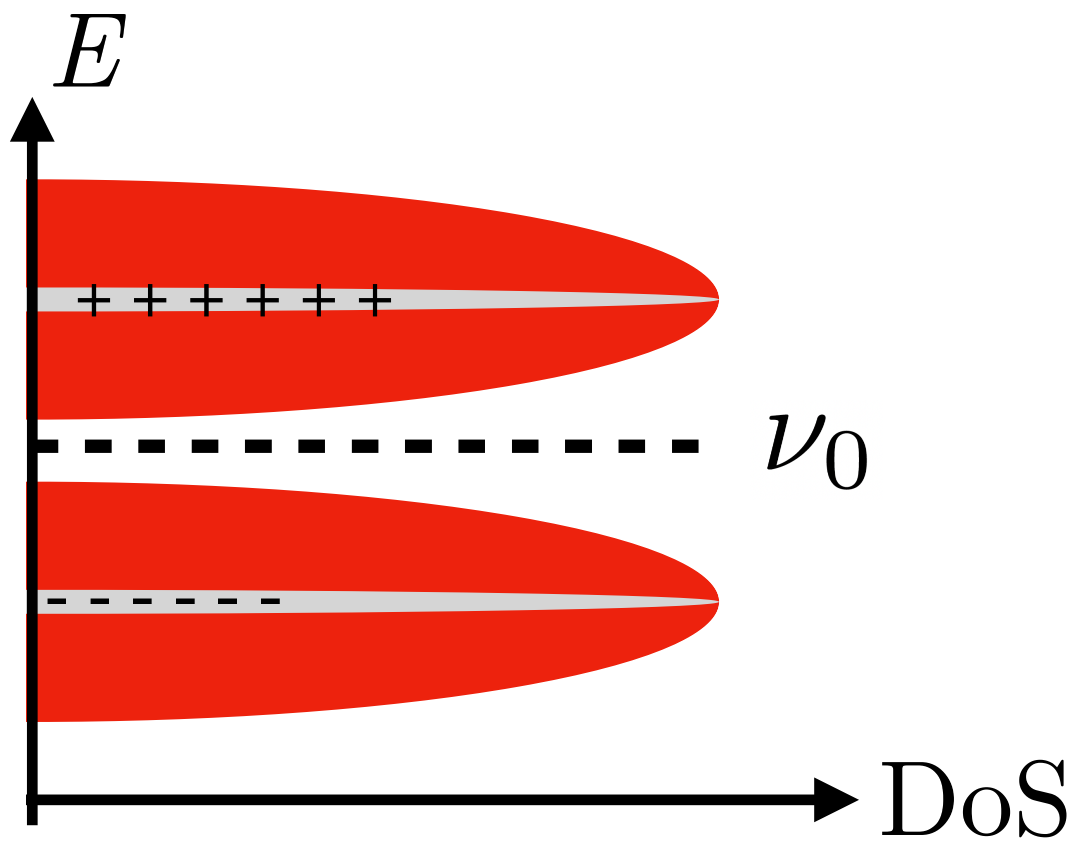
Full Anderson localization & non-trivial topology

Applies to fully localized insulators, i.e.,

$\sigma_{xx} = 0 \ \forall \nu$

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



IQHE, Chern insulators, QSHE, 3D TIs...

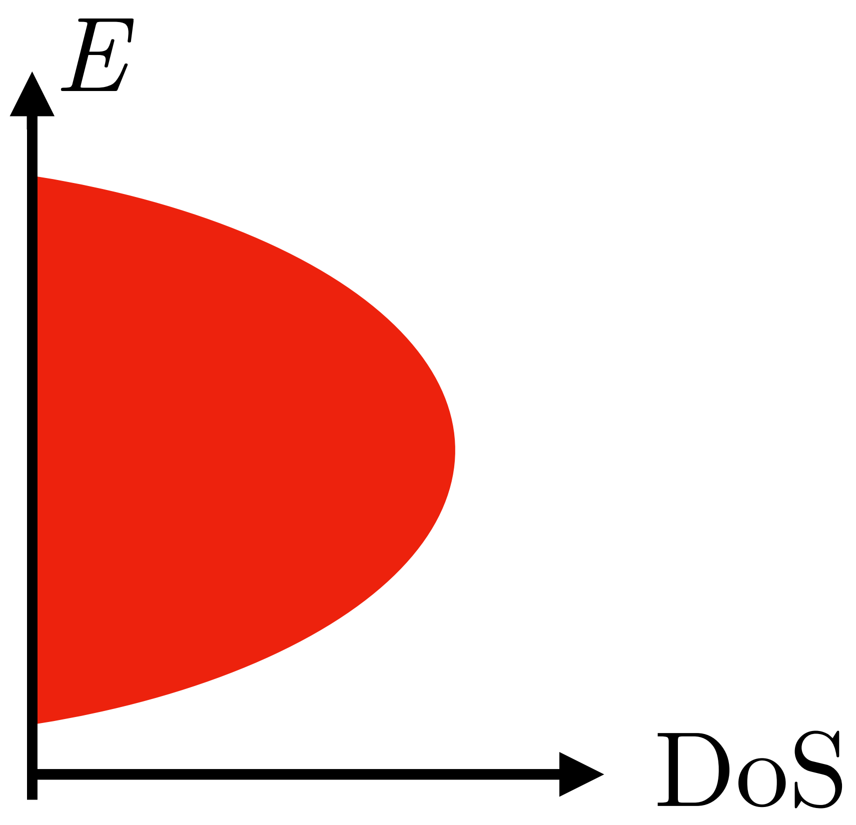
Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Phase transition: $\sigma_{xx} \neq 0$ (For fixed filling ν_0)

Topologically Localized Insulators (TLIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?



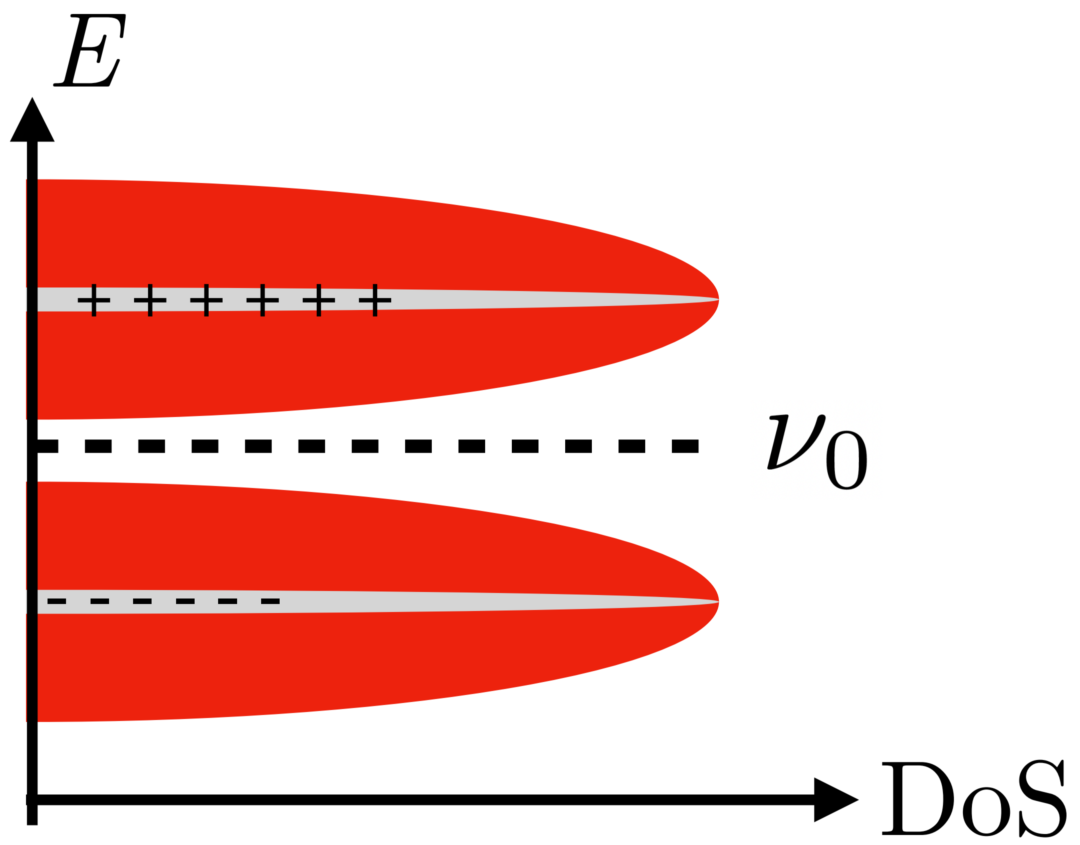
Full Anderson localization & non-trivial topology

Applies to fully localized insulators, i.e., $\sigma_{xx} = 0 \ \forall \nu$

Phase transition: $\exists \nu$ such that $\sigma_{xx} \neq 0$

Topological Insulators (TIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0



IQHE, Chern insulators, QSHE, 3D TIs...

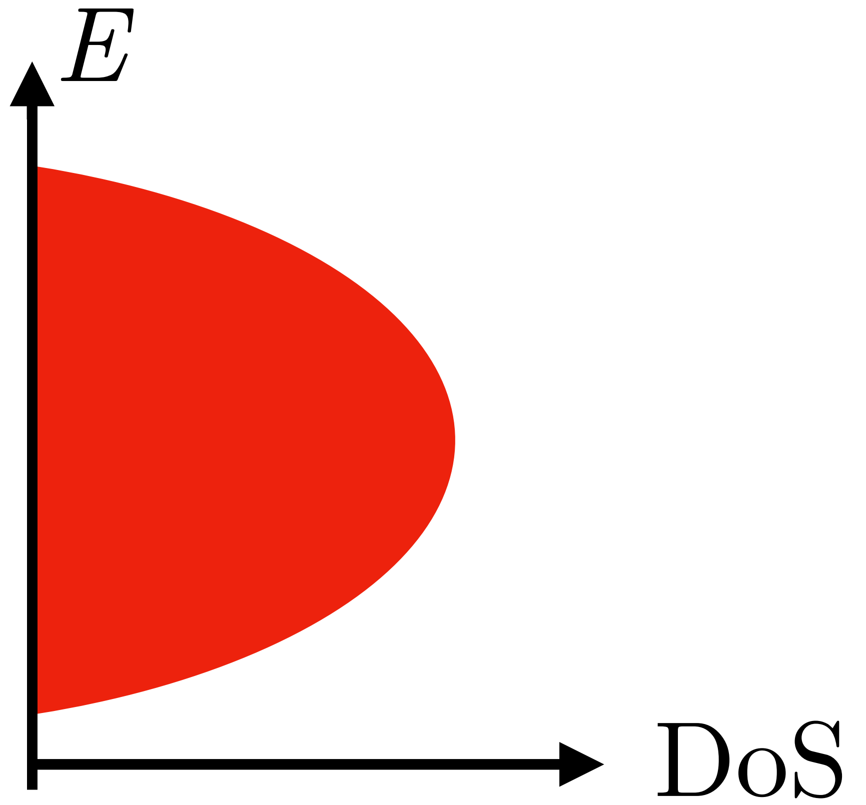
Applies to insulators, i.e., $\sigma_{xx} = 0$

(For fixed filling ν_0)

Phase transition: $\sigma_{xx} \neq 0$ (For fixed filling ν_0)

Topologically Localized Insulators (TLIs)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?



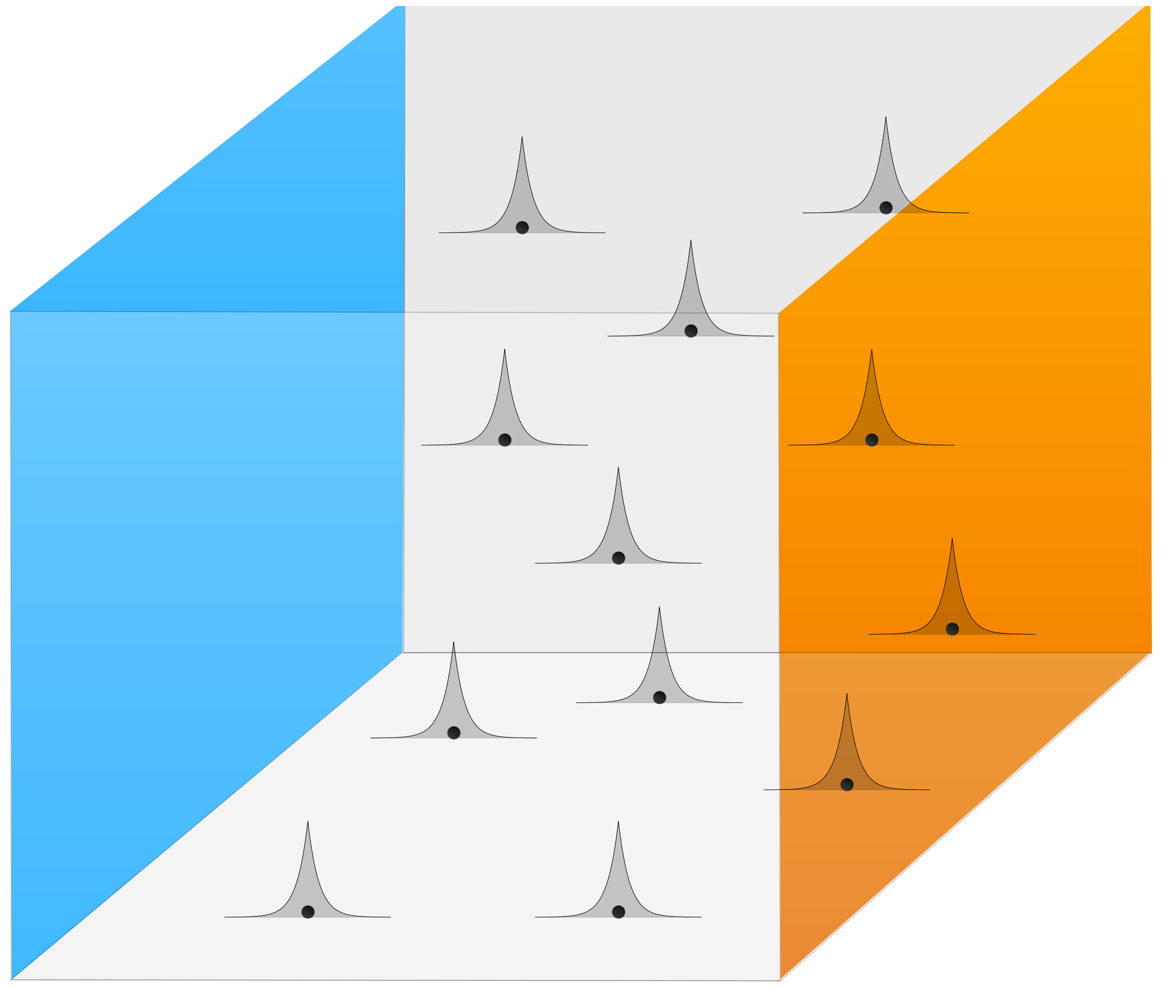
Full Anderson localization & non-trivial topology

Applies to fully localized insulators, i.e.,

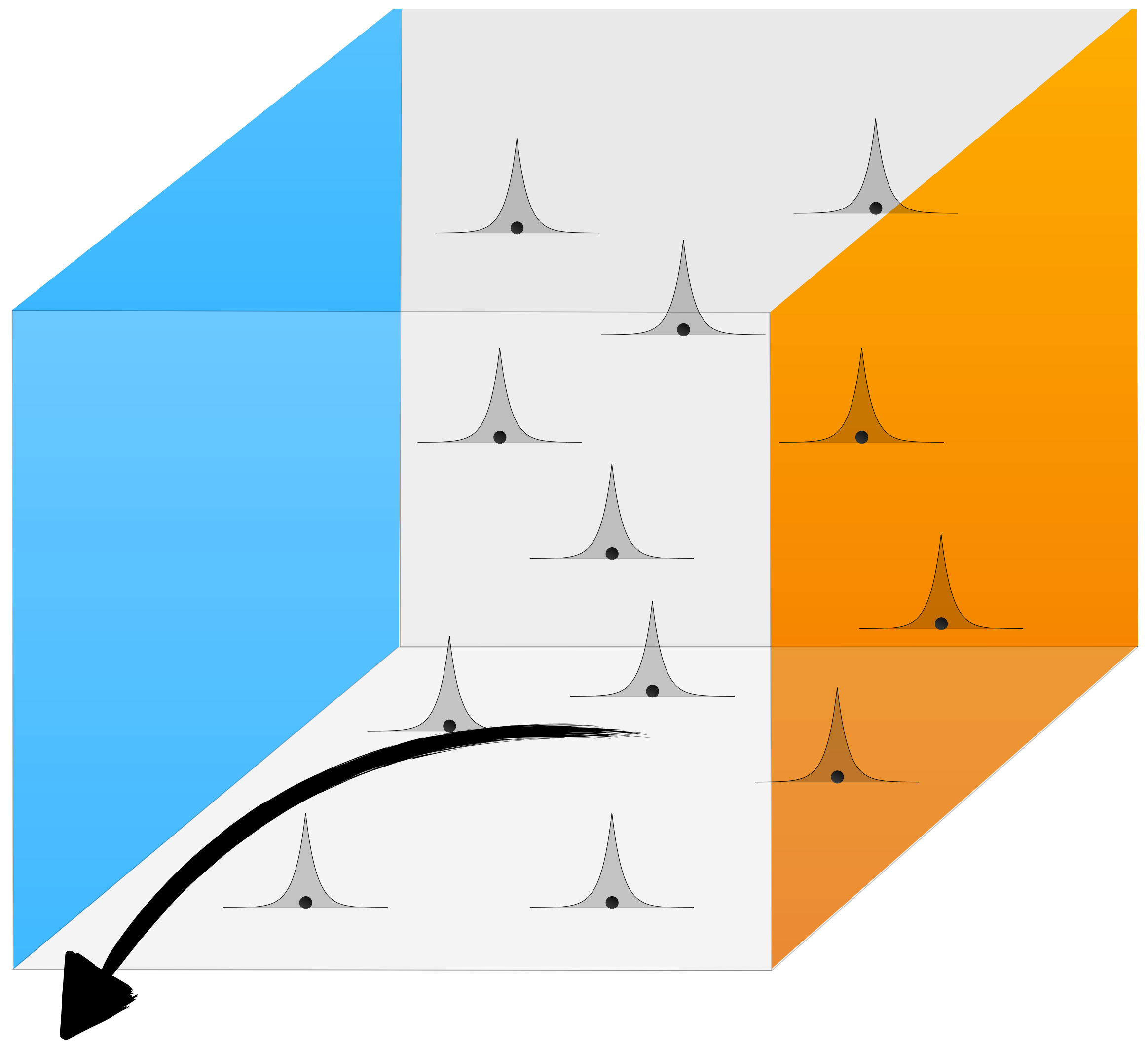
$\sigma_{xx} = 0 \ \forall \nu$

Phase transition: $\exists \nu$ such that $\sigma_{xx} \neq 0$

Topologically Localized Insulators

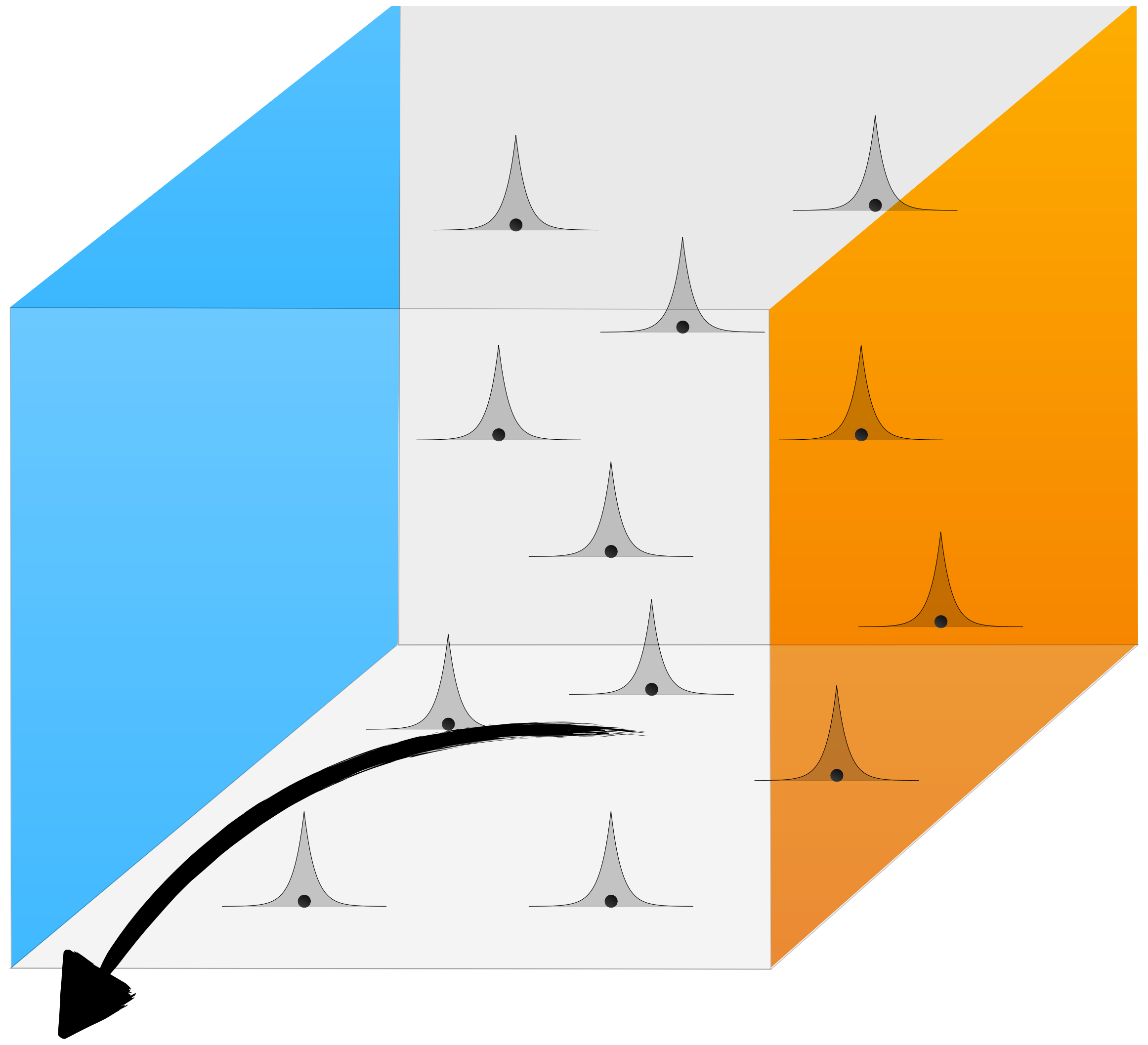


Topologically Localized Insulators



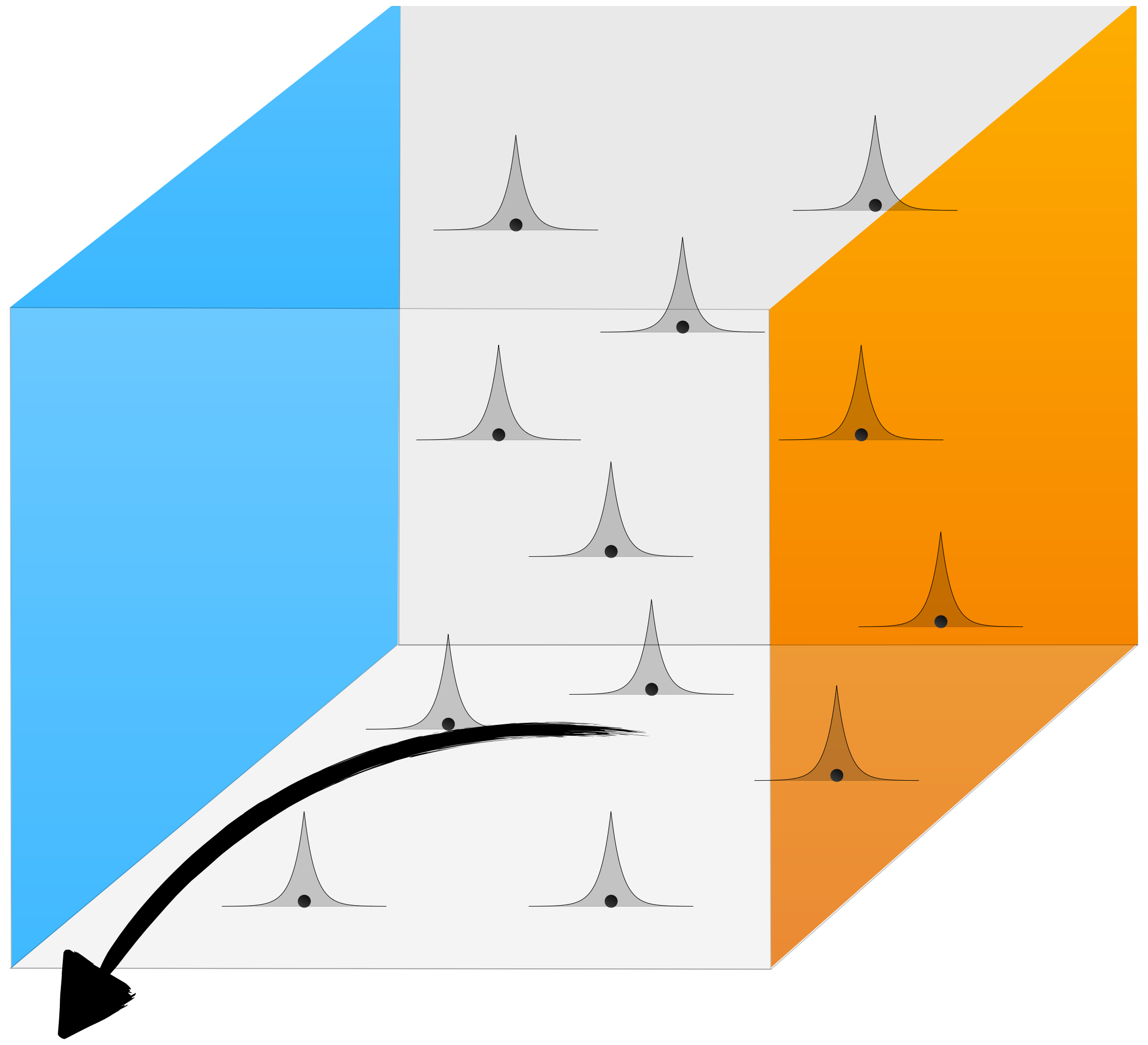
$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$

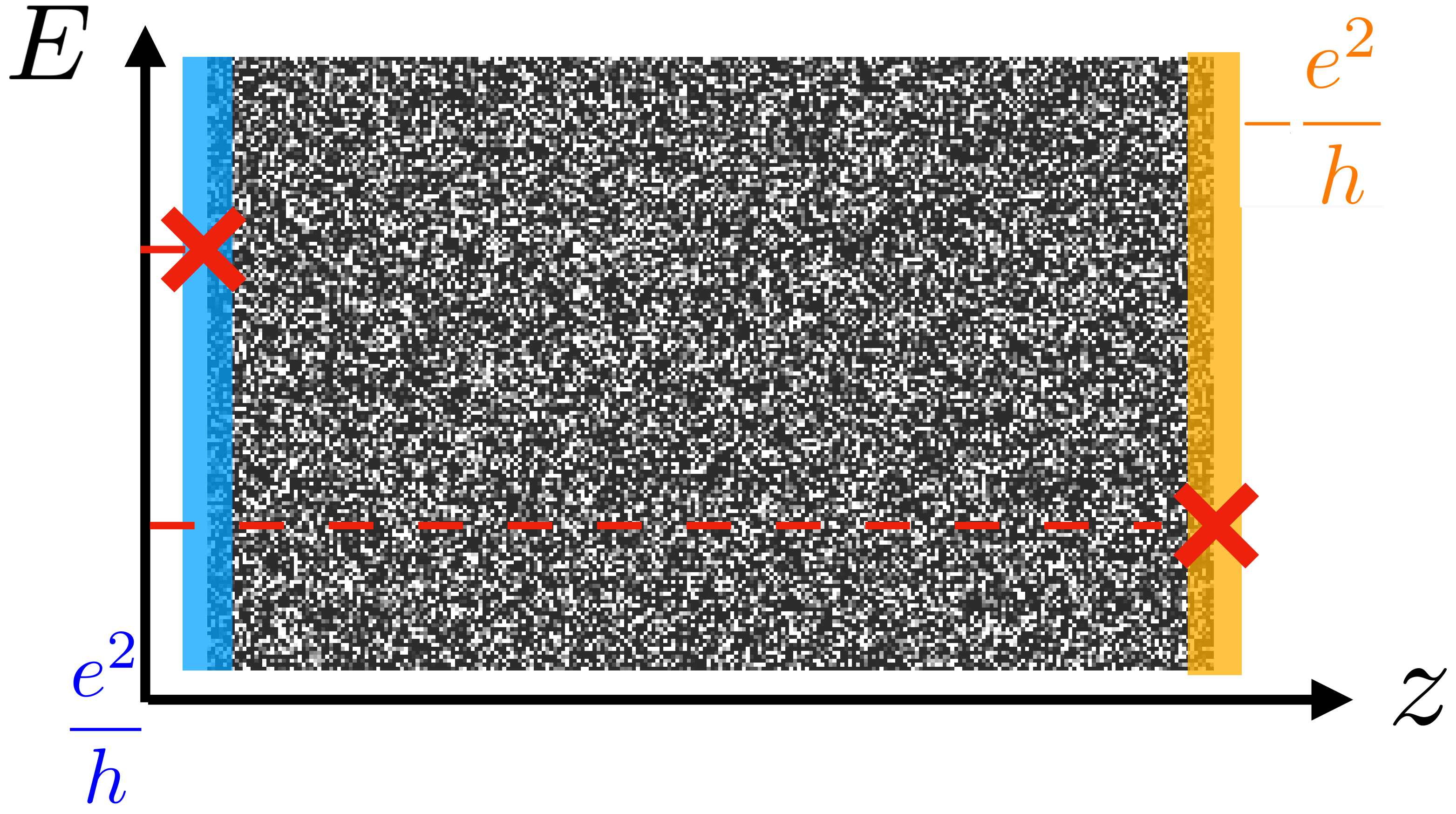
Topologically Localized Insulators



$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$ $\quad \alpha_{\mathrm{ME}}$ quantized to integer value

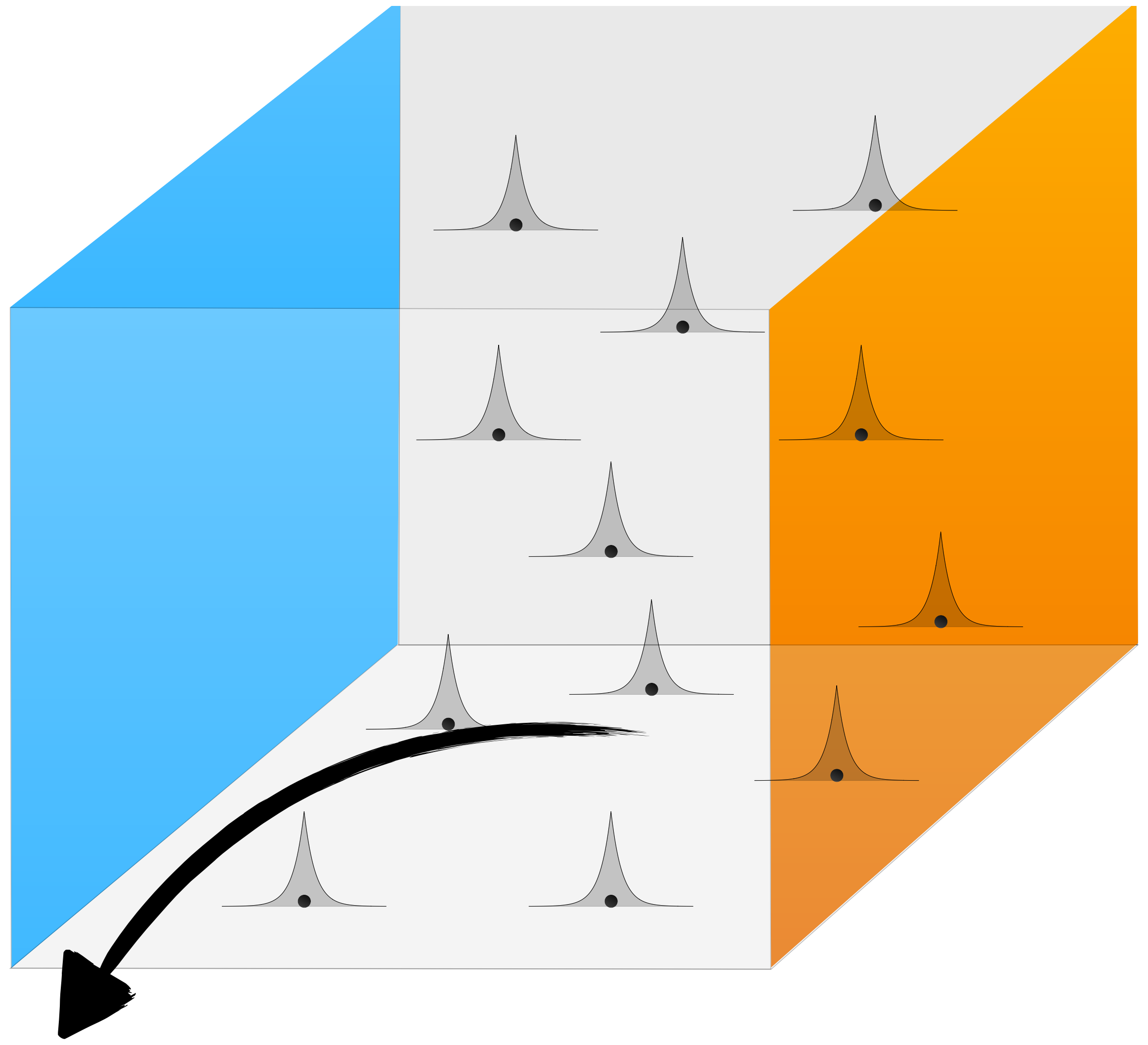
Topologically Localized Insulators





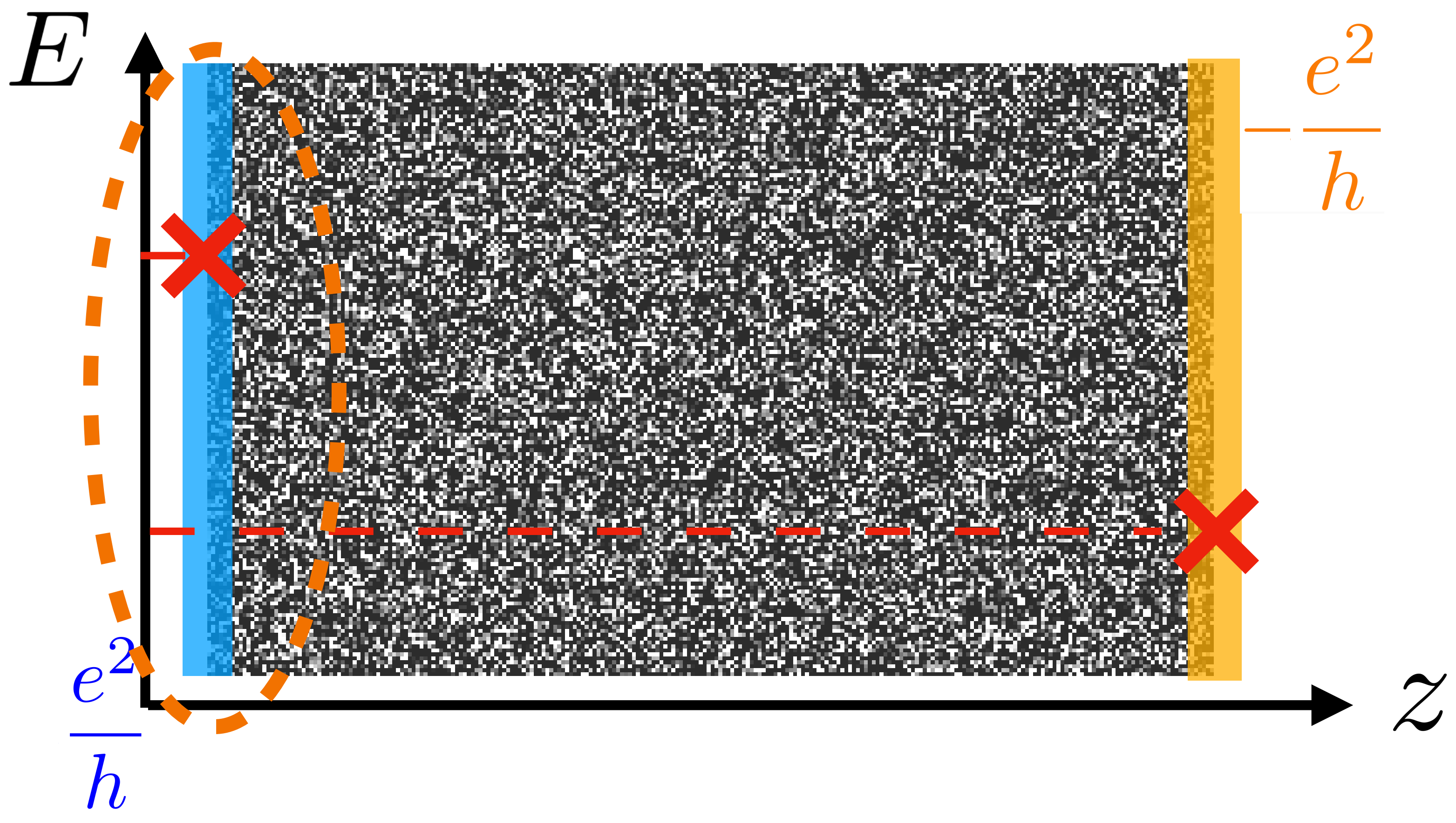
$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$ α_{ME} quantized to integer value

Topologically Localized Insulators



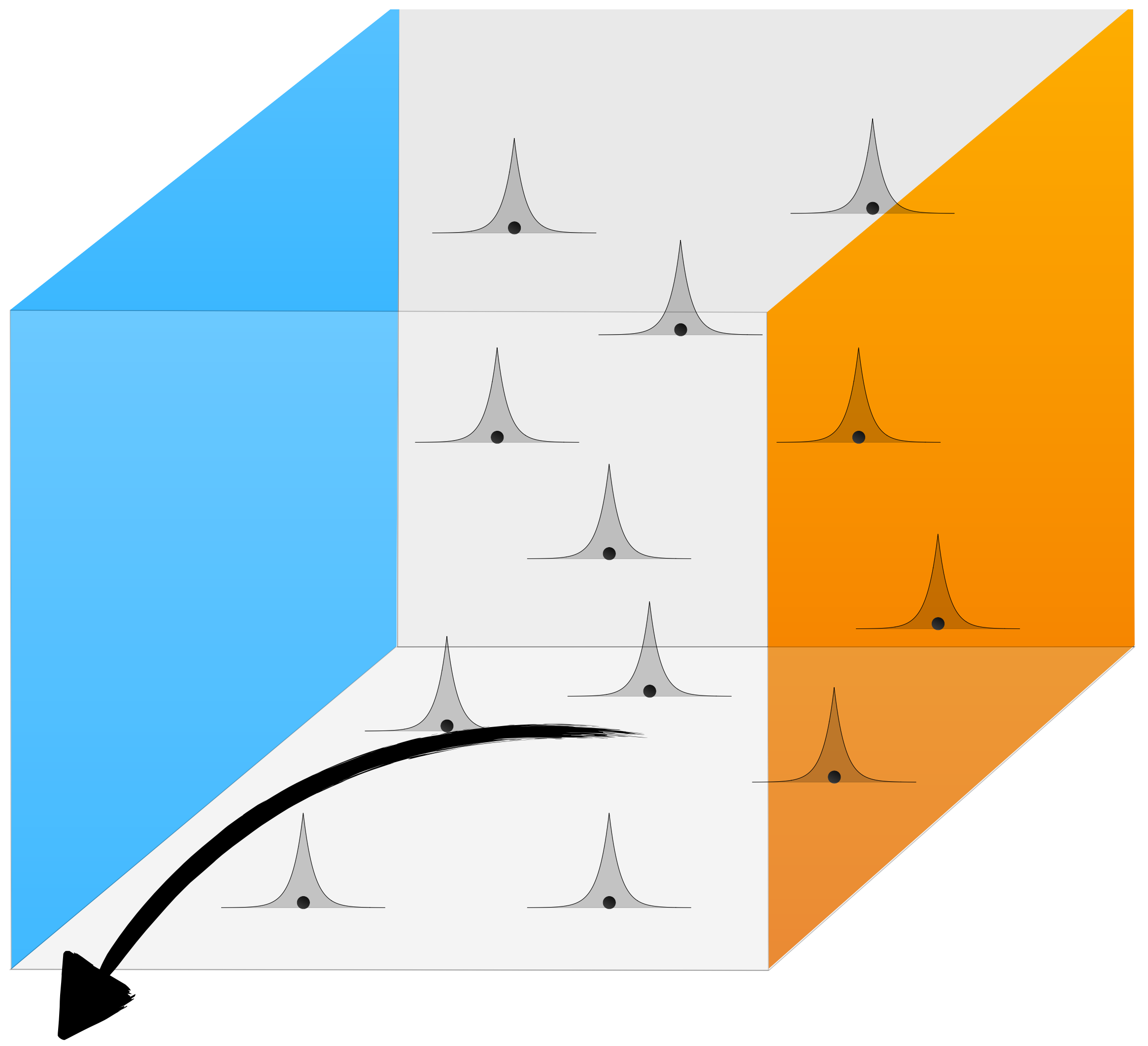
$\vec{P} = \hat{\alpha}_{\mathrm{ME}}\vec{B}$ α_{ME} quantized to integer value

OBC in z-direction



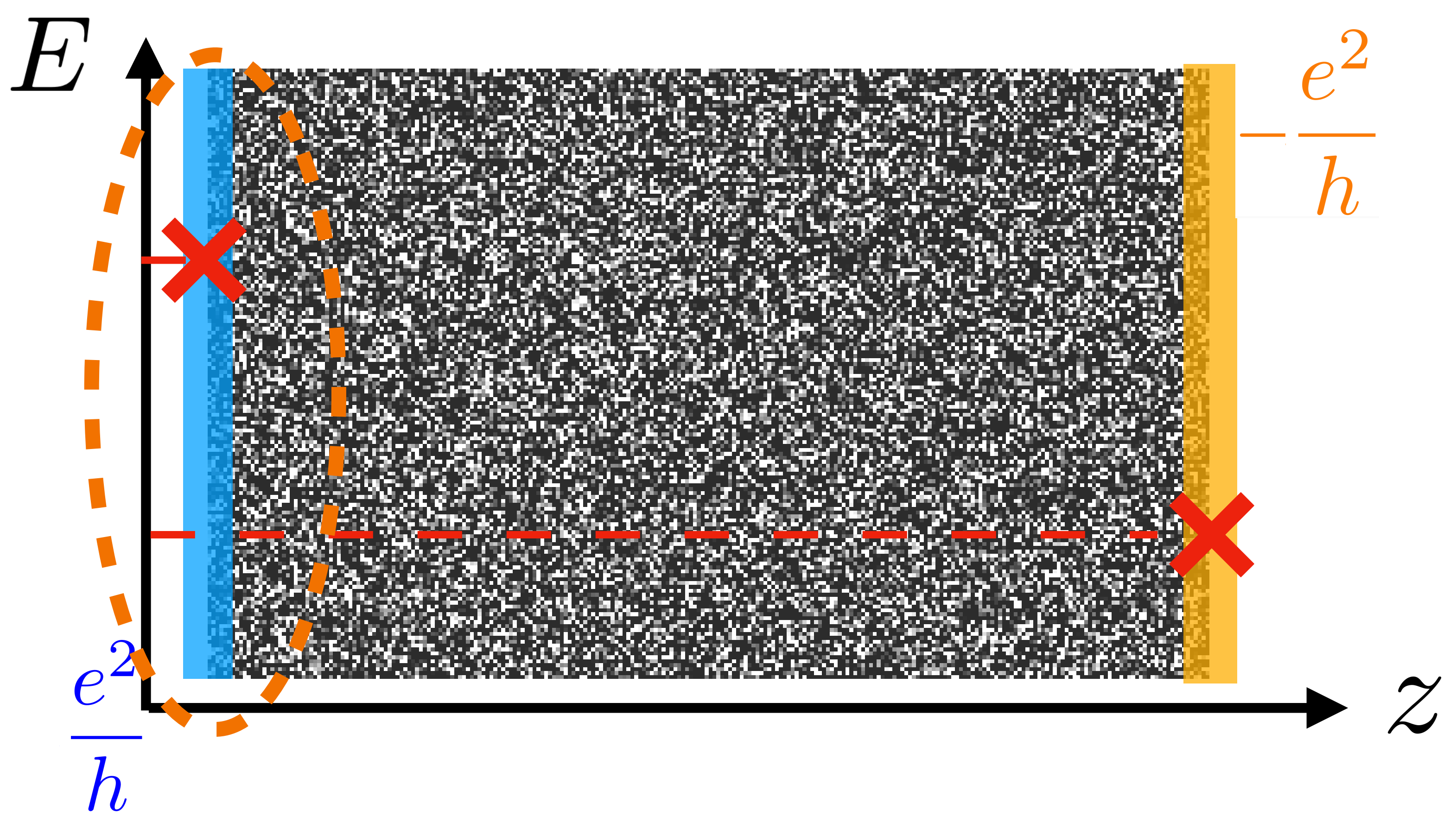
"Half" Chern insulator

Topologically Localized Insulators



$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$ α_{ME} quantized to integer value

OBC in z-direction

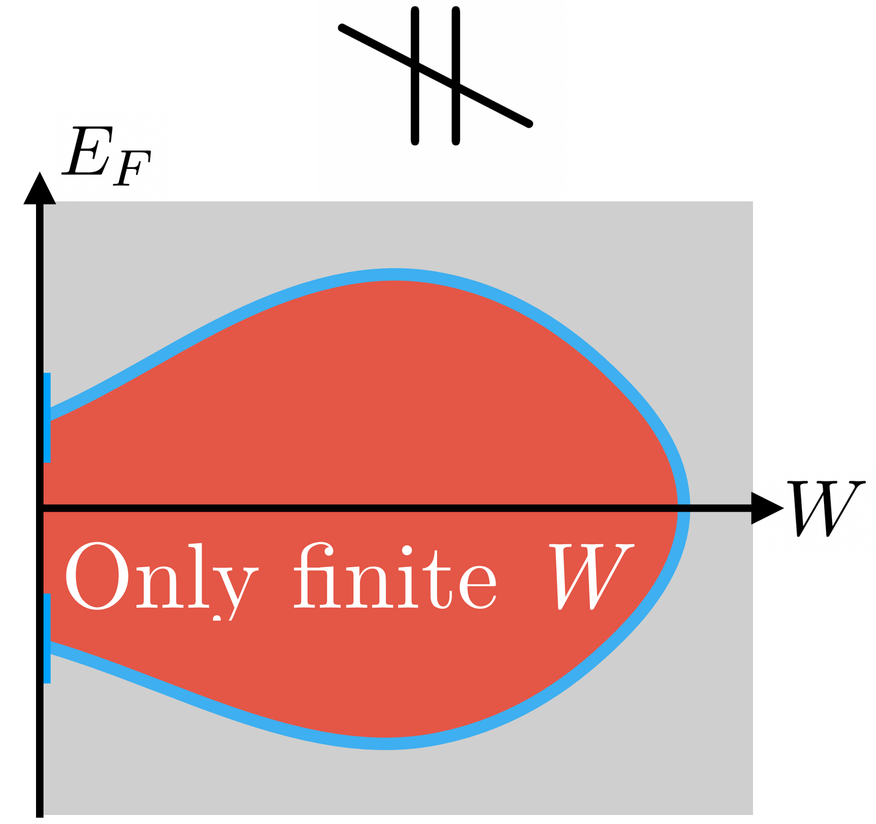


“Half” Chern insulator

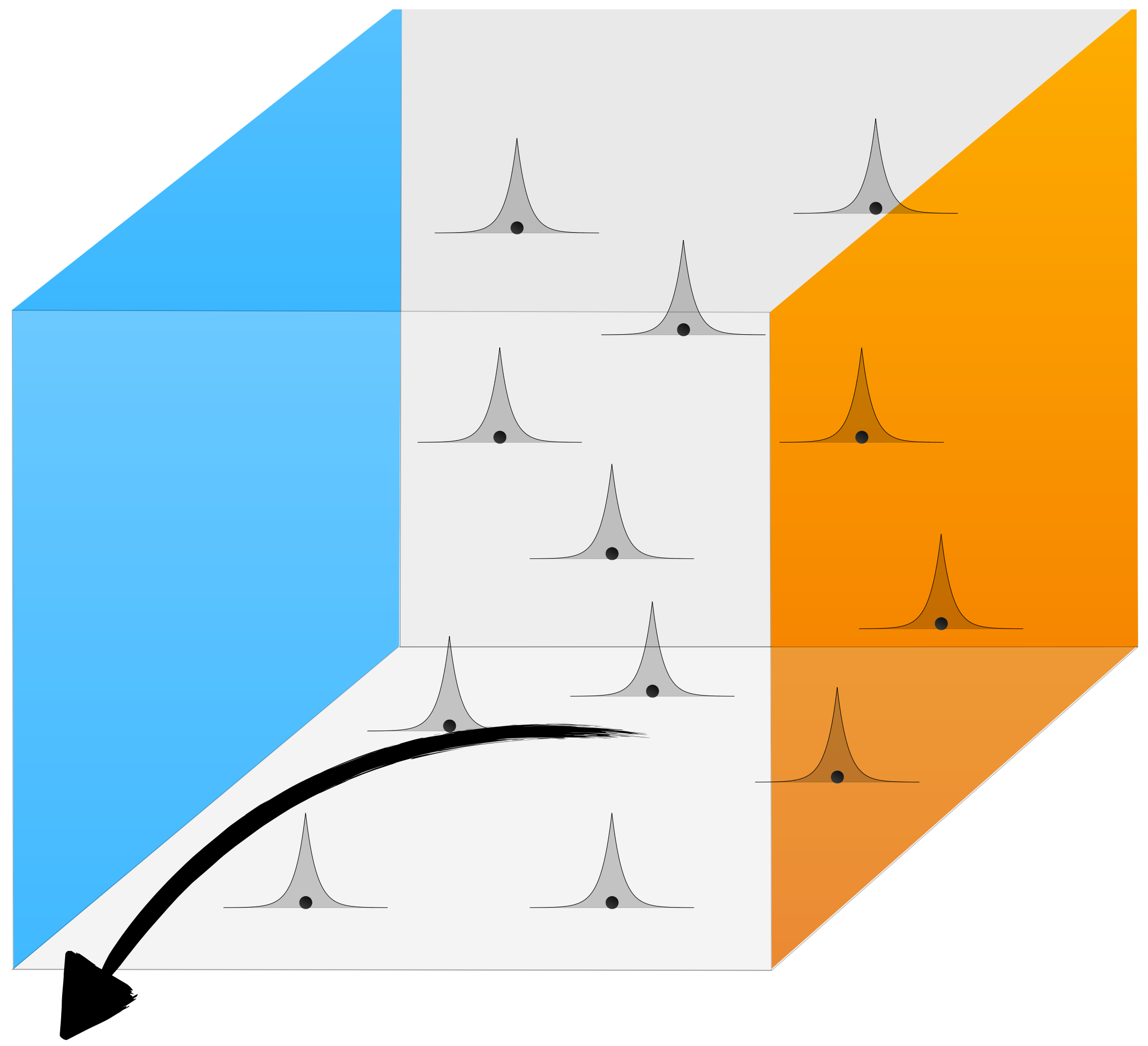
Quantized Hall conductivity

$\sigma_{xy} = n\frac{e^2}{h}$ for any W

on boundary

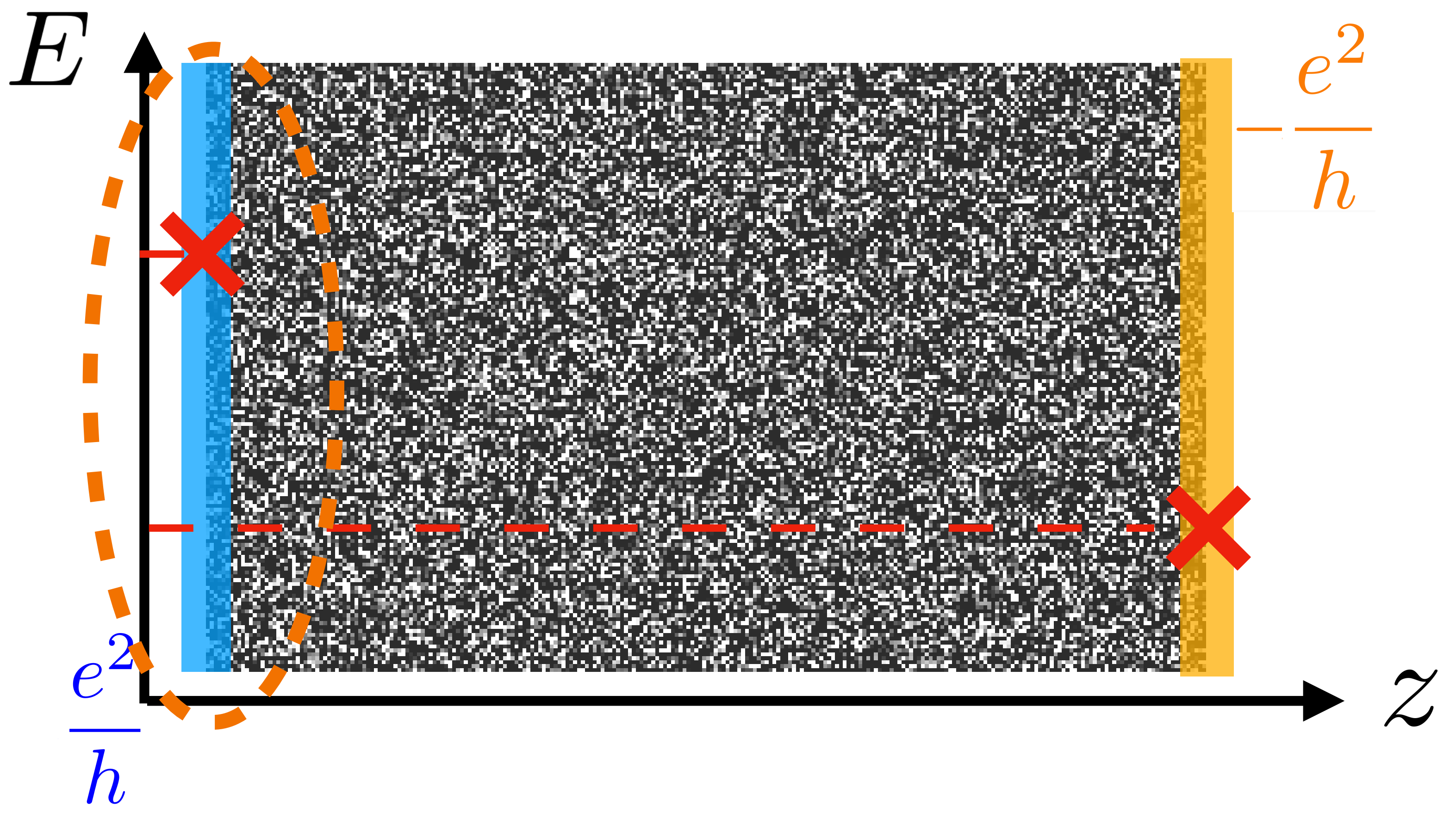


Topologically Localized Insulators



$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$ α_{ME} quantized to integer value

OBC in z-direction

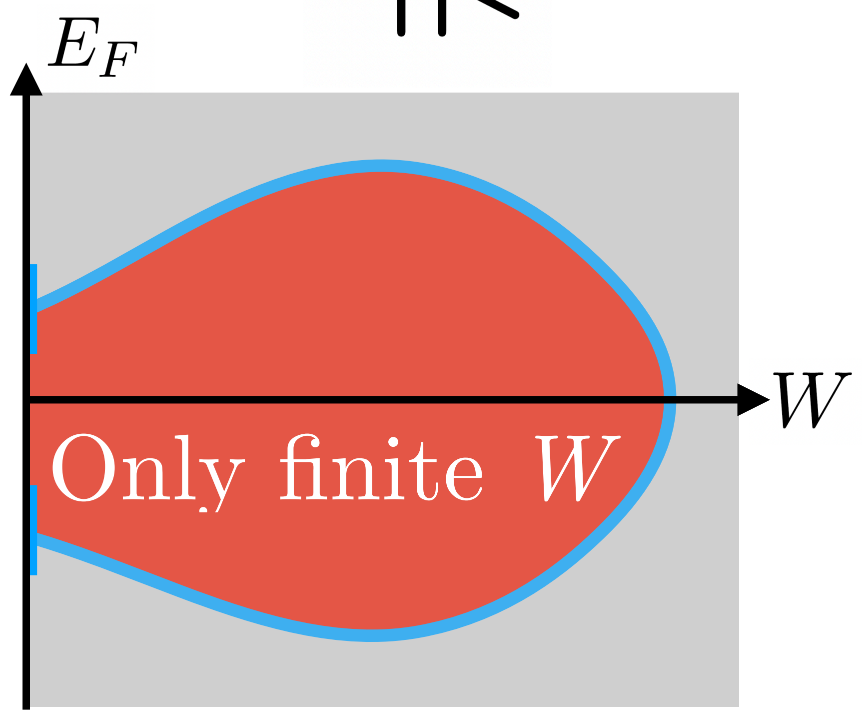


"Half" Chern insulator

Quantized Hall conductivity

$\sigma_{xy} = n\frac{e^2}{h}$ for any W

on boundary



Three properties of TLIs:

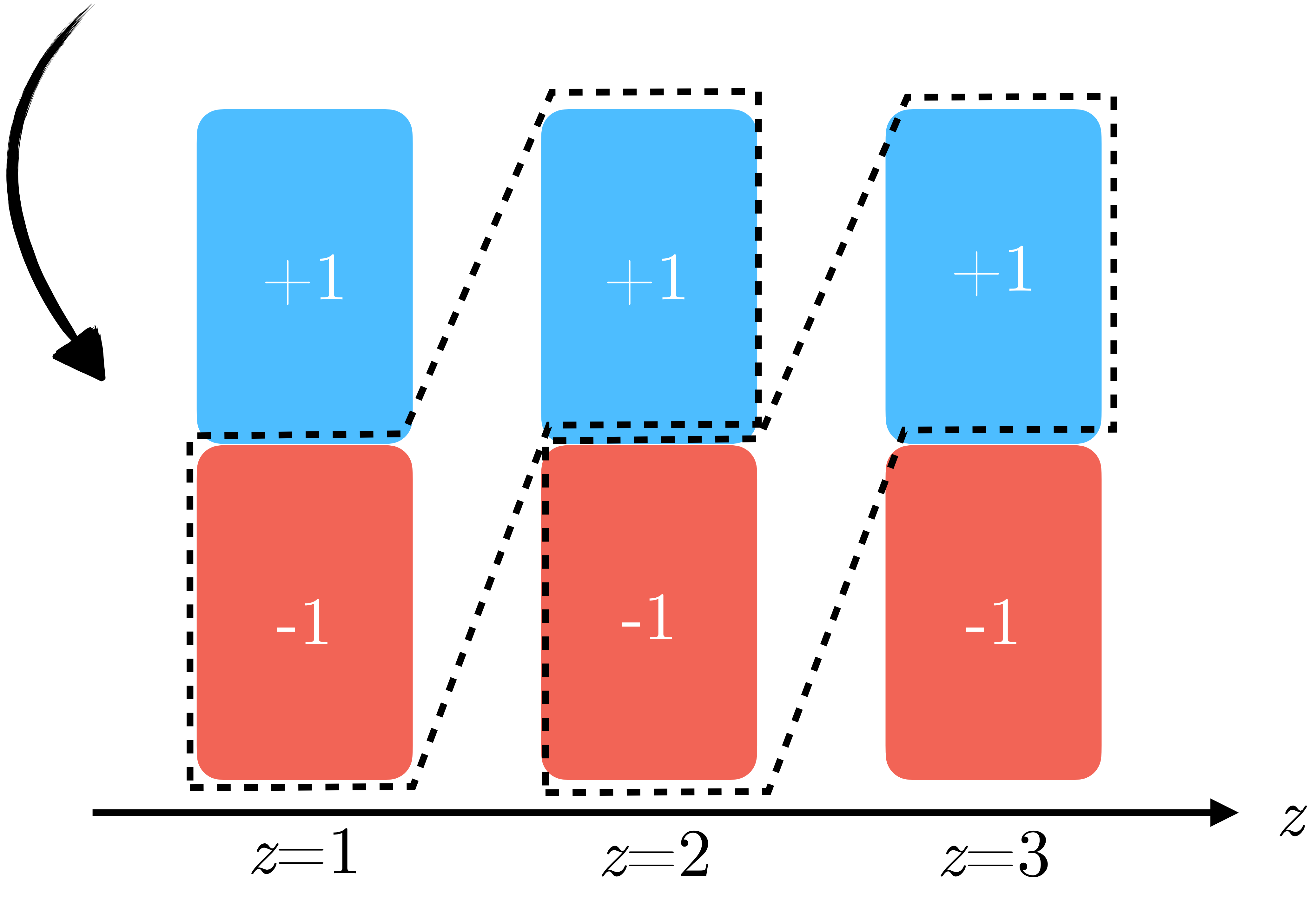
I) Bulk fully localized

II) Quantized magnetoelectric pol. tensor

III) Anomalous boundary with non-zero Chern number

A concrete TLI model

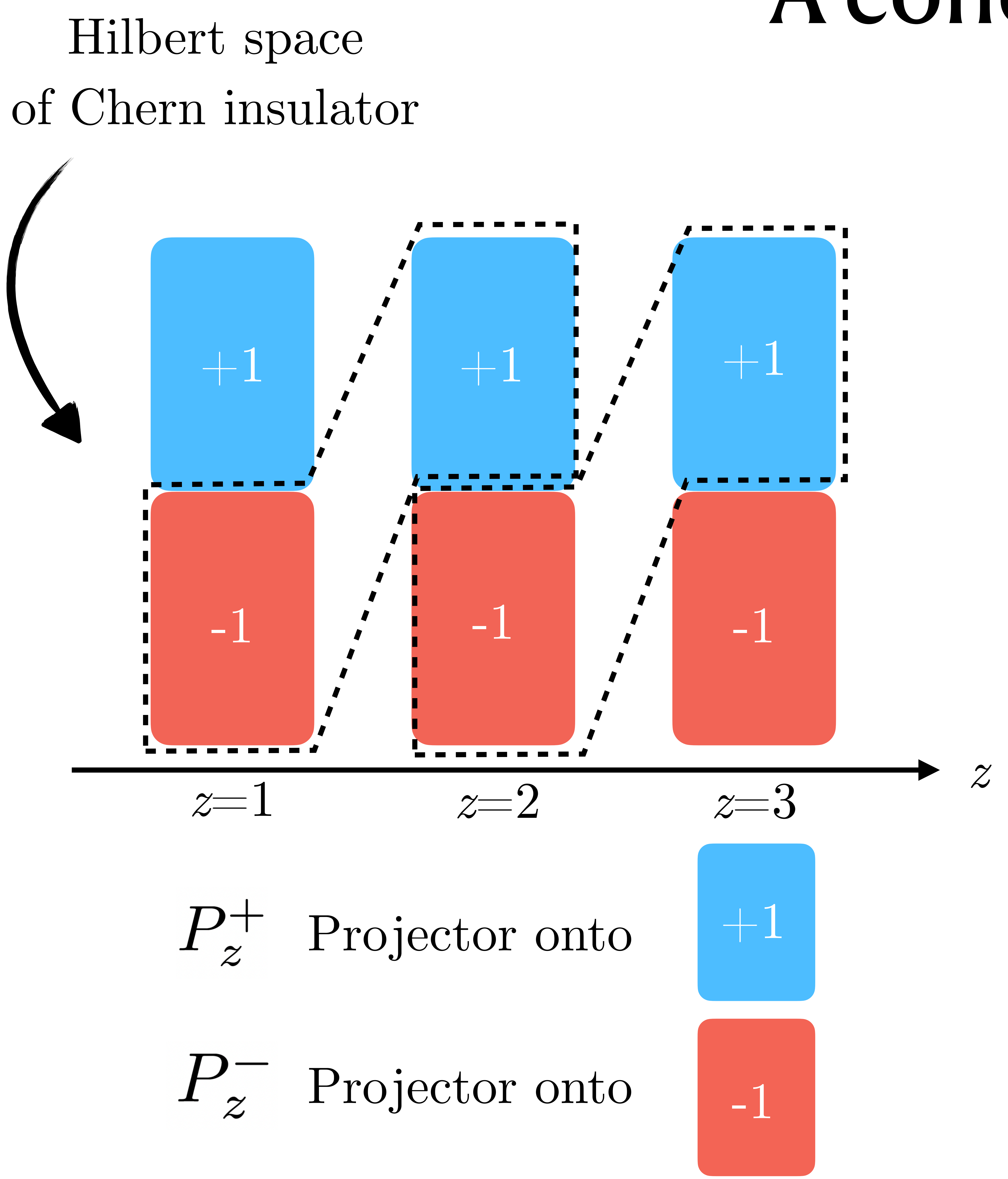
Hilbert space
of Chern insulator



P_z^+ Projector onto 

P_z^- Projector onto 

A concrete TLI model



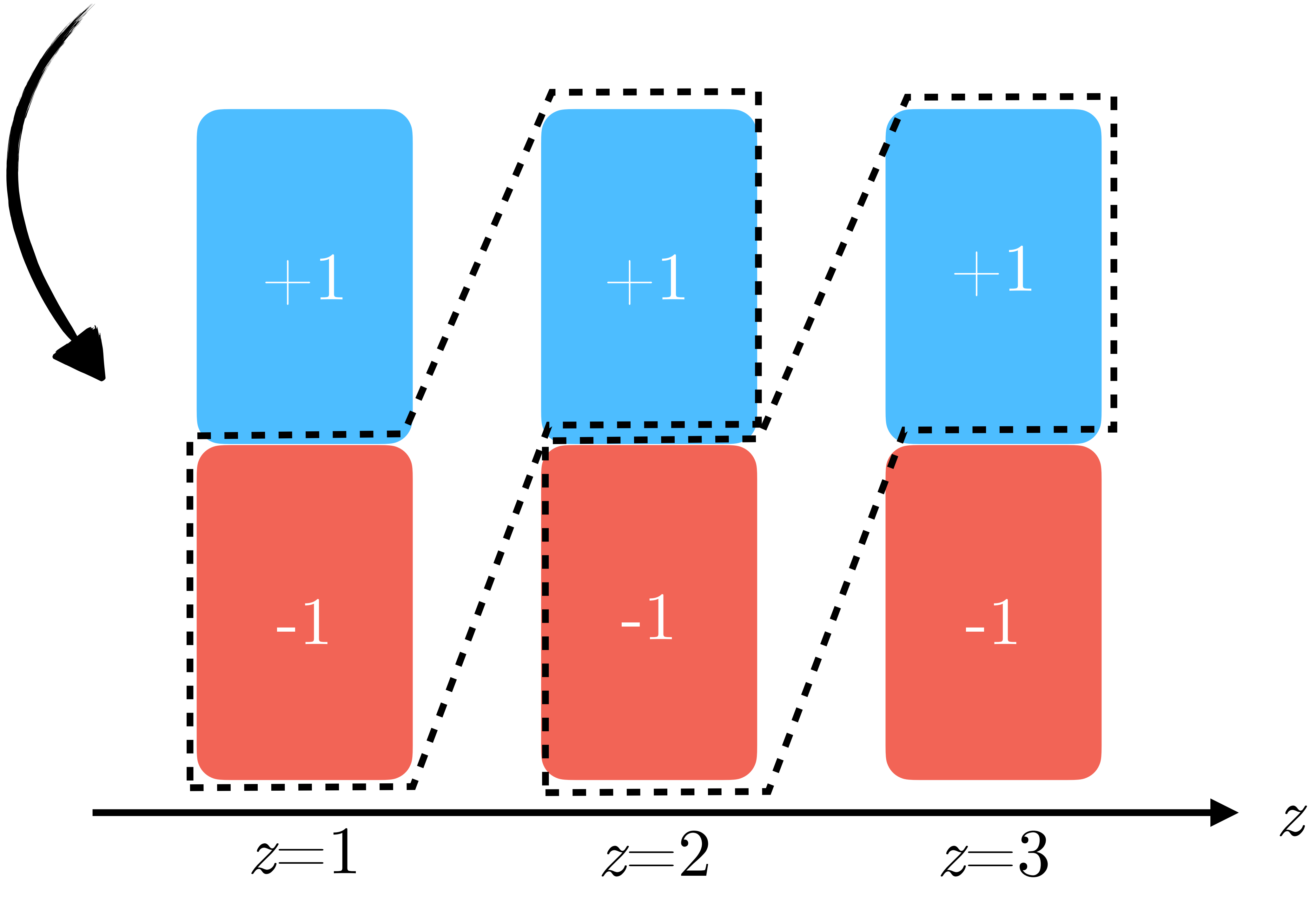
$$H = \sum_z (P_z^- + P_{z+1}^+) H_W (P_z^- + P_{z+1}^+)$$

$$H_W = \sum_{\vec{R},\alpha} W_{\vec{R}\alpha} \left|g_{\vec{R}\alpha}\right\rangle \left\langle g_{\vec{R}\alpha}\right| , \qquad W_{\vec{R}\alpha} \in [-W, W]$$

with $\left|g_{\vec{R}\alpha}\right\rangle = |\vec{R}\alpha\rangle + |(\vec{R}+\hat{e}_z)\alpha\rangle$

A concrete TLI model

Hilbert space of Chern insulator



P_z^+ Projector onto +1

P_z^- Projector onto -1

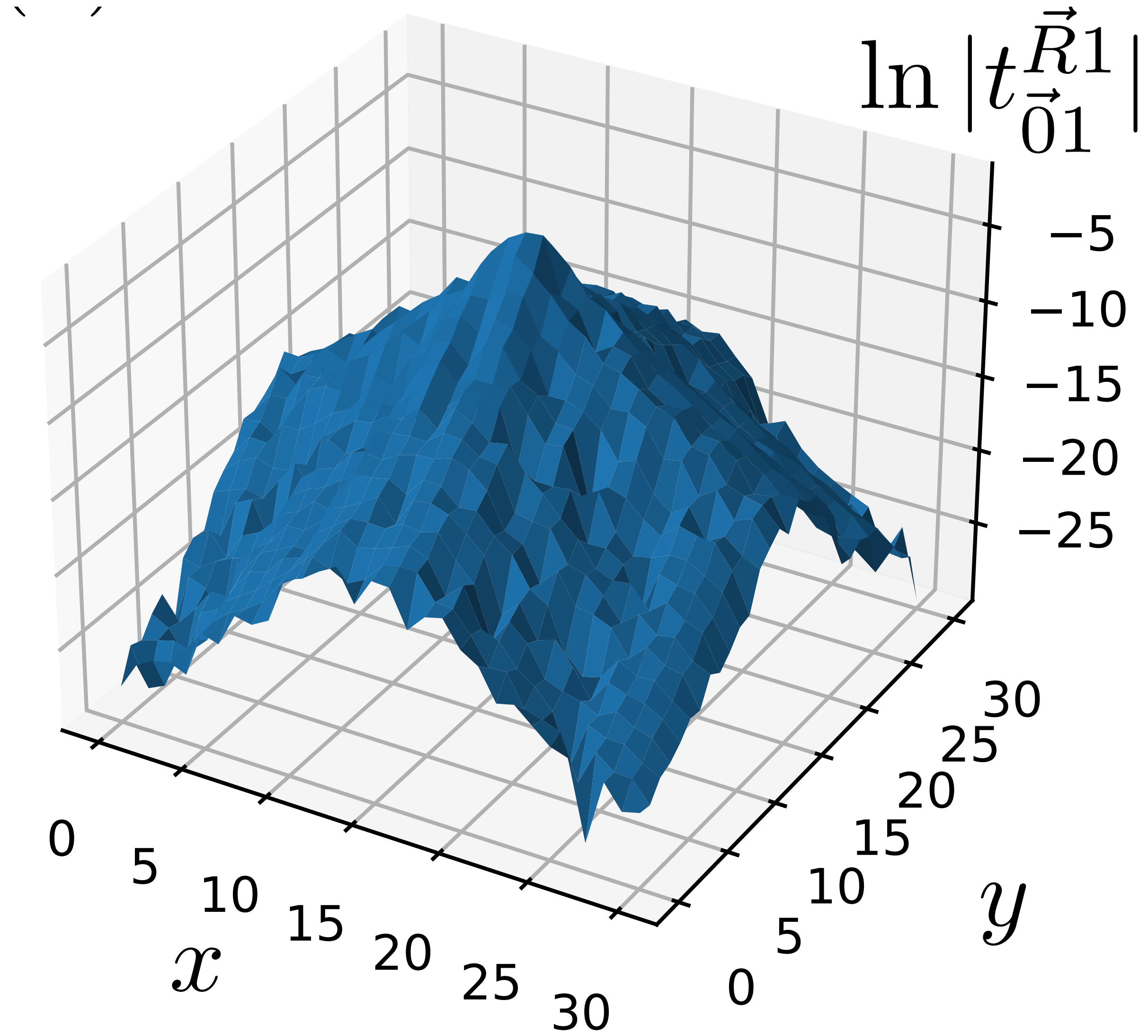
$$H = \sum_z (P_z^- + P_{z+1}^+) H_W (P_z^- + P_{z+1}^+)$$

$$H_W = \sum_{\vec{R},\alpha} W_{\vec{R}\alpha} \left|g_{\vec{R}\alpha}\right\rangle \left\langle g_{\vec{R}\alpha}\right| , \qquad W_{\vec{R}\alpha} \in [-W, W]$$

with $\left|g_{\vec{R}\alpha}\right\rangle = |\vec{R}\alpha\rangle + |(\vec{R}+\hat{e}_z)\alpha\rangle$

$$H = \sum_{\vec{R},\alpha,\vec{R}',\alpha'} t^{\vec{R}\alpha}_{\vec{R}'\alpha'} |\vec{R}\alpha\rangle\langle\vec{R}'\alpha'|$$

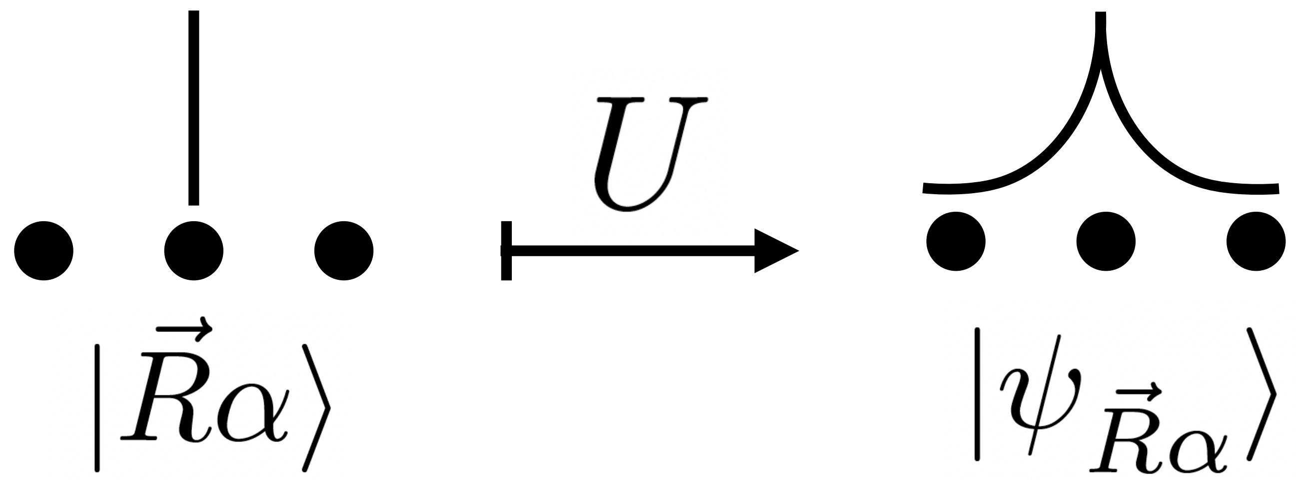
Exponentially decaying hoppings



Bulk and surface topology

Fully localized phase: Eigenstate $|\psi_{\vec{R}\alpha}\rangle$ localized around lattice vector $|\vec{R}\alpha\rangle$

U such that

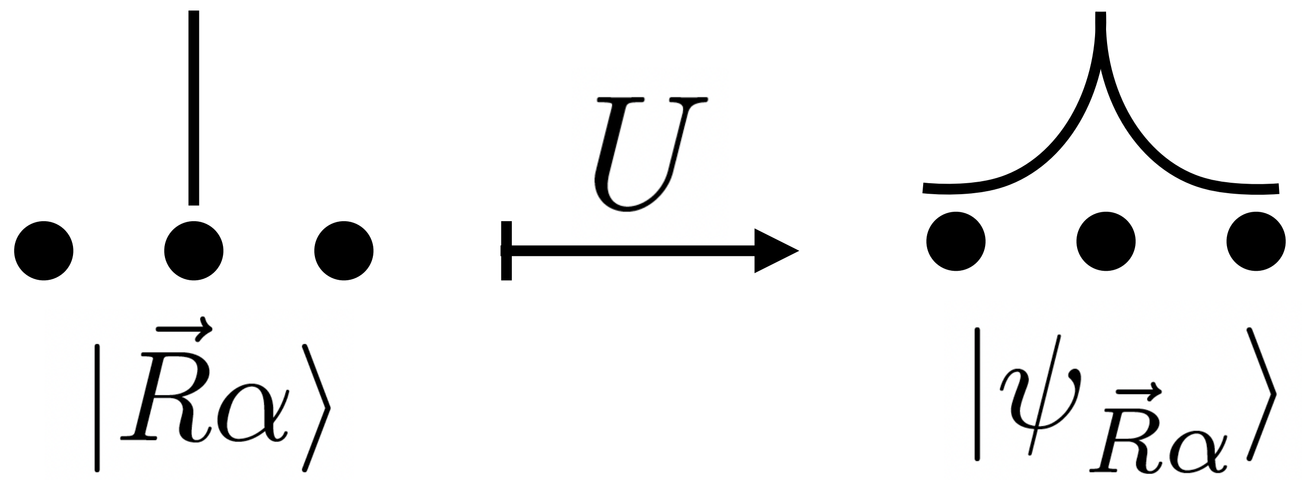


U is not unique!

Bulk and surface topology

Fully localized phase: Eigenstate $|\psi_{\vec{R}\alpha}\rangle$ localized around lattice vector $|\vec{R}\alpha\rangle$

U such that



U is not unique!

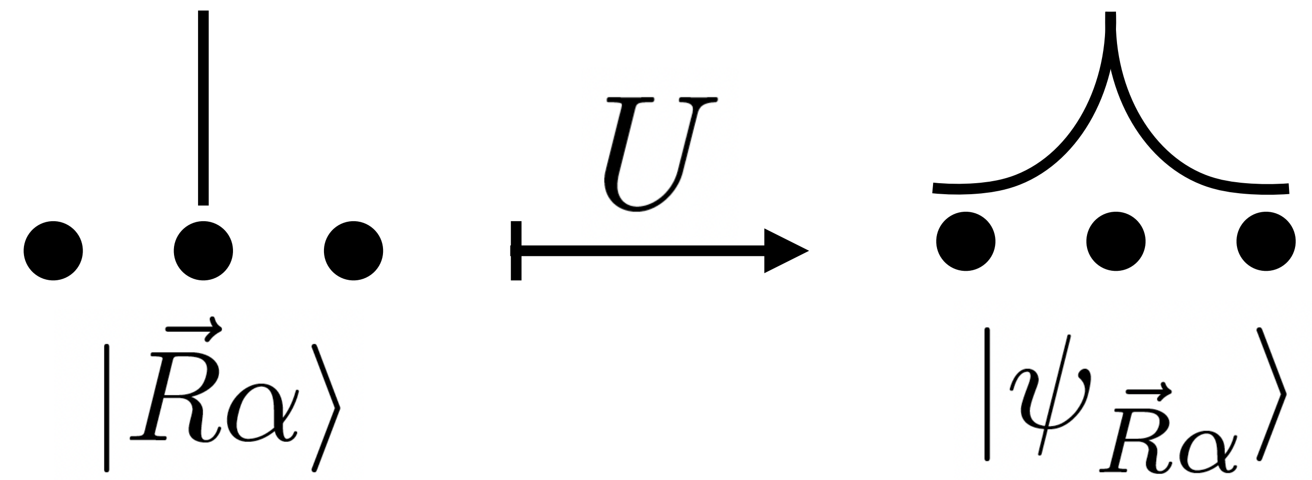
Bulk invariant: Third winding number of the unitary $\nu[U] = \alpha_{\mathrm{ME}}$

$$\nu[U] = \frac{i\pi}{3}\frac{1}{N_x N_y N_z}\epsilon^{ijk}\mathrm{tr}\left(U^{-1}[\hat{X}_i, U]U^{-1}[\hat{X}_j, U]U^{-1}[\hat{X}_k, U]\right) \in \mathbb{Z} \quad \text{if } \langle\vec{R}'\alpha'|U|\vec{R}\alpha\rangle \sim e^{-\gamma|\vec{R}-\vec{R}'|}$$

Bulk and surface topology

Fully localized phase: Eigenstate $|\psi_{\vec{R}\alpha}\rangle$ localized around lattice vector $|\vec{R}\alpha\rangle$

U such that



U is not unique!

Bulk invariant: Third winding number of the unitary $\nu[U] = \alpha_{\mathrm{ME}}$

$$\nu[U] = \frac{i\pi}{3}\frac{1}{N_x N_y N_z}\epsilon^{ijk}\mathrm{tr}\left(U^{-1}[\hat{X}_i, U]U^{-1}[\hat{X}_j, U]U^{-1}[\hat{X}_k, U]\right) \in \mathbb{Z} \;\text{ if } \langle \vec{R}'\alpha'|\, U\, |\vec{R}\alpha\rangle \sim e^{-\gamma|\vec{R}-\vec{R}'|}$$

Surface invariant: Chern number of the projector onto d.o.f of the surface

$$\mathrm{Ch}[\mathcal{P}] = \frac{2\pi i}{N_x N_y}\mathrm{Tr}\left(\mathcal{P}\left[\left[\hat{X}_1, \mathcal{P}\right], \left[\hat{X}_2, \mathcal{P}\right]\right]\right)$$

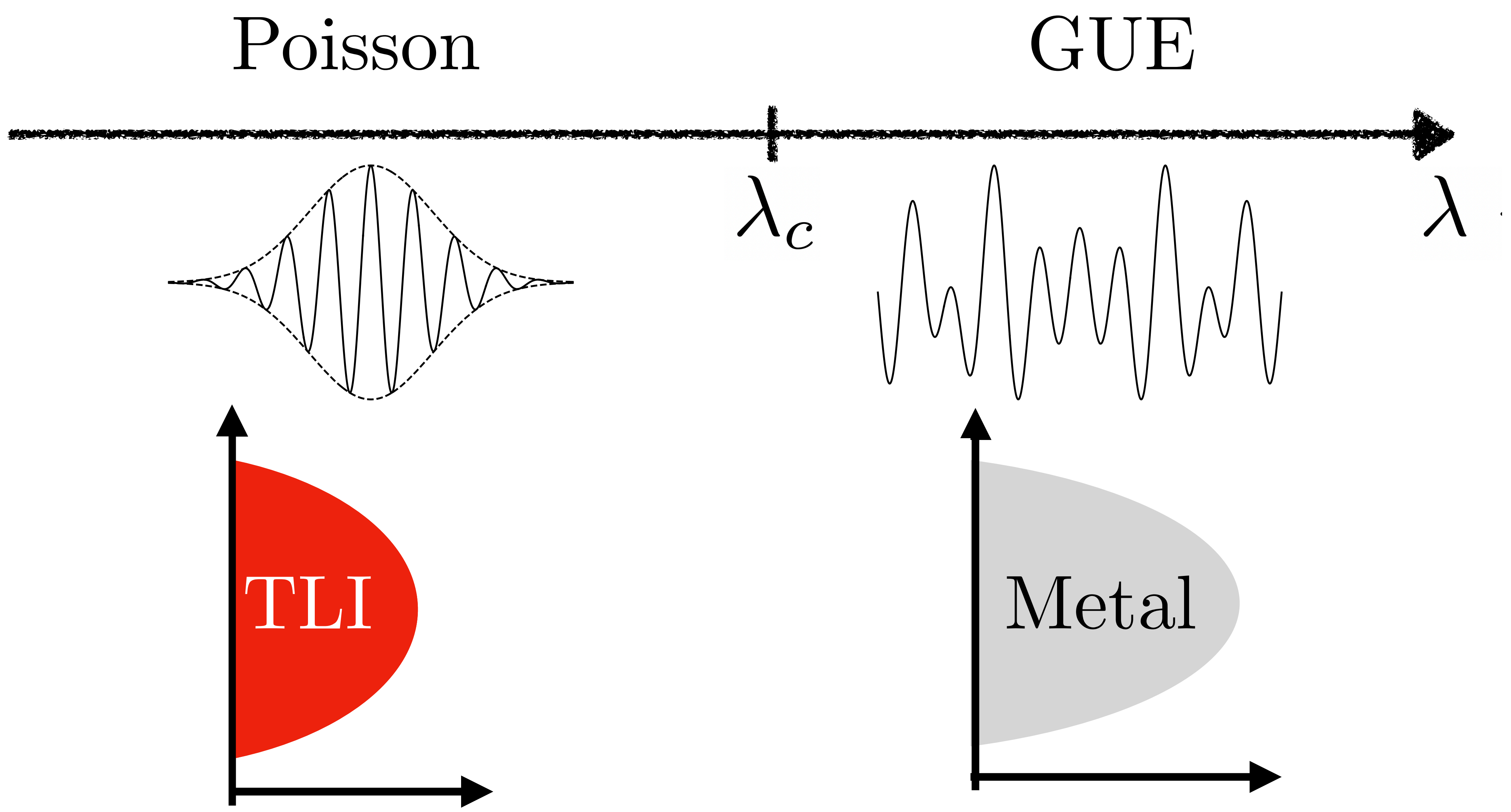
Bulk fully localized $\rightarrow$ surfaces well decoupled

TLI to metal transition

Nearest neighbour hopping perturbation $H_V(\lambda) = H + \lambda V$ → 3D Anderson transition

TLI to metal transition

Nearest neighbour hopping perturbation $H_V(\lambda) = H + \lambda V$ → 3D Anderson transition



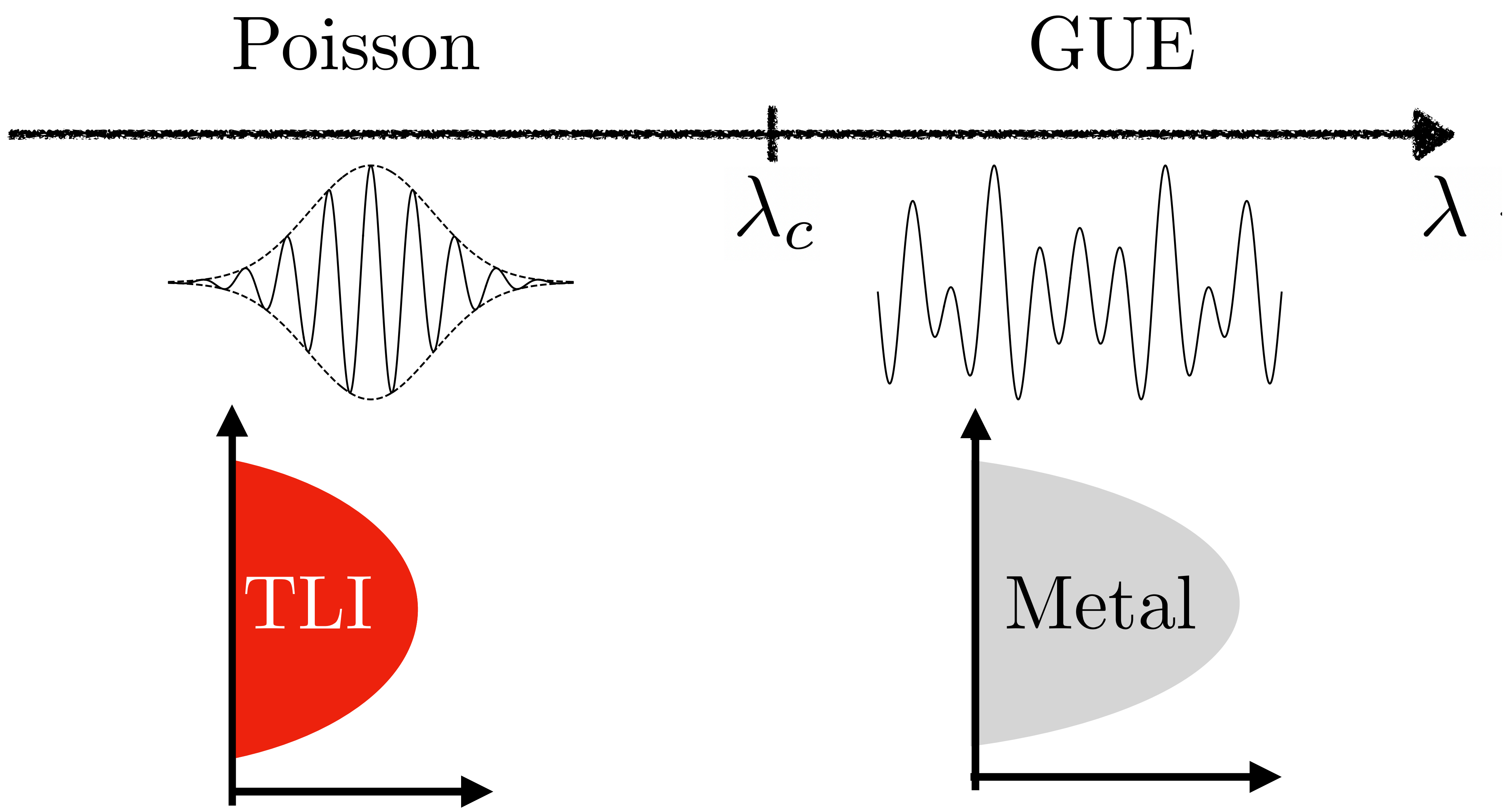
TLI to metal transition

Nearest neighbour hopping perturbation $H_V(\lambda) = H + \lambda V$ → 3D Anderson transition

Level spacing statistics

$s_n = E_{n+1} - E_n$ → Level spacing ratio $r_n = \min\{s_n, s_{n+1}\}/\max\{s_n, s_{n+1}\}$

Averaged $r = \langle\langle r_n \rangle_n\rangle_W$



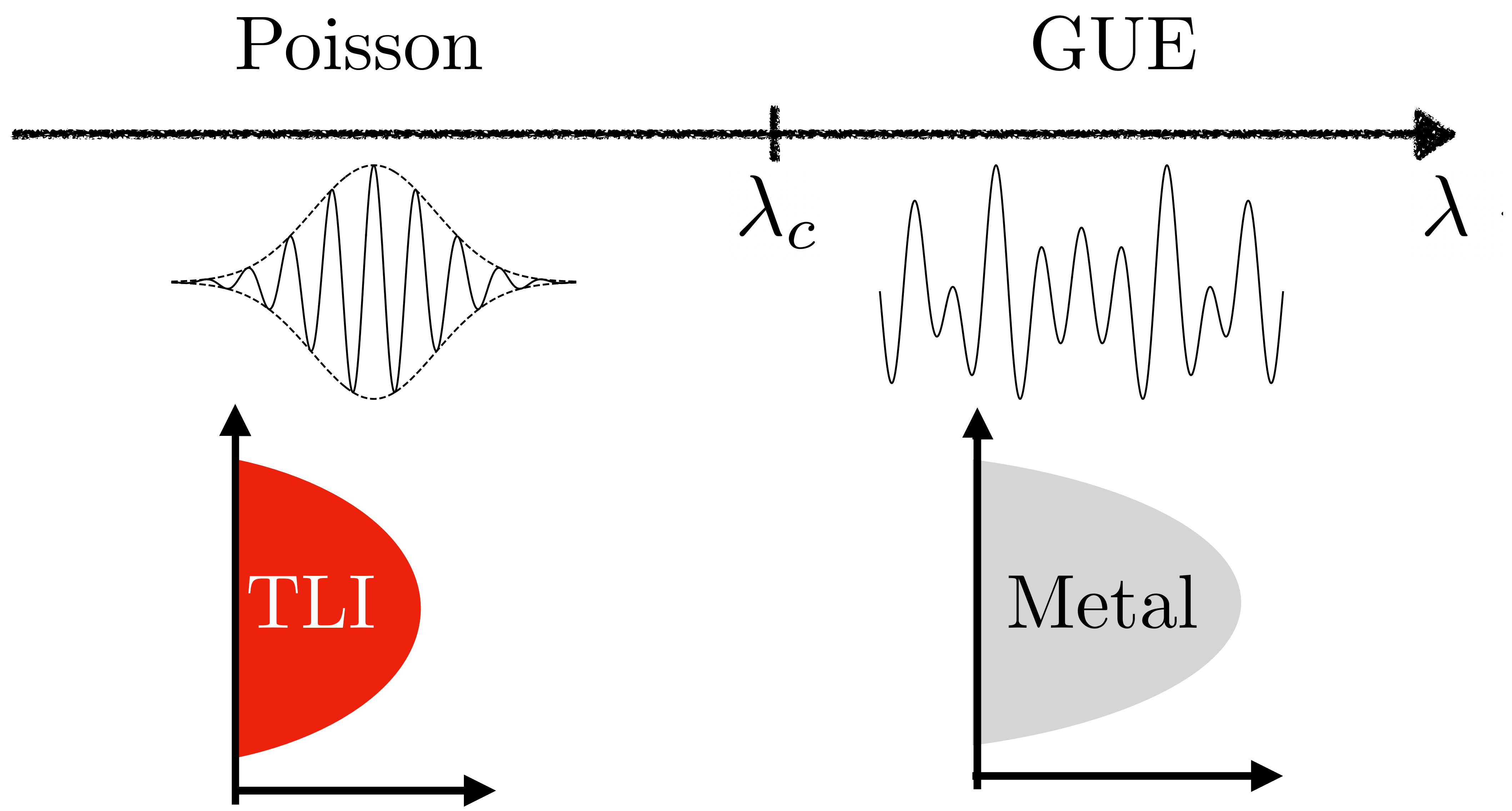
TLI to metal transition

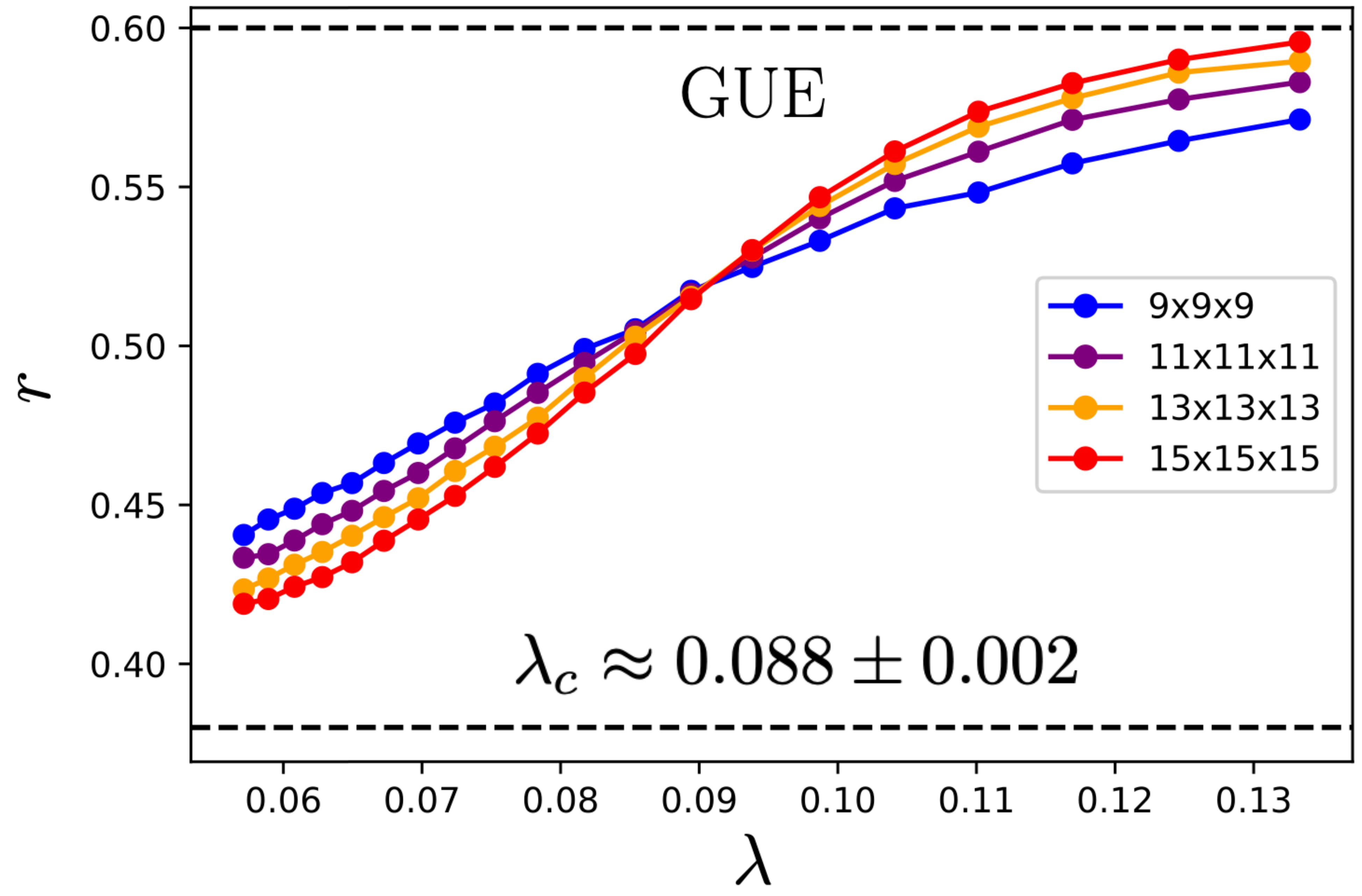
Nearest neighbour hopping perturbation $H_V(\lambda) = H + \lambda V$ → 3D Anderson transition

Level spacing statistics

$s_n = E_{n+1} - E_n$ → Level spacing ratio $r_n = \min\{s_n, s_{n+1}\}/\max\{s_n, s_{n+1}\}$

Averaged $r = \langle\langle r_n\rangle_n\rangle_W$





TLI to metal transition

Nearest neighbour hopping $H_V(\lambda) = H + \lambda V$

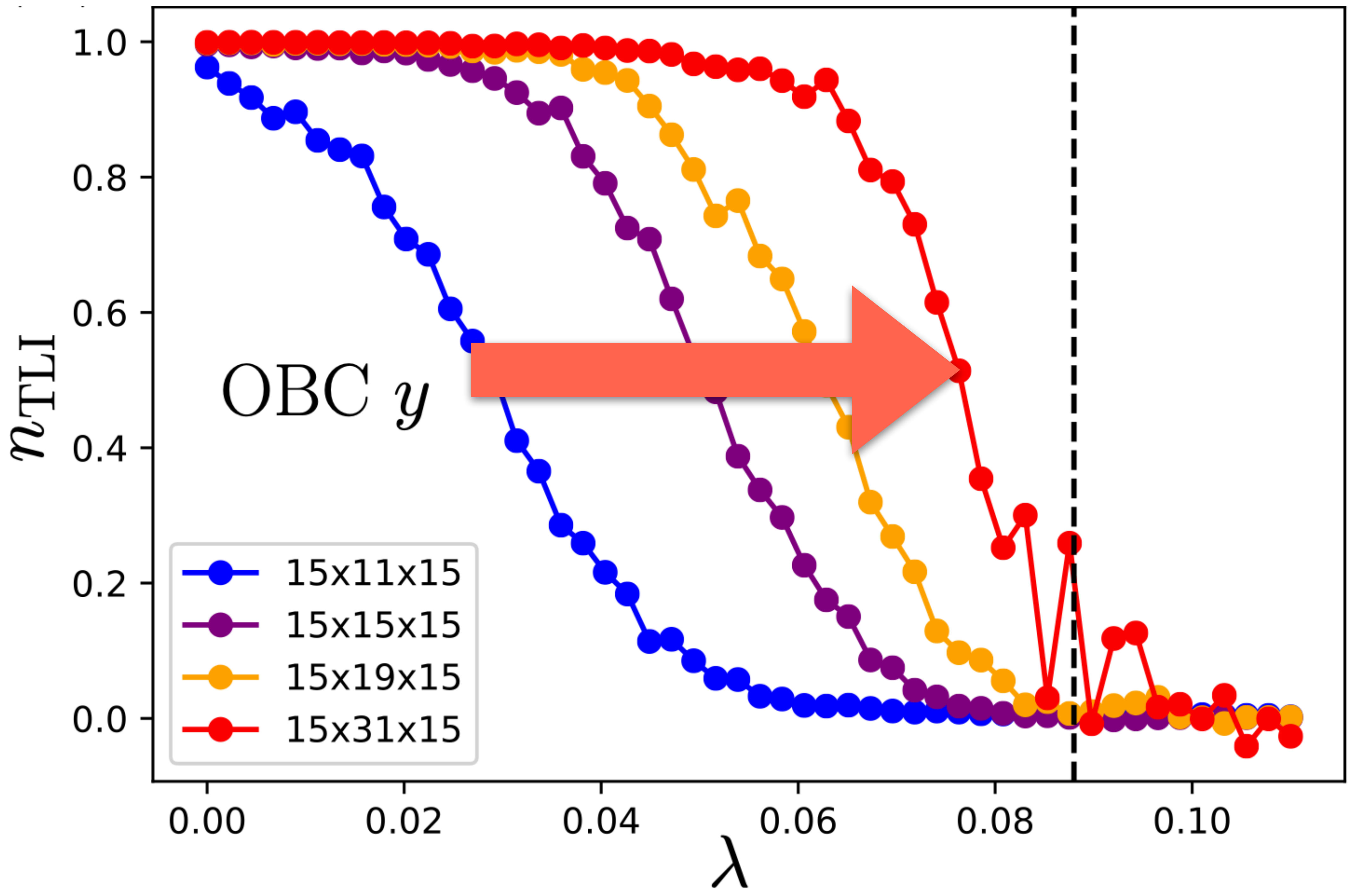
3D Anderson transition: if $\lambda < \lambda_c$, localized phase

TLI to metal transition

Nearest neighbour hopping $H_V(\lambda) = H + \lambda V$

3D Anderson transition: if $\lambda < \lambda_c$, localized phase

Surface Chern number $n_{\text{TLI}} = \langle \text{Ch}[\mathcal{P}_W^{\text{surf}}] \rangle_W$

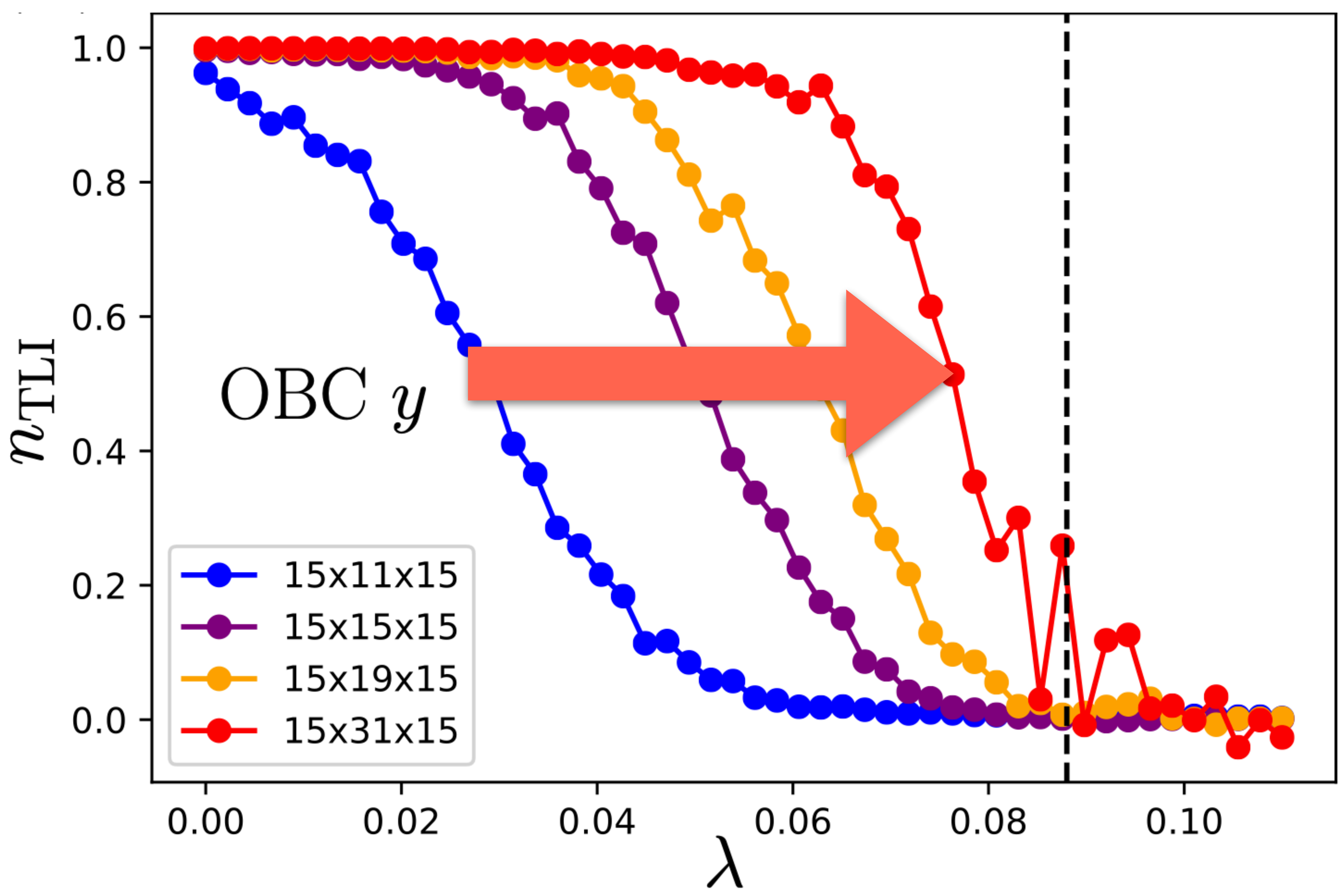


TLI to metal transition

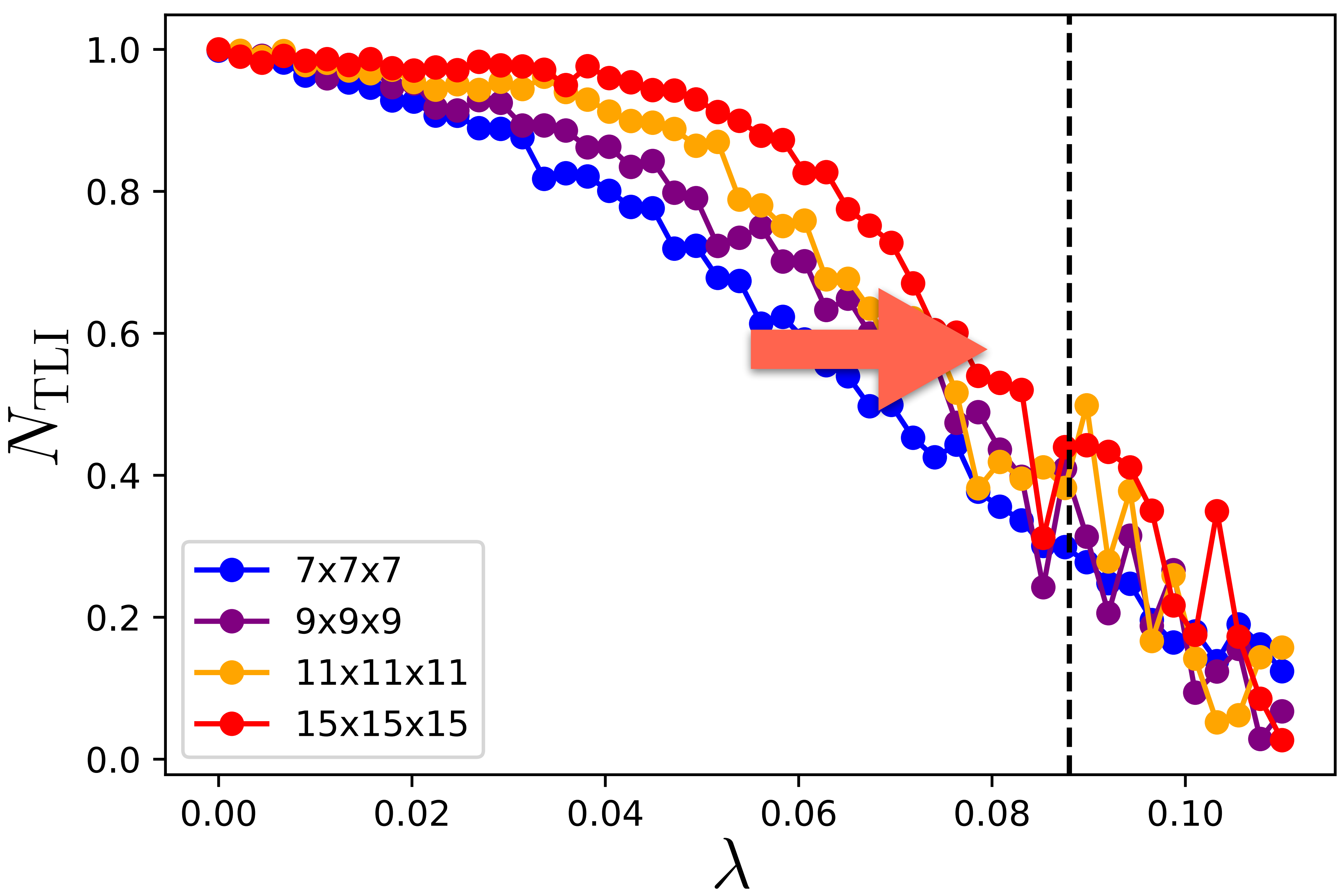
Nearest neighbour hopping $H_V(\lambda) = H + \lambda V$

3D Anderson transition: if $\lambda < \lambda_c$, localized phase

Surface Chern number $n_{\mathrm{TLI}} = \langle \mathrm{Ch}[\mathcal{P}_W^{\mathrm{surf}}] \rangle_W$



Bulk invariant $N_{\mathrm{TLI}} = \langle \nu[U_W] \rangle_W$



Classification of TLIs in class A

Idea: associate a chiral Hamiltonian to the local unitary U $\Rightarrow$ $\tilde{H} = \begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}$ Class AIII

Up to (local) permutations P of the eigenvectors and phase matrix D (Not affecting TLI invariant)

***B.L**, T. Neupert, P. Brouwer, L. Trifunovic, To appear*

Classification of TLIs in class A

Idea: associate a chiral Hamiltonian to the local unitary U $\Rightarrow$ $\tilde{H} = \begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}$ Class AIII

Up to (local) permutations P of the eigenvectors and phase matrix D (Not affecting TLI invariant)

TLI classifying group is $K_{\mathrm{A}}^{\mathrm{TLI}}(d) = K_{\mathrm{AIII}}(d)/K'_{\mathrm{A}\to\mathrm{AIII}}(d)$

$$\tilde{H}' = \begin{pmatrix} 0 & DP \\ P^\dagger D^\dagger & 0 \end{pmatrix}$$

(Dimer Hamiltonian)

***B.L**, T. Neupert, P. Brouwer, L. Trifunovic, To appear*

Classification of TLIs in class A

Idea: associate a chiral Hamiltonian to the local unitary U $\Rightarrow$ $\tilde{H} = \begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}$ Class AIII

Up to (local) permutations P of the eigenvectors and phase matrix D (Not affecting TLI invariant)

TLI classifying group is $K_{\mathrm{A}}^{\mathrm{TLI}}(d) = K_{\mathrm{AIII}}(d)/K'_{\mathrm{A}\to\mathrm{AIII}}(d)$ $\tilde{H}' = \begin{pmatrix} 0 & DP \\ P^\dagger D^\dagger & 0 \end{pmatrix}$

(Dimer Hamiltonian)

$$K_{\mathrm{A}}^{\mathrm{TLI}}(d) = \begin{cases} 0 & \text{if } d = 0 \text{ or } d = 1, \\ K_{\mathrm{AIII}}(d) & \text{if } d > 1. \end{cases}$$

***B.L**, T. Neupert, P. Brouwer, L. Trifunovic, To appear*

Classification of TLIs in class A

Idea: associate a chiral Hamiltonian to the local unitary U → $\tilde{H} = \begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}$ Class AIII

Up to (local) permutations P of the eigenvectors and phase matrix D (Not affecting TLI invariant)

TLI classifying group is $K_{\mathrm{A}}^{\mathrm{TLI}}(d) = K_{\mathrm{AIII}}(d)/K'_{\mathrm{A}\to\mathrm{AIII}}(d)$

$$\tilde{H}' = \begin{pmatrix} 0 & DP \\ P^\dagger D^\dagger & 0 \end{pmatrix}$$

(Dimer Hamiltonian)

$$K_{\mathrm{A}}^{\mathrm{TLI}}(d) = \begin{cases} 0 & \text{if } d = 0 \text{ or } d = 1, \\ K_{\mathrm{AIII}}(d) & \text{if } d > 1. \end{cases}$$

→ Non-trivial TLIs in $d = 3,\ 5,\ 7,\ 9,\ etc.$

***B.L**, T. Neupert, P. Brouwer, L. Trifunovic, To appear*

Classification of topologically localized phases

Topologically localized insulators and superconductors (not captured by the tenfold way):

AZ	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	1	2	3	4	5	6	7	8
A	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	1	1	1	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0
DIII	-1	1	1	0	0	0	0	$\mathbb{Z}$	0	0	0
AII	-1	0	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	-1	-1	1	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	0	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0
CI	1	-1	1	0	0	0	0	$2\mathbb{Z}$	0	0	0

***B.L**, T. Neupert, P. Brouwer, L. Trifunovic, To appear*