The ABJM Hagedorn temperature

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What is the Hagedorn temperature?

The Hagedorn temperature is the lowest temperature at which the ABJM/AdS_4-string thermal partition function diverges. At this temperature the lightest string mode winding the thermal circle becomes tachyonic. This signals a deconfinement phase transition from a "string star" to a thermal string gas [1, 2].

Gauge theory

From the gauge theory side the Hagedorn temperature was computed to the first subleading order by [3]. It is computed from the two-loop dilatation operator for the ABJM theory by

$$\frac{\delta T_H}{T_H} = \sqrt{2}\lambda^2 \langle D_2(x_H)\rangle. \qquad (1)$$

The result is

$$T_H = \frac{1}{4\log(1+\sqrt{2})}\left(1 + 4(\sqrt{2}-1)\lambda^2 + \mathcal{O}(\lambda^4)\right). \qquad (2)$$

String theory

At leading order the condition for the lightest string mode around the thermal circle becoming tachyonic can be computed in flat space:

$$m^2 = \left(\frac{\beta}{2\pi\alpha'}\right)^2 - \frac{2}{\alpha'} \quad \Rightarrow \quad T_H = \frac{1}{\sqrt{8\pi^2\alpha'}}. \qquad (3)$$

To compute higher orders, curvature corrections have to be included. Expanding the zero point energy as $-\frac{2}{\alpha'} + \frac{\beta^2}{2\pi^2\alpha'}\Delta c + \mathcal{O}(\alpha')$ and treating the winding string mode as a scalar in a supergravity background, we can get three subleading terms ($\alpha' = \frac{1}{\pi\sqrt{2\lambda}}$)

$$T_H = \frac{(\lambda/2)^{1/4}}{2\sqrt{\pi}} + \frac{3}{8\pi} + \frac{3+2\Delta c}{8\pi^{3/2}}\left(\frac{\lambda}{2}\right)^{-1/4} + \frac{165}{512\pi^2}\left(\frac{\lambda}{2}\right)^{-1/2} + \mathcal{O}((\alpha')^{3/2}). \qquad (4)$$

In the pp-wave limit one can evaluate Δc to be $-\frac{15}{36}\log(2) \approx -0.288812$ [4].

QSC basics for AdS_4 Hagedorn

In the symmetric sector the finite difference equation we solve is [5]

$$Q^{+}_{a|i} + \mathbf{P}_{ab}\kappa^{bc}Q^{-}_{c|j}\mathbb{K}^{j}_{i} = 0. \qquad (5)$$

For the asymptotics we make the Ansatz inspired by [6] ($y = (-\exp(-\frac{1}{2T_H}))$)

$$\mathbf{P}_A \simeq \begin{pmatrix} u \\ 1 \\ u^4 \\ u^3 \\ u^2 \\ u^2 \end{pmatrix}, \qquad Q_{a|i} \simeq \begin{pmatrix} y^{iu} & u & 1 & y^{-iu} \\ y^{iu}u^2 & u^3 & u^2 & y^{-iu}u^2 \\ y^{iu}u & u^2 & u & y^{-iu}u \\ y^{iu}u^3 & u^4 & u^3 & y^{-iu}u^3 \end{pmatrix}. \qquad (6)$$

The $\mathbf{P}_A$ have a short square-root cut on the real axis, while the functions $\mathbf{Q}_I = -\frac{1}{2}(\overline{\Sigma}_I)^{ij}Q^{a+}{}_{|i}P_{ab}Q^{a+}{}_{|i}$ have a long cut and the $Q_{a|i}$ are upper-half-plane analytic. Using parity we construct the analytically continued $\mathbf{Q}$-functions

$$\widetilde{\mathbf{Q}}_I(u) = \mathcal{L}_I^{J}(u)\mathbf{Q}_J(-u). \qquad (7)$$

From the asymptotics of the $\mathbf{Q}$, $\mathcal{L}$ is fixed to be upper triangular.

References

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[5] D. Bombardelli, A. Cavaglià, D. Fioravanti, N. Gromov and R. Tateo, The full Quantum Spectral Curve for AdS_4/CFT_3, JHEP 09 (2017) 140 [arXiv:1701.00473].

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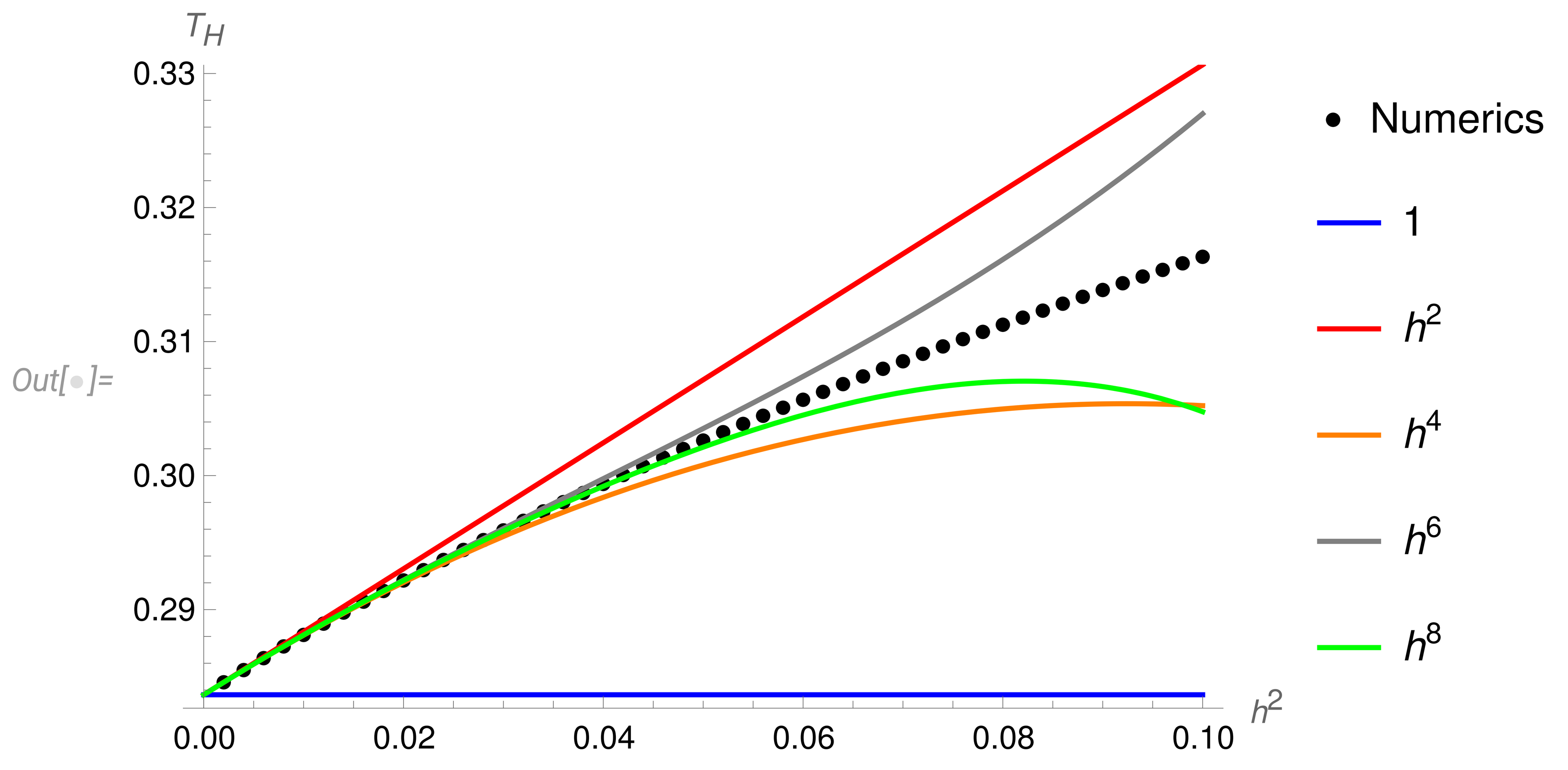
Perturbative solution at weak coupling

Starting from a perturbative Ansatz for $\mathbf{P}_A$ we solve order by order in h^2 equation (5) for $Q_{a|i}$. To completely fix the solution we impose gauge constraints and gluing at the cut for the $\mathbf{Q}_I$:

$$\frac{\widetilde{\mathbf{Q}}_I - \mathbf{Q}_I}{x - \frac{1}{x}} = \text{regular at the cut.} \qquad (8)$$

This determines the Hagedorn temperature perturbatively and we find

$$T_H = \frac{1}{4\log(1+\sqrt{2})} + \frac{(\sqrt{2}-1)}{\log(1+\sqrt{2})}h^2 + \left(-4(1+2\sqrt{2})\mathrm{Li}(1)\left(\frac{1}{(1+\sqrt{2})^2}\right) - \frac{2(1+2\sqrt{2})\mathrm{Li}(2)\left(\frac{1}{(1+\sqrt{2})^2}\right)}{\log(1+\sqrt{2})} + 7\sqrt{2} - 8\right)h^4 + (21.77821\ldots)h^6 - (222.29969\ldots)h^8 + \mathcal{O}(h^{10}). \qquad (9)$$



Numerical solution for strong coupling

We use the truncated Ansatz

$$\mathbf{P}_A = A_A(xh)^{r_A}\left(1 + \sum_{n=1}^{K} c_{A,2n-r_A}\frac{h^{2(n-r_A)}}{x^{2n}}\right), \qquad Q_{a|i} = y^{s_i iu}u^{p_{a|i}}\sum_{n=0}^{N} b^{(n)}_{a|i}u^{-n} \qquad (10)$$

and the gluing matrix

$$\mathcal{L}_I^{J} = \begin{pmatrix} e^{-2\pi u} & 0 & -16i\sinh\left(\frac{1}{2T_H}\right)^3 & 0 & 0 \\ 0 & -e^{-2\pi u} & 0 & 16i\sinh\left(\frac{1}{2T_H}\right)^3 & 0 \\ 0 & 0 & -e^{2\pi u} & 0 & 0 \\ 0 & 0 & 0 & e^{2\pi u} & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \qquad (11)$$

To get the Hagedorn temperature we numerically minimize the function:

$$F(\{c_{A,n}\}, y) = \sum_{v\in I}\sum_{J=3}^{4}\left(\left|\frac{\mathbf{Q}_J(v+i0)}{\widetilde{\mathbf{Q}}_J(v-i0)} - 1\right|^2 + \left|\frac{\widetilde{\mathbf{Q}}_J(v+i0)}{\mathbf{Q}_J(v-i0)} - 1\right|^2\right). \qquad (12)$$

Curve fitting to the results of the numerics, we find:

$$T_H(\lambda) = \frac{\lambda^{1/4}}{2^{5/4}\sqrt{\pi}} + \frac{3}{8\pi} - (0.0308 \pm 0.0004)\lambda^{-1/4} + (0.046 \pm 0.002)\lambda^{-1/2} - (0.020 \pm 0.005)\lambda^{-3/4} + \mathcal{O}(\lambda^{-1}). \qquad (13)$$

We get that for AdS_4

$$\Delta c = -2.0786 \pm 0.0015 \approx -3\log(2) = -2.0794\ldots. \qquad (14)$$

For AdS_5 we find $\Delta c \approx -4\log(2)$.