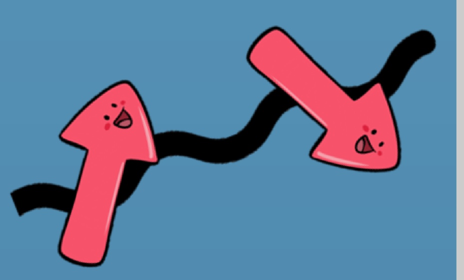

The Hubbard model and beyond

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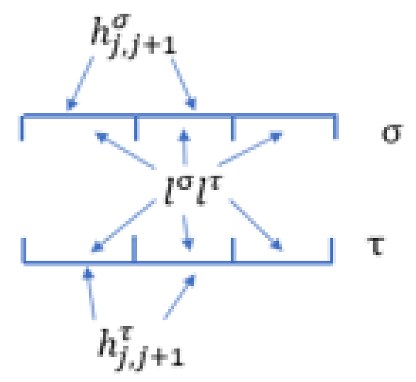
Based on 2301.01612 and 2305.01922

First range 3 elliptic deformation of the Hubbard model

$$\mathbb{H}_3 = \sum_j h^{\sigma}_{j,j+1} + h^{\tau}_{j,j+1} + U l^{\sigma}_{j,j+1,j+2} l^{\tau}_{j,j+1,j+2},$$
$$h_{j,j+1} = i(\sigma^+_j \sigma^-_{j+1} - \sigma^-_j \sigma^+_{j+1})$$
$$l_{j,j+1,j+2} = Z_{j+1} + \kappa\,(X_j + X_{j+2})\,X_{j+1} - \kappa^2 X_j Z_{j+1} X_{j+2}$$

U coupling constant κ deformation parameter



Where is the Hubbard model?

$\kappa = 0$ Hubbard model

Jordan-Wigner transformation $\sigma \to c^{\uparrow}$ $\tau \to c^{\downarrow}$

$$\mathbb{H}_{Hub} = \sum_j \left[(c^{\uparrow}_j)^{\dagger} c^{\uparrow}_{j+1} + (c^{\uparrow}_{j+1})^{\dagger} c^{\uparrow}_j + (c^{\downarrow}_j)^{\dagger} c^{\downarrow}_{j+1} + (c^{\downarrow}_{j+1})^{\dagger} c^{\downarrow}_j + \frac{U}{4}(1 - 2n^{\uparrow}_j)(1 - 2n^{\downarrow}_j) \right],$$

$n = c^{\dagger} c$, $c^{\dagger}$, c fermionic creation and annihilation operators

Open quantum system interpretations: general mapping

Map between Hermitian $\mathbb{H}$ and Lindblad superoperator

$$\mathbb{H} = \sum_j h^{\sigma} + h^{\tau} + U l^{\sigma} l^{\tau}$$

Conditions: $h = -h^T$, $l = l^* = l^T$, $l^2 = l^{\dagger} l = \mathbb{I}$

Complexify $U \to i\,U$

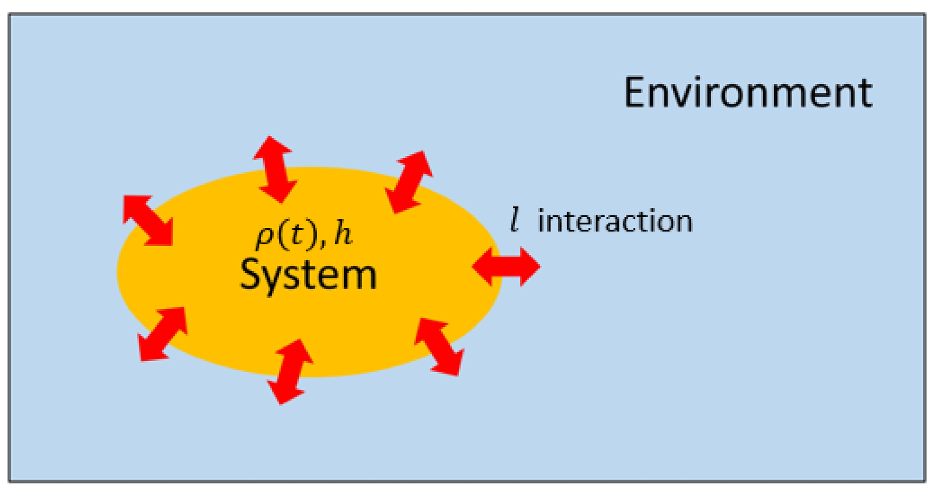
$$\mathbb{H} \;\to\; \mathcal{L} = -i h^{\sigma}_{j,j+1} + i h^{\tau T} + U\left(l^{\sigma} l^{\tau *} - \frac{1}{2} l^{\sigma\dagger} l^{\sigma} - \frac{1}{2} l^{\tau T} l^{\tau *} \right),$$

Lindblad superoperator

Approximation:

- Markovian evolution

$\tau_{environment} \ll \tau_{system}$

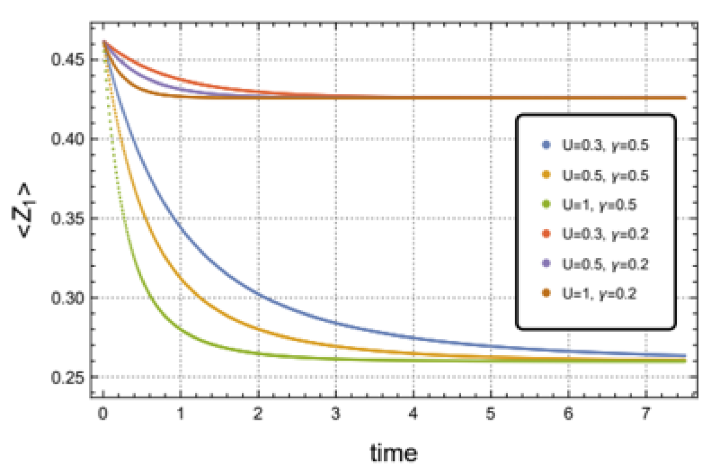


$$\dot{\rho}(t) = \mathcal{L}\rho(t) \;\to\; \underbrace{\dot{\rho}(t) = i[\rho, h]}_{\text{von-Neumann equation}} + U \underbrace{\left[\ell \rho \ell^{\dagger} - \frac{1}{2}\{\ell^{\dagger}\ell, \rho\} \right]}_{\text{dissipator}}$$

Physical properties

Hidden "strong symmetries" Q : $[Q,h] = [Q,l] = 0$

	Q	# NESS	NESS
$\boldsymbol{\kappa = 0}$	$Q_0 = \sum_i Z\_i$	$L+1$	$\rho_0 \sim e^{\alpha\, Q_0}$
$\boldsymbol{\kappa \neq 0}$	$Q_{\kappa} = T(\kappa)^{\dagger}\, Q_0\, T(\kappa)$	$L+1$	$\rho_{\kappa} \sim T(\kappa)^{\dagger}\, \rho_0\, T(\kappa)$



$$T(\kappa) = \mathrm{Tr}_A(A_L(\kappa) A_{L-1}(\kappa) \dots A_1(\kappa)), \quad [T(\kappa), T(\kappa')] = 0$$

Non-unique NESS: memory - Long time limit of observable depend on initial state

Corresponding range 2 model

Bond-site transformation $X_j \to X_{j-\frac{1}{2}} X_{j+\frac{1}{2}}$, $Z_i Z_{j+1} \to Z_{j+1/2}$

$$\begin{cases} \uparrow\uparrow \\ \downarrow\downarrow \end{cases} \to \uparrow \qquad \begin{cases} \uparrow\downarrow \\ \downarrow\uparrow \end{cases} \to \downarrow$$

$\kappa = \tan\frac{\theta}{2}$

$$\mathbb{H}_2 = \sum_j [h^{\sigma}_{j,j+1} + h^{\tau}_{j,j+1} + u L^{\sigma}_{j,j+1} L^{\tau}_{j,j+1}]$$

$$h_{j,j+1} = \frac{dn}{k}(\sigma^+_j \sigma^-_{j+1} - \sigma^-_j \sigma^+_{j+1}), \qquad L_{j,j+1} = \begin{pmatrix} \sin\theta & 0 & 0 & \cos\theta \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \cos\theta & 0 & 0 & -\sin\theta \end{pmatrix}$$

Integrability

Range 2 model

R-matrix : unusual functional dependence $\sin\frac{1}{2}\Big[\mathrm{am}(u|k^2) - \mathrm{am}(v|k^2)\Big]$

Range 3 model

$$\check{R}\text{-matrix (2 site)} \underbrace{\to}_{\text{bond-site}} \text{Lax operator (3 site)} \underbrace{\to}_{\text{RLL}} \check{R}\text{-matrix (4 site)}$$

$$\check{R}_{34,56}(u_1,u_2)\check{R}_{12,34}(u_1,u_3)\check{R}_{34,56}(u_2,u_3) = \check{R}_{12,34}(u_2,u_3)\check{R}_{34,56}(u_1,u_3)\check{R}_{12,34}(u_1,u_2)$$

Remarks

Why is this model new?

- First integrable deformation of Hubbard spanning 3 sites
- Unusual functional dependence in the R-matrix
- κ terms break $U(1)$ symmetry: XYZ-type deformation

Another deformation

Model B2 of PRL 126, 240403 (2021)

$$\mathbb{H}_{B2} = \sum_j h^{\sigma}_{j,j+1} + h^{\tau}_{j,j+1} + U l^{\sigma}_{j,j+1} l^{\tau}_{j,j+1},$$
$$h_{j,j+1} = i(\sigma^+_j \sigma^-_{j+1} - \sigma^-_j \sigma^+_{j+1})$$
$$l_{j,j+1} = Z_j + \eta\,(X_j X_{j+1} + Y_j Y_{j+1}) + \eta^2 Z_{j+1}$$
$$\eta = 0 \text{ Hubbard model} \qquad (1)$$

[Murakami '98, Medvedyeva Essler Prosen '16, Ziolkowska Essler '19]

Open questions

- Solution of the model: Nested Bethe Ansatz + XYZ-methods
- Construction of integrable spin chains with open boundary conditions
- Implication of the Hermitian-Lindbladian mapping in both integrable and *chaotic* case
- Analytical computation of Liouville gap

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