ANALYTIC BOOTSTRAP FOR THE LOCALIZED MAGNETIC FIELD

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Conformal line defects

- Extended excitations are important probes of Quantum Field Theories. They probe aspects of the theory that are not accessible to correlation functions of local operators, e.g. the global structure of the gauge group [Aharony, Seiberg, Tachikawa '13].
- Some of these questions can be made very precise in the context of defect Conformal Field Theories, where the rich interplay between bulk and defect finds an explicit realization in the defect crossing equation [Billo', Goncalves, Lauria, Meineri '16].

Localized magnetic fields in the $O(N)$ model

- We study the $O(N)$ model in $d = 4 - \epsilon$ and consider a relevant deformation localized on a straight line ($\sim$ localized magnetic field)

$$S = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}m^2(\phi_i)^2 + \frac{\lambda}{4!}(\phi_i\phi_i)^2\right] + h_0 \int_D d\tau\, |\dot{x}(\tau)| \phi_1(x(\tau))$$

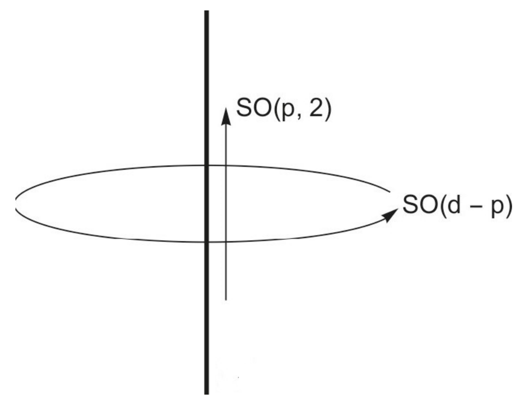
- The deformation triggers a RG flow which admits an infrared fixed point

$$\lambda_* = \frac{48\pi^2}{N+8}\epsilon + \mathcal{O}(\epsilon^2) \quad h_* = \sqrt{N+8} + \frac{4N^2 + 45N + 170}{4(N+8)^{\frac{3}{2}}}\epsilon + \mathcal{O}(\epsilon^2)$$

The theory at the fixed point describes a conformal line defect, that can potentially be realized in quantum simulators and in some liquid mixtures [Cuomo, Komargodski, Mezei '21].

- The defect breaks part of the bulk conformal and internal symmetries

$$SO(d+1,1) \to SO(2,1) \times SO(d-1)$$
$$O(N) \to O(N-1)$$



- Our aim is to compute correlators of the bulk fundamental fields ϕ_i in presence of the defect and extract the CFT data of bulk and defect operators at one loop order in the ϵ- expansion [Bianchi, DB and de Sabbata '22] [Gimenez-Grau '22].

Bulk correlators in presence of the defect

- Bulk operators can acquire non trivial 1-point functions in presence of the defect

$$\langle \phi_i(x) \rangle = \delta_{i1} \frac{a_\phi}{|x_\perp|^{\Delta_\phi}}$$

- They can also couple to defect operators

$$\langle \phi_i(x) \hat{O}(y) \rangle = \frac{\hat{b}^i_{\phi\hat{O}}}{|x_\perp|^{\Delta_\phi - \hat{\Delta}_{\hat{O}}}(|x_\perp|^2 + y^2)^{\hat{\Delta}}}$$

- The 2-point functions depend on two spacetime cross-ratios $z, \bar{z}$

$$\langle \phi_i(x)\phi_j(y) \rangle = \frac{F_{ij}(z,\bar{z})}{|x_{1\perp}|^{\Delta_\phi}|x_{2\perp}|^{\Delta_\phi}}$$

They can be expanded using the bulk and defect Operator Product Expansions.

- The bulk channel expansion reads

$$\langle \phi_i(x)\phi_j(y) \rangle = \frac{F_S(z,\bar{z})\delta_{ij} + F_T(z,\bar{z})\left(\delta_{i1}\delta_{j1} - \frac{1}{N}\delta_{ij}\right)}{|x_\perp|^{\Delta_\phi}|y_\perp|^{\Delta_\phi}}$$

$$F_R(z,\bar{z}) = \left(\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})}\right)^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\phi\phi\mathcal{O}_R} a_{\mathcal{O}_R} f_{\Delta,\ell}(z,\bar{z})$$

where the sum runs over bulk primary operators of dimension Δ and spin ℓ in the $O(N)$ representation R.

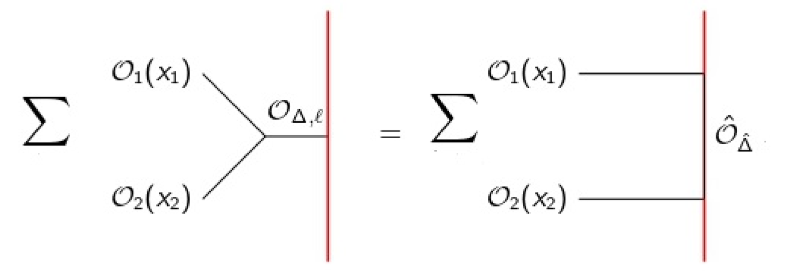
- The defect channel expansion reads

$$\langle \phi_i(x)\phi_j(y) \rangle = \frac{\hat{F}_S(z,\bar{z})\delta_{i1}\delta_{j1} + \hat{F}_V(z,\bar{z})\left(\delta_{ij} - \delta_{i1}\delta_{j1}\right)}{|x_\perp|^{\Delta_\phi}|y_\perp|^{\Delta_\phi}}$$

$$\hat{F}_R(z,\bar{z}) = \sum_{\hat{\Delta},s} b^2_{R,\hat{\Delta},s} \hat{f}_{\hat{\Delta},s}$$

where the sum runs over defect primary operators of dimension $\hat{\Delta}$ and transverse spin s in the $O(N-1)$ representation R.

- Demanding that the two expansions reproduce the same correlator imposes constraints on the CFT data of defect and bulk exchanged operators



Inversion formula and dispersion relation

- The defect CFT data of exchanged operators can be extracted from 2-point functions using an inversion formula [Lemos, Liendo, Meineri, Sarkar '17].

$$b_{\hat{\Delta},s} = \int_0^1 d^2z\, I_{\hat{\Delta},s}(z,\bar{z})\, \text{Disc}F(z,\bar{z})$$

where the discontinuity is defined as

$$\text{Disc}F(z,\bar{z}) = F(z,\bar{z}+i\epsilon) - F(z,\bar{z}-i\epsilon)$$

and $b_{\hat{\Delta},s}$ encodes the CFT data in its poles and residues

$$b_{\hat{\Delta},s} = -\sum_{\hat{O}} \frac{\hat{b}^2_{\phi\hat{O}}}{\hat{\Delta} - \hat{\Delta}_{\hat{O}}}$$

- A similar formula allows to extract the bulk CFT data [Liendo, Linke, Schomerus '19]

$$c(\Delta,\ell) = c^t(\Delta,\ell) + (-1)^\ell c^u(\Delta,\ell),$$

$$c^t(\Delta,\ell) = \int_0^1 d^2z\, H_{\Delta,\ell}(z,\bar{z})\text{dDisc}\left(\left(\frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}}\right)^{\Delta_\phi} F(z,\bar{z})\right)$$

where the double discontinuity is defined as

$$\text{dDisc}F(z,\bar{z}) = F(z,\bar{z}) - \frac{1}{2}F^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2}F^{\circlearrowright}(z,\bar{z})$$

- The full 2-point function can be directly reconstructed from its discontinuity using a dispersion relation, which can be derived from a contour deformation argument [Bianchi, DB '22], [Barrat, Gimenez-Grau, Liendo '22].

$$F(r,w) = \int_0^r \frac{dw'}{2\pi i}\left(\frac{1}{w'-w} + \frac{1}{w' - \frac{1}{w}} - \frac{1}{w'}\right)\text{Disc}F(r,w')$$

$$z = rw \quad \bar{z} = \frac{r}{w}$$

- The discontinuity can be computed from the bulk channel OPE expansion of the 2-point function. Sometimes, typically in perturbation theory, only a few operators contribute to the discontinuity. This is why these formulae are so powerful.

Results

- Admirably, at one loop in the ϵ-expansion, the discontinuity of $\langle\phi_i(x)\phi_j(y)\rangle$ depends only on two bulk anomalous dimension!

$$\gamma^{(1)}_{S,0,\ell} = \frac{N+2}{N+8}\delta_{0,\ell}, \qquad \gamma^{(1)}_{T,0,\ell} = \frac{2}{N+8}\delta_{0,\ell}.$$

They were already known from previous work on the theory without the defect.

- Just from these two anomalous dimensions we were able to reconstruct the 2-point function and extract an infinite amount of defect and bulk CFT data [Bianchi, DB and de Sabbata '22]

$$\hat{\gamma}^{(1)}_{S,m,s} = \frac{1-s}{(2s+1)}\delta_{m,0} \qquad \hat{b}^{2(1)}_{S,m,s} = \frac{-2(s-1)H_s - 3H_{s+\frac{1}{2}}}{2(2s+1)}\delta_{m,0}$$

$$\hat{\gamma}^{(1)}_{V,m,s} = -\frac{s}{(2s+1)}\delta_{m,0} \qquad \hat{b}^{2(1)}_{V,m,s} = -\frac{(2s+1)\left(2sH_s + H_{s-\frac{1}{2}}\right) + 2}{2(2s+1)^2}\delta_{m,0}$$

$$a\lambda^{(1)}_{T,0,\ell} = Na\lambda^{(1)}_{S,0,\ell} = -\frac{2^{-\ell-7}\Gamma\left(\frac{\ell}{2}+\frac{1}{2}\right)^3(N+8)}{\pi\Gamma\left(\frac{\ell}{2}+1\right)\Gamma\left(\ell+\frac{1}{2}\right)}\Big(-32H_{\frac{\ell}{2}-\frac{1}{2}} + 35H_{\ell-\frac{1}{2}}$$
$$+ 19\psi^{(0)}(\ell) - 38\psi^{(0)}(2\ell) - 19\gamma + 38\log(2) + 16\frac{N^2-3N-22}{(N+8)^2}\Big)$$

Outlook

- It could be interesting to compute the same observables to higher order in the ϵ-expansion or consider other correlators.
- One could also repeat the same analysis for other interesting line defects in the $O(N)$ model, such as the spin impurity in the O(3) model [work in progress with Bianchi, de Sabbata and Gimenez-Grau].
- One could also consider more general line defects, which are also important in the context of AdS_2/CFT_1 [work in progress with V. Forini]