A Large Twist Limit for Any Operator in $\mathcal{N} = 4$ SYM

Gwenaël Ferrando

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Introduction and Motivation

- holography: explicit dictionary, many tests but no proof,
- ideal example: $\mathcal{N}=4$ SYM in the planar limit, but still too complicated, many results remain conjectural,
- further simplification: fishnet theory. Origin of integrability is better understood, holography has been derived. [Gürdoğan and Kazakov (2015)] [Gromov, Kazakov, Korchemsky, Negro, and Sizov (2018)] [Gromov and Sever (2019)]

How to progressively go back to $\mathcal{N}=4$ SYM?

Outline

1. A Few Facts About the Fishnet Theory
2. A Short Operator: $\mathrm{Tr}(FZ)$
3. Mixing Between Operators and Between Scaling Limits

A Few Facts About the Fishnet Theory

From $\mathcal{N} = 4$ SYM to The Fishnet Theory

Start from γ-deformed $\mathcal{N} = 4$ SYM:

$$\mathcal{L} = -N\mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D^{\mu}\phi_i^{\dagger}D_{\mu}\phi^{i} + \psi^{\dagger}_{\dot{\alpha}A}\not{D}^{\dot{\alpha}\alpha}\psi^{A}_{\alpha}\right] + \mathcal{L}_{int}\,,$$

where

$$D_{\mu} = \partial_{\mu} + \mathrm{i}\,g[A_{\mu},\cdot]\,,$$

$$F_{\mu\nu} = -\frac{\mathrm{i}}{g}[D_{\mu}, D_{\nu}] = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + \mathrm{i}\,g[A_{\mu}, A_{\nu}]\,,$$

and

$$\mathcal{L}_{int} = N\,g^2\,\mathrm{Tr}\left[2\ \mathrm{e}^{-\mathrm{i}\,\epsilon^{ijk}\gamma_k}\,\phi_i^{\dagger}\phi_j^{\dagger}\phi^{i}\phi^{j} - \frac{1}{2}\left\{\phi_i^{\dagger},\phi^{i}\right\}\left\{\phi_j^{\dagger},\phi^{j}\right\}\right]$$
$$+\ \text{Yukawa interactions}\,.$$

Set $\gamma_1 = \gamma_2 = 0$ and take the double-scaling limit

$$e^{-\,i\,\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi_1^2 = \frac{g^2\, e^{-\,i\,\gamma_3}}{8\pi^2} \quad \text{fixed}\,.$$

Denoting $\phi_1 = X\,, \phi_2 = Z$, the fishnet Lagrangian is

$$\mathcal{L}_{\text{fishnet}} = -N\,\text{Tr}\left(\partial^\mu X^\dagger \partial_\mu X + \partial^\mu Z^\dagger \partial_\mu Z - (4\pi)^2 \xi_1^2 X^\dagger Z^\dagger X Z\right)\,.$$

[Gürdoğan and Kazakov (2015)]

Single, chiral interaction vertex:

We will work in the planar limit $N \to +\infty$.

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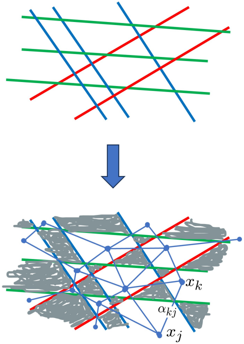
- Holographic dual derived from first principles: chain of point particles with local interactions. [Gromov and Sever (2019)]

Aside: Loom for CFTs



Generalization of fishnet CFT based on arbitrary Baxter lattice (set of intersecting lines)

Same properties: non-unitary, conformal, integrable

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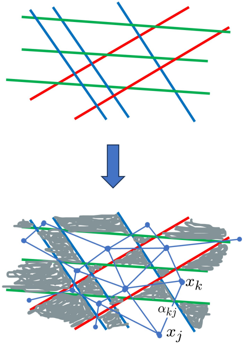

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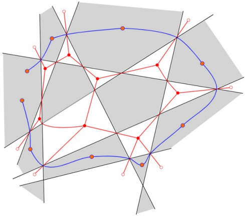


Feynman diagrams exhibit Yangian invariance

[Chicherin, Kazakov, Loebbert, Müller, Zhong (2017)]

[Corcoran, Loebbert, and Miczajka (2021)]

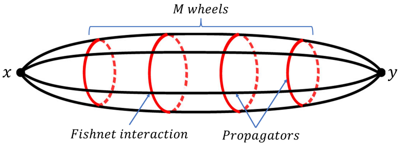
[Duhr, Klemm, Loebbert, Nega, and Porkert (2022)]

[Kazakov, Levkovich-Maslyuk, and Mishnyakov (2023)]

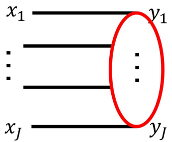
Graph-Building Operators

Conformal dimension of $Tr(Z^J(x))$: the 2-point function has an iterative structure.

$$\left\langle Tr(Z^J(x))\, Tr(Z^{\dagger J}(y))\right\rangle \leftrightarrow \sum_{M=0}^{+\infty} \xi_1^{2JM}$$



The graph-building operator $\widehat{H}$ is an integral operator with kernel



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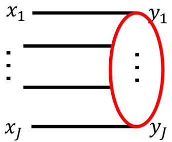

Its action on an arbitrary function Φ is

$$\left[\widehat{H}\Phi\right](x_1,\ldots,x_J) = \int \frac{\Phi(y_1,\ldots,y_J)}{\prod_{k=1}^{J}(x_k-y_k)^2 y_{k,k+1}^2} \mathrm{d}^4y_1\ldots\mathrm{d}^4y_J$$

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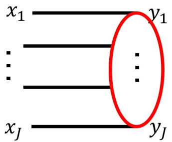

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The 2-point function is essentially reduced to the computation of

$$\sum_{M=0}^{+\infty}\xi_1^{2MJ}\widehat{H}^M=\frac{1}{1-\xi_1^{2J}\widehat{H}}\,.$$

$\Longrightarrow$ one needs to diagonalise $\widehat{H}$

Physical Eigenvectors

Eigenvectors of $\widehat{H}$ with eigenvalue $E = \xi_1^{-2J}$ represent primary operators of the fishnet theory (and their descendants). This is given by the representation of the conformal group $(\Delta(\xi_1^2), \ell, \bar{\ell})$ under which the eigenvector tranforms.

Example: $J = 2$, eigenvectors can be written explicitly, physical states correspond to symmetric traceless tensors of arbitrary rank $\ell \geqslant 0$, their dimensions are

$$\Delta_{\ell,\pm} = 2 + \sqrt{(\ell+1)^2 + 1 \pm 2\sqrt{(\ell+1)^2 + 4\xi_1^4}}\,.$$

[Grabner, Gromov, Kazakov, and Korchemsky (2017)]

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- The previous results are exact. In particular, for $\ell = 0$,

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- On the other hand, $\Delta_{0,+}$ is the dimension of $Tr(Z\Box Z) + \ldots$ which we do not know exactly because there is mixing.

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- The fishnet theory is a logarithmic CFT: the dilatation operator is not diagonalisable.
- Neither fermions nor gauge boson in the fishnet theory.

[Gürdoğan and Kazakov (2015)]

How can one incorporate back these protected or logarithmic operators?

New Double-Scaling Limits

Operator-dependent limit:

$$e^{-\,i\,\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi_n^2 = \frac{g^2\, e^{-\,i\,\frac{\gamma_3}{n}}}{8\pi^2} \quad \text{fixed}$$

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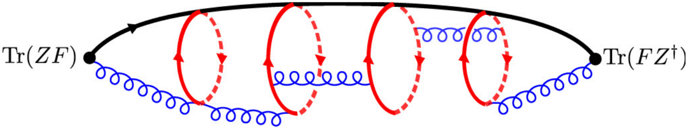
Example: for $\mathrm{Tr}(ZF)$, one must take $n = 2$ and the only diagrams that remain are



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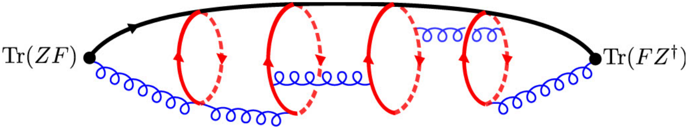


Following the procedure outlined previously, we find that

$$\Delta_{\mathrm{Tr}(FZ)} \underset{g\to 0\,,\xi_2 \text{ fixed}}{\longrightarrow} 2 + \sqrt{5 - 4\sqrt{1 + \xi_2^4}}\,.$$

General Situation: Mixing

If we turn to longer operators, such as $\text{Tr}(FZ^J)$ for $J > 1$, then $n = 1 + 1/J$.

But there is some form of mixing with $\text{Tr}(XX^\dagger Z^J)$ (same double-scaling limit) and $\text{Tr}(Z^J)$ (fishnet limit).

The relevant graph-building operator is a 3×3 matrix. We will show that it is integrable.

A Short Operator: $\mathrm{Tr}(FZ)$

Feynman Diagrams

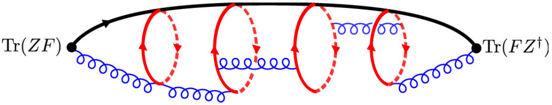
Double-scaling limit:

$$e^{-i\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi_2^2 = \frac{g^2\, e^{-i\frac{\gamma_3}{2}}}{64\pi^4} \quad \text{fixed}\,.$$

Relevant interactions:

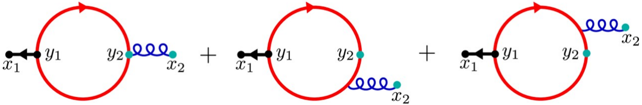
$$-\,i\,N_c g\,\mathrm{Tr}\big(\partial_\mu X^\dagger [A^\mu, X] + \partial_\mu X [A^\mu, X^\dagger]\big)\,,$$
$$2N_c g^2\,\mathrm{Tr}\big(X^\dagger A_\mu X A^\mu\big)\,, \quad \text{and} \quad 2N_c g^2\, e^{-i\gamma_3}\,\mathrm{Tr}\big(X^\dagger Z^\dagger X Z\big)\,.$$

Typical diagram:



Graph-Building Operator

$\widehat{H}_A$ depends on the gauge:



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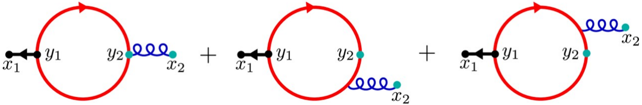

However, there exists a gauge-independent operator $\widehat{H}_F$ acting on antisymmetric tensors $\Psi_F^{\mu\nu}$ and such that: if $\Psi_F^{\mu\nu} = \partial_2^\mu \Psi_A^\nu - \partial_2^\nu \Psi_A^\mu$, then

$$\left[\widehat{H}_F \Psi_F\right]^{\mu\nu} = \partial_2^\mu \left[\widehat{H}_A \Psi_A\right]^\nu - \partial_2^\nu \left[\widehat{H}_A \Psi_A\right]^\mu .$$

$\Longrightarrow \left\langle \mathrm{Tr}(ZF)(x)\, \mathrm{Tr}\left(Z^\dagger F\right)(y)\right\rangle$ is gauge-independent in the double-scaling limit.

One can invert $\widehat{H}_F$:

$$\left[\widehat{H}_F^{-1}\Psi_F\right]^{\mu\nu} = \frac{1}{16}\left(\partial_2^{\mu} x_{12}^4 \Box_1 \partial_2^{\rho} \Psi_{F,\rho}{}^{\nu} - (\mu \leftrightarrow \nu)\right) .$$

Eigenvectors are fixed by the conformal covariance of the operator: three-point functions involving a scalar of dimension 1 and a rank-2 antisymmetric tensor of dimension 2.

Spectrum:

- $(\Delta_{\ell,\pm}, \ell, \ell)$ for $\ell \geqslant 1$ with

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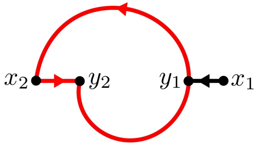
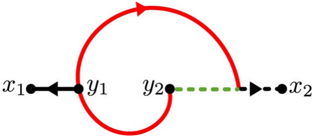
The dimension of $\mathrm{Tr}(ZF)$ is $\Delta'_{0,-}$.

Other Short Operators

We performed a similar analysis for the following operators:

$$\mathrm{Tr}\left(XX^\dagger Z\right) \quad \text{and} \quad \mathrm{Tr}\left(X^\dagger XZ\right) \implies n = 2$$
$$\mathrm{Tr}(\psi_4 Z) \quad \text{or} \quad \mathrm{Tr}\left(\psi_1^\dagger Z\right) \implies n = \frac{4}{3}$$
$$\mathrm{Tr}(\psi_2 Z) \quad \text{or} \quad \mathrm{Tr}\left(\psi_3^\dagger Z\right) \implies n = 4$$





Mixing Between Operators and Between Scaling Limits

Fishnet Contributions

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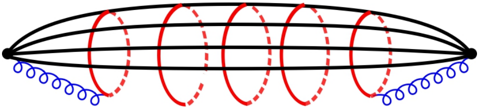
Let us consider the 2-pt function $\big\langle \mathrm{Tr}\big(Z^J F\big)(x)\, \mathrm{Tr}\big((Z^\dagger)^J F\big)(y)\big\rangle$. When $e^{-\mathrm{i}\,\gamma_3} \to +\infty$, the dominant contributions are



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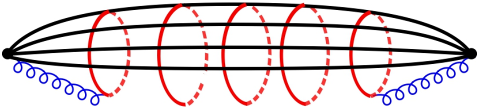
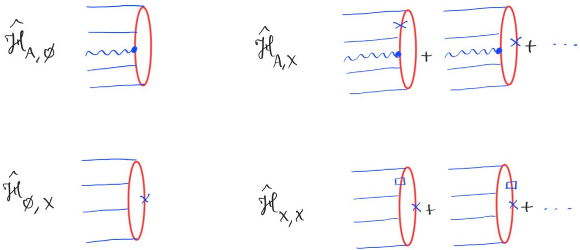


But $\mathrm{Tr}\big(Z^J F\big)$ is absent from the fishnet theory, so more graphs need to be taken into account.

Mixing

There is still an iterative structure: the graph-building operator is actually a matrix $\widehat{\mathcal{H}}$ with one row (and one column) for each state that participate in the mixing.

In our case, there are 3 intermediate states: $\mathrm{Tr}(Z^J)$, $\mathrm{Tr}(Z^J F)$ and $\mathrm{Tr}(Z^J X X^\dagger)$.



$\widehat{\mathcal{H}}$ is defined such that 2-point functions are essentially matrix elements of $\frac{1}{1-\widehat{\mathcal{H}}}$

Example:

$$\left\langle \mathrm{Tr}(A^\mu(x_0)Z(x_1)\dots Z(x_J))\,\mathrm{Tr}\big(Z^\dagger(z_J)\dots Z^\dagger(z_1)\big)\right\rangle$$
$$= -\frac{\mathrm{i}}{2}\int \frac{\langle x_0, x_1, \dots, x_J|\left(\frac{1}{1-\widehat{\mathcal{H}}}\right)^{\mu}_{A\emptyset}|y_1,\dots,y_J\rangle}{(4\pi^2)^J\prod_{i=1}^J(y_i-z_i)^2}\,\frac{\prod_{i=1}^J \mathrm{d}^4 y_i}{\pi^{2J}}\,.$$

The problem is still to diagonalise $\widehat{\mathcal{H}}$, and physical states correspond to those with eigenvalue equal to 1.

Double-Scaling Limit

$$e^{-\mathrm{i}\,\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi_{1+1/J}^2 = \frac{g^2\, e^{-\mathrm{i}\,\frac{J}{J+1}\gamma_3}}{8\pi^2} \quad \text{fixed}$$

Each matrix element scales differently:

$$\widehat{\mathcal{H}} = \xi_{1+1/J}^{2(J+1)} \begin{pmatrix} g^{-2}\widehat{\mathcal{H}}_{\emptyset\emptyset} & g^{-1}\widehat{\mathcal{H}}_{\emptyset A} & g^{-1}\widehat{\mathcal{H}}_{\emptyset X} \\ g^{-1}\widehat{\mathcal{H}}_{A\emptyset} & \widehat{\mathcal{H}}_{AA} & \widehat{\mathcal{H}}_{AX} \\ g^{-1}\widehat{\mathcal{H}}_{X\emptyset} & \widehat{\mathcal{H}}_{XA} & \widehat{\mathcal{H}}_{XX} \end{pmatrix}.$$

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Some eigenvalues will diverge, some will go to zero. We focus on those which remain finite:

$$\widehat{\mathcal{H}}\Psi = E\Psi\,, \quad \text{with} \quad E = E_0 + O(g)\,, \quad E_0 \neq 0\,.$$

At leading order, only the above 3×3 submatrix is relevant. Writing

$$\Psi = \begin{pmatrix} \Psi_{\emptyset,0}(x_1, \ldots, x_J) \\ \Psi^{\mu}_{A,0}(x_0, x_1, \ldots, x_J) \\ \Psi_{X,0}(x_0, x_1, \ldots, x_J) \end{pmatrix} + O(g)\,,$$

we get $\Psi_{\emptyset,0} = 0$ and

$$\xi_{1+1/J}^{2(J+1)} \widehat{\mathfrak{H}} \begin{pmatrix} \Psi_{F,0} \\ \Psi_{X,0} \end{pmatrix} = E_0 \begin{pmatrix} \Psi_{F,0} \\ \Psi_{X,0} \end{pmatrix}$$

for $\Psi^{\mu\nu}_{F,0} = \partial_0^{\mu} \Psi^{\nu}_{A,0} - \partial_0^{\nu} \Psi^{\mu}_{A,0}$, and some 2×2 matrix $\widehat{\mathfrak{H}}$ depending on all 9 matrix elements of $\widehat{\mathcal{H}}$.

$\widehat{\mathfrak{H}}$ is a complicated matrix of integral operators but it is local and gauge invariant (contrary to $\widehat{\mathcal{H}}$) and can be inverted:

$$\widehat{\mathfrak{H}}^{-1} = \begin{pmatrix} \theta \cdot \partial_0 \, x_{J0}^2 x_{10}^2 \, \partial_0 \cdot \partial^{(\theta)} & 2\,\theta \cdot \partial_0 \left(\frac{\theta \cdot x_{J0}}{x_{J0}^2} - \frac{\theta \cdot x_{10}}{x_{10}^2} \right) x_{J0}^2 x_{10}^2 \\ 2 \left(\frac{x_{10} \cdot \partial^{(\theta)}}{x_{10}^2} - \frac{x_{J0} \cdot \partial^{(\theta)}}{x_{J0}^2} \right) x_{J0}^2 x_{10}^2 \, \partial_0 \cdot \partial^{(\theta)} & \partial_{0,\mu} \, x_{J0}^2 x_{10}^2 \, \partial_0^{\mu} + 8\, x_{10} \cdot x_{J0} \end{pmatrix} \times \frac{\prod_{i=1}^{J-1} x_{i,i+1}^2 \prod_{i=1}^{J} \Box_i}{(-4)^{J+1}} \,,$$

where θ^μ is a polarisation vector such that $\{\theta^\mu, \theta^\nu\} = 0$. It encodes the tensor structure: $\Psi^{\mu\nu} \mapsto \Psi = \theta^\mu \theta^\nu \Psi_{\mu\nu}$.

Integrability

We can construct a transfer matrix

$$T(u) = \mathrm{tr}_6 \left(L^{(\rho_0)}_{Y_0}(u) L^{(1,0,0)}_{Y_1}(u) \cdots L^{(1,0,0)}_{Y_J}(u) \right)$$

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$$T(0) = (-1)^{J+1} \widehat{\mathfrak{H}}^{-1} \, .$$

We have checked that the 6×6 Lax matrices are solution to the RLL equation

$$R_{12}(u-v) L^{(\rho_0)}_{Y,1}(u) L^{(\rho_0)}_{Y,2}(v) = L^{(\rho_0)}_{Y,2}(v) L^{(\rho_0)}_{Y,1}(u) R_{12}(u-v) \, ,$$

where $R_{12}(u)$ is the usual $O(5,1)$-invariant R-matrix.

[Zamolodchikov and Zamolodchikov (1979)]

The Lax matrices for sites $1, \ldots, J$ are the usual ones for scalar representations:

$$L^{(1,0,0)}_{Y,MN}(u) = u^2 \eta_{MN} - u \left(Y_M \partial_{Y^N} - Y_N \partial_{Y^M} \right) - \frac{1}{2} Y_M Y_N \Box_Y \,.$$

Embedding space: $1 \leqslant M \leqslant 6$, metric $\eta^{MN} = \text{diag}(1, 1, 1, 1, 1, -1)$, and $Y^M Y_M = 0$.

But the representation at site 0 is reducible and the Lax matrix appears to be new:

$$L^{(\rho_0)}_{Y,MN}(u) = u^2\eta_{MN} - u\, q^{(\rho_0)}_{MN} + \mathcal{L}_{Y,MN}\,,$$

where the conformal generators are

$$q^{(\rho_0)}_{MN} = \begin{pmatrix} Y_M\partial_{Y^N} - Y_N\partial_{Y^M} + \Theta_M\partial_{\Theta^N} - \Theta_N\partial_{\Theta^M} & 0 \\ 0 & Y_M\partial_{Y^N} - Y_N\partial_{Y^M} \end{pmatrix}$$

and the operator $\mathcal{L}_Y$ is

$$\mathcal{L}^{MN}_Y = -\frac{1}{2}\begin{pmatrix} (\Theta\cdot\partial_Y)\, Y^M Y^N\, (\partial_Y\cdot\partial_\Theta) & (\Theta\cdot\partial_Y)\left[Y^M\Theta^N - Y^N\Theta^M\right] \\ \left[Y^N\partial^M_\Theta - Y^M\partial^N_\Theta\right](\partial_Y\cdot\partial_\Theta) & \tfrac{1}{2}\left[Y^M\Box_Y Y^N + Y^N\Box_Y Y^M\right] + 2\eta^{MN} \end{pmatrix}.$$

Conclusion

- Twisting the correlators, one can devise a double-scaling limit for any operator in $\mathcal{N} = 4$ SYM such that an iterative structure emerges.

  [Cavaglià, Grabner, Gromov, and Sever (2020)]

- In most cases, this involves mixing with other operators, including fishnet operators. But integrability is always present.

- Regarding holography, the fishchain picture appears to be generic.

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- In most cases, this involves mixing with other operators, including fishnet operators. But integrability is always present.
- Regarding holography, the fishchain picture appears to be generic.
- The graph-building operator $\widehat{\mathcal{H}}$ can also be used to study corrections in g. For instance, corrections to the fishnet limit.
- It would be interesting to study three-point functions of operators with different double-scaling limits.

Thank you for your attention!