Multipoint Correlators on the Wilson Line Defect CFT

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RTG 2575: Rethinking Quantum Field Theory

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2112.10780, 2210.14916 with J. Barrat, P. Liendo and J. Plefka

2307.xxxxx with D. Artico, J. Barrat

23xx.xxxxx with J. Barrat, G. Bliard, P. Ferrero, C. Meneghelli

Motivation

Multipoint Correlators

Wilson Line Defect CFT

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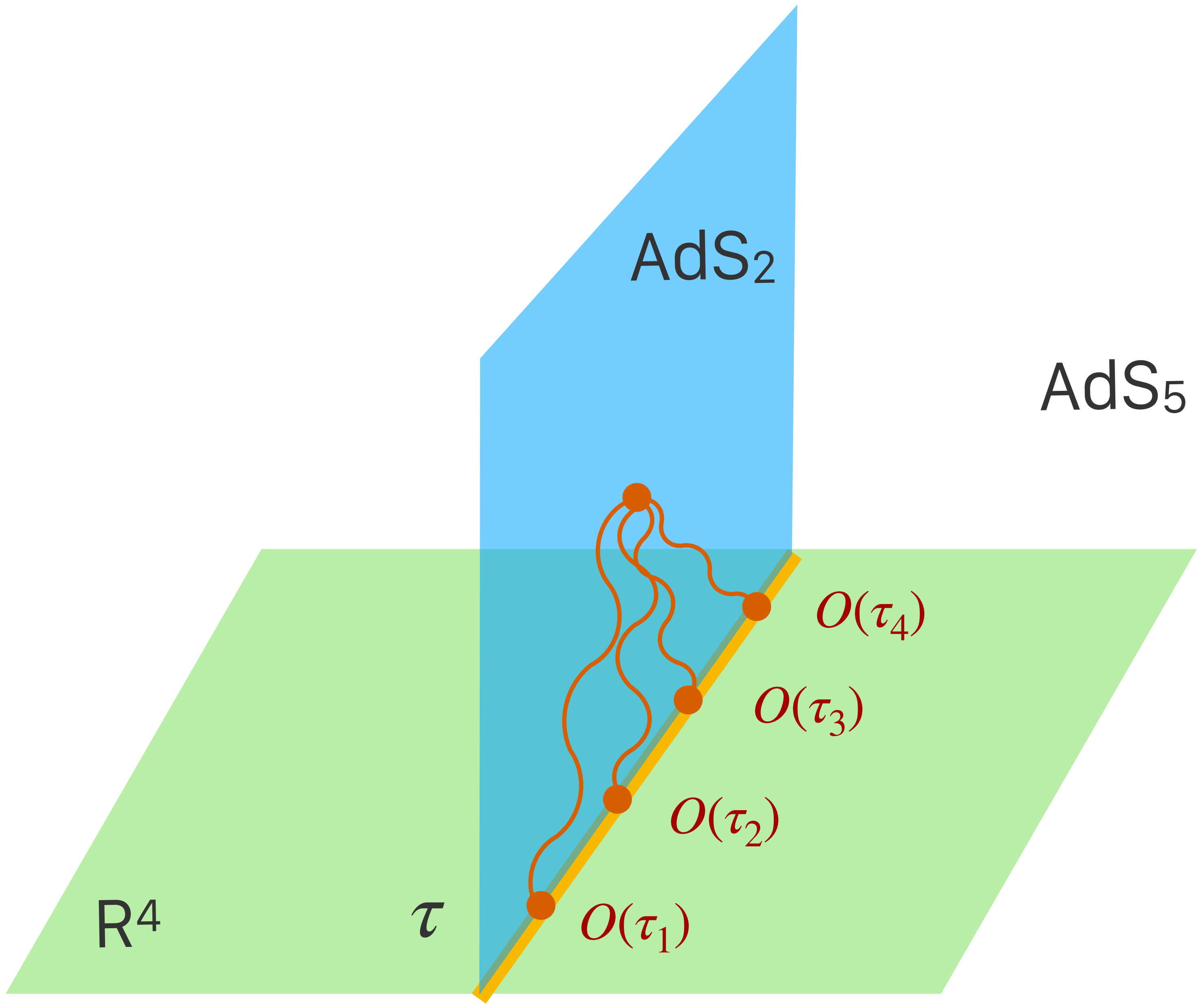
Multipoint Correlators   Wilson Line Defect CFT

- Conformal, integrable, supersymmetric, holographic dual, non-local, unitary



Motivation

Multipoint Correlators   Wilson Line Defect CFT

- Conformal, integrable, supersymmetric, holographic dual, non-local, unitary
- Interplay of different techniques
  - Supersymmetric localization
    [Pestun, 2007]
    [Drukker, Giombi, Ricci & Trancanelli, 2008]
    [Giombi & Pestun, 2008]
    [Giombi, Komatsu & Offertaler, 2021]
  - Witten diagrams
    [Giombi, Roiban & Tseytlin, 2017]
    [Beccaria, Giombi & Tseytlin, 2019]
  - Conformal Bootstrap
    [Liendo, Meneghelli & Mitev, 2018]
    [Ferrero & Meneghelli, 2021]
  - Integrability
    [Grabner, Gromov & Julius, 2020]
  - Bootstrability
    [Cavaglià, Gromov, Julius & Preti; 2021, 2022]
  - Large charge
    [Giombi, Komatsu & Offertaler; 2021, 2022]

Motivation

Multipoint Correlators   Wilson Line Defect CFT

- Conformal, integrable, supersymmetric, holographic dual, non-local, unitary
- Interplay of different techniques (bootstrap, integrability, localization, ..)
- Simpler but not trivial

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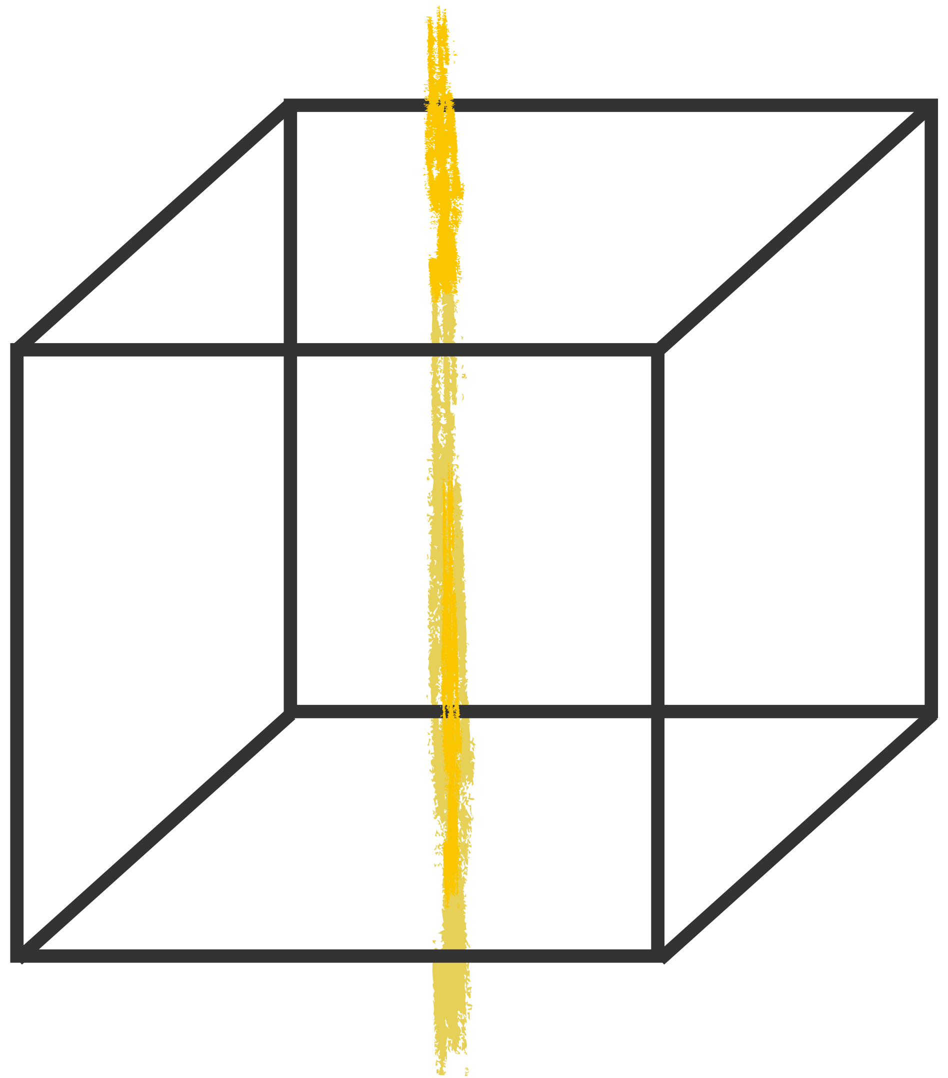
- Interplay of different techniques (bootstrap, integrability, localization, ..)
- Simpler but not trivial
- Defect CFT



Motivation

Multipoint Correlators Wilson Line Defect CFT

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Multipoint Correlators | Wilson Line Defect CFT

- Not much is known

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- Non-perturbative constraints

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Multipoint Correlators   Wilson Line Defect CFT

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- Prime target for bootstrap → Contain information about lower-point correlators
- How to tackle higher d theories
- Non-perturbative constraints
- Functions that we expect in multipoint correlators

1/2-BPS Wilson Line

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$$\mathcal{W}_\ell \equiv \frac{1}{N} \operatorname{tr} P \exp \int_{-\infty}^{\infty} d\tau \left(i \dot{x}_\mu A^\mu + |\dot{x}| \phi^6 \right)$$

[J. Maldacena; 1998]

1/2-BPS Wilson Line

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[J. Maldacena; 1998]

$\mathcal{N}=4$ SYM	$SO(4,2)$	$SO(6)_R$	$\phi^1, \phi^2, \phi^3, \phi^4, \phi^5, \phi^6$
↓	↓	↓	↓
$\mathcal{N}=4$ SYM with Wilson line	$SO(3) \times SO(2,1)$	$SO(5)_R$	$\phi^1, \phi^2, \phi^3, \phi^4, \phi^5, \phi^6$

Operators

Scalar operators, single-trace representation of the algebra

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$$\langle \phi^{I_1} \dots \phi^{I_n} \rangle_{1d} := \frac{1}{N} \left\langle \operatorname{tr} \mathcal{P} \left[\phi^{I_1} \dots \phi^{I_n} \exp \int_{-\infty}^{\infty} d\tau \, (i \dot{x}^\mu A_\mu + |\dot{x}| \phi^6) \right] \right\rangle_{4d}$$

[Drukker, Kawamoto; '06]

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[Drukker, Kawamoto; '06]

→ Protected $\Delta = \Delta_0 + \gamma(\lambda)$ + Non-protected $\gamma^{(1)}_{\phi^6} = \frac{1}{4\pi^2}$

$\phi^1, \phi^2, \phi^3, \phi^4, \phi^5$ ϕ^6

[Grabner, Gromov, Julius; '20]

Correlators

CFT Data: $\{\Delta, c_{ijk}\}$

2-point functions: $\langle \phi_{\Delta_1}(\tau_1)\phi_{\Delta_2}(\tau_2)\rangle = \dfrac{c_{12}}{\tau_{12}^{2\Delta}}\,, \qquad \Delta_1 = \Delta_2 \equiv \Delta$

3-point functions: $\langle \phi_{\Delta_1}(\tau_1)\phi_{\Delta_2}(\tau_2)\phi_{\Delta_3}(\tau_3)\rangle = \dfrac{c_{123}}{(\tau_{12}^2)^{\Delta_{123}}\,(\tau_{23}^2)^{\Delta_{231}}\,(\tau_{13}^2)^{\Delta_{132}}}$

$\Delta_{ijk} = \Delta_i + \Delta_j - \Delta_k$

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$\Delta_{ijk} = \Delta_i + \Delta_j - \Delta_k$

$$\langle \phi^{I_1} \ldots \phi^{I_n} \rangle = \mathcal{K}_{\Delta_{I_1} \ldots \Delta_{I_n}} \mathcal{A}^{I_1 \ldots I_n}(\chi_i, r_i, s_i, t_{ij})$$

Correlators

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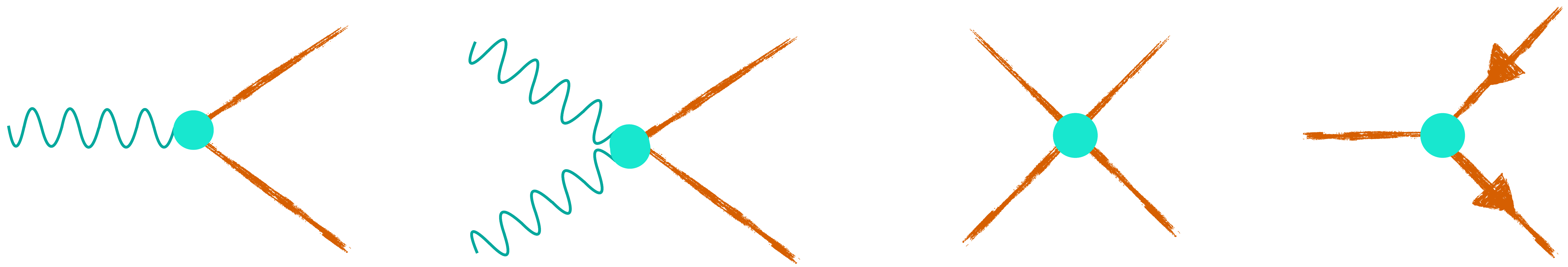
Pinching: $\left\langle \phi^{I_1}(\tau_1)\,\phi^{I_2}(\tau_2)\,\phi^{I_3}(\tau_3)\,\phi^{I_4}(\tau_4)\right\rangle \xrightarrow{\tau_4\to\tau_3} \left\langle \phi^{I_1}(\tau_1)\,\phi^{I_2}(\tau_2)\,\phi^{I_3}(\tau_3)\,\phi^{I_4}(\tau_3)\right\rangle$

Bulk action and propagators

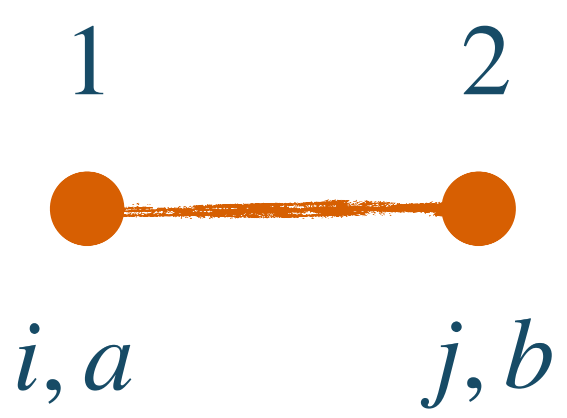
$$S = \frac{1}{g^2}\int d^4x\,\mathrm{tr}\left\{\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi_i D^\mu\phi^i - \frac{1}{2}\left[\phi_i,\phi_j\right]\left[\phi^i,\phi^j\right] + i\bar{\psi}\gamma^\mu D_\mu\psi + \bar{\psi}\Gamma^i\left[\phi_i,\psi\right] + \partial_\mu\bar{c}D^\mu c + (\partial_\mu A^\mu)^2\right\}$$

Vertices :



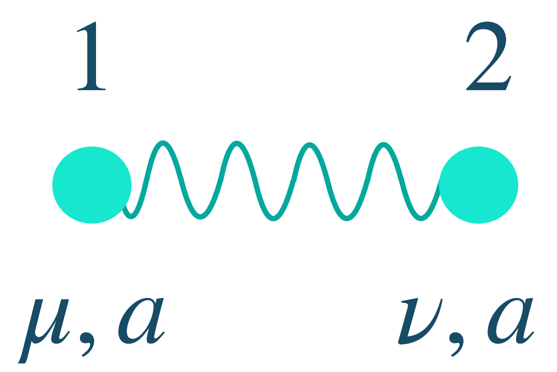
Propagators :

- Scalars



$$= g^2\delta_{ij}\delta^{ab}I_{12}$$

- Gluons



$$= g^2\delta_{\mu\nu}\delta^{ab}I_{12}$$

$$I_{ij} := \frac{1}{(2\pi)^2\tau_{ij}^2}$$

Outline

Motivation

Main characters of this story

Wilson Line

Operators

Recursion relations to derive multipoint correlation functions at NLO

- protected operators
- non-protected operators

Multipoint Ward identities

- NNLO 4-pt correlator
- bootstrap 5pt at strong coupling

Conclusions and outlook

Recursion Relations

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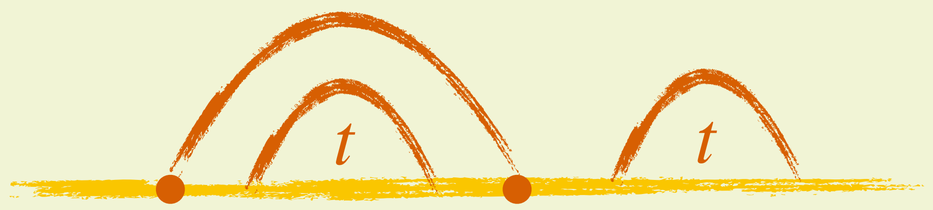
Recursion relations

Leading order

$$\langle \phi^{i_1} \dots \phi^{i_n} \rangle^{(0)}_{i=1,\dots,5} = \sum$$



Recursion relations

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EVEN

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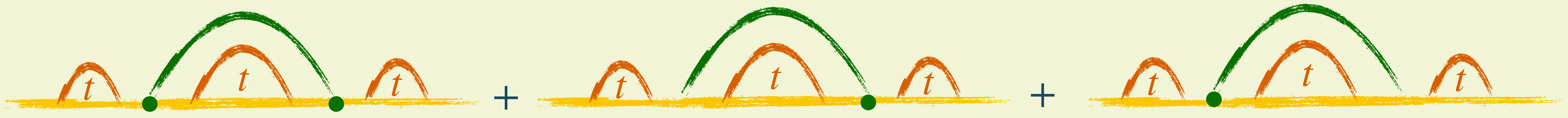
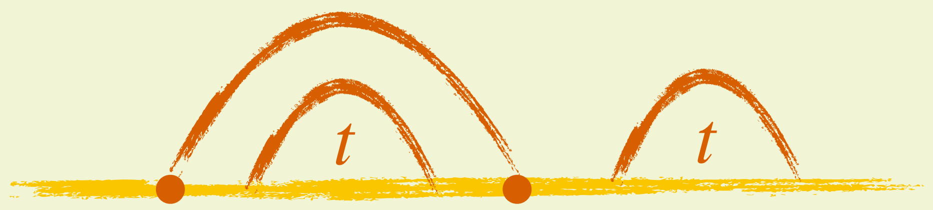


ODD

Recursion relations

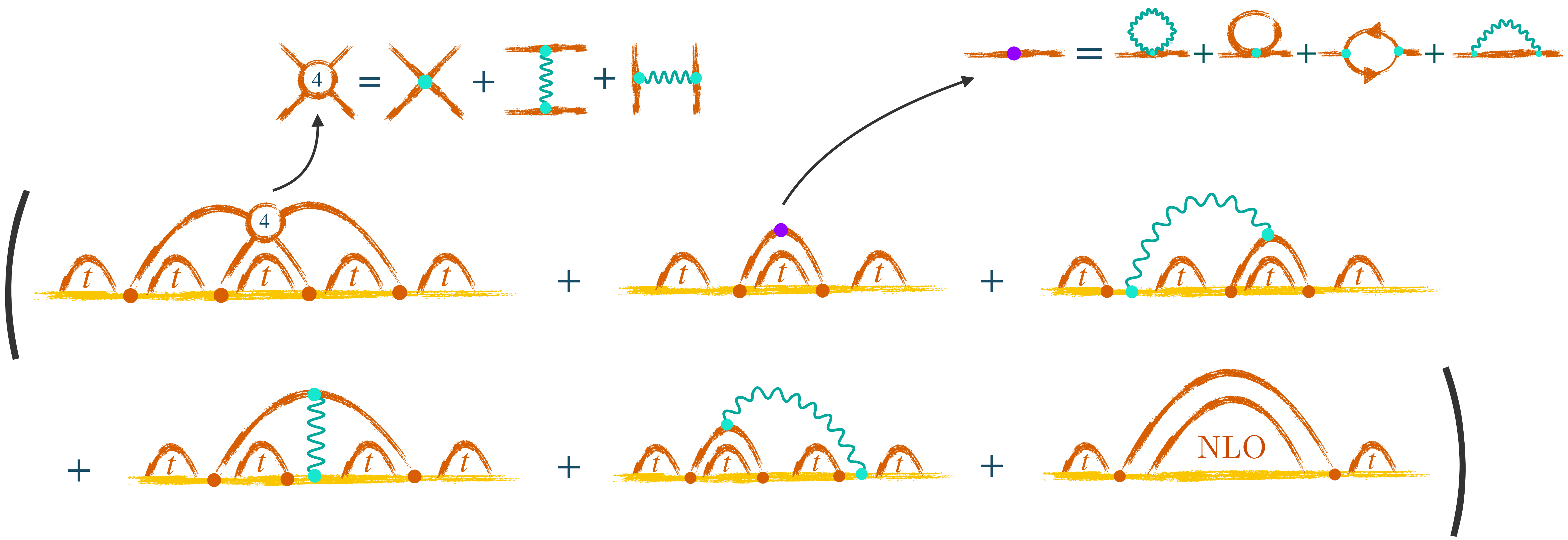
Leading order

$$\langle \phi^{i_1} \dots \phi^{i_n} \rangle^{(0)}_{i=1,\dots,5} = \sum$$



Next-to-leading order

$$\left\langle \phi^{i_1} \dots \phi^{i_n} \right\rangle^{(1)} = \sum \Bigg($$

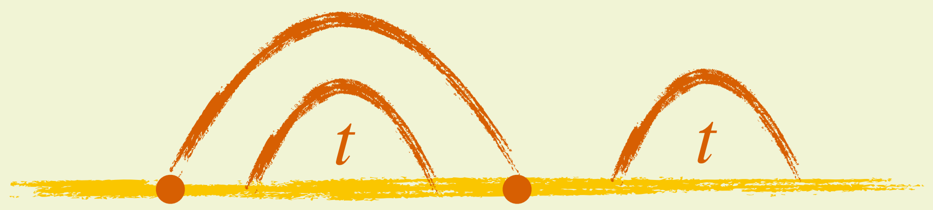


$$\Bigg)$$

Recursion relations

Leading order

$$\langle \phi^{i_1} \dots \phi^{i_n} \rangle^{(0)} = \sum_{i=1,\dots,5}$$



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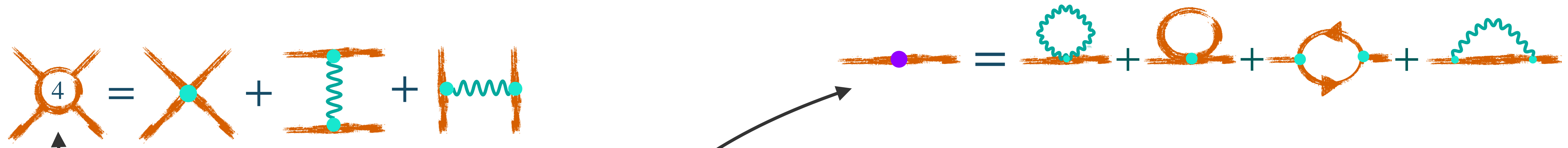
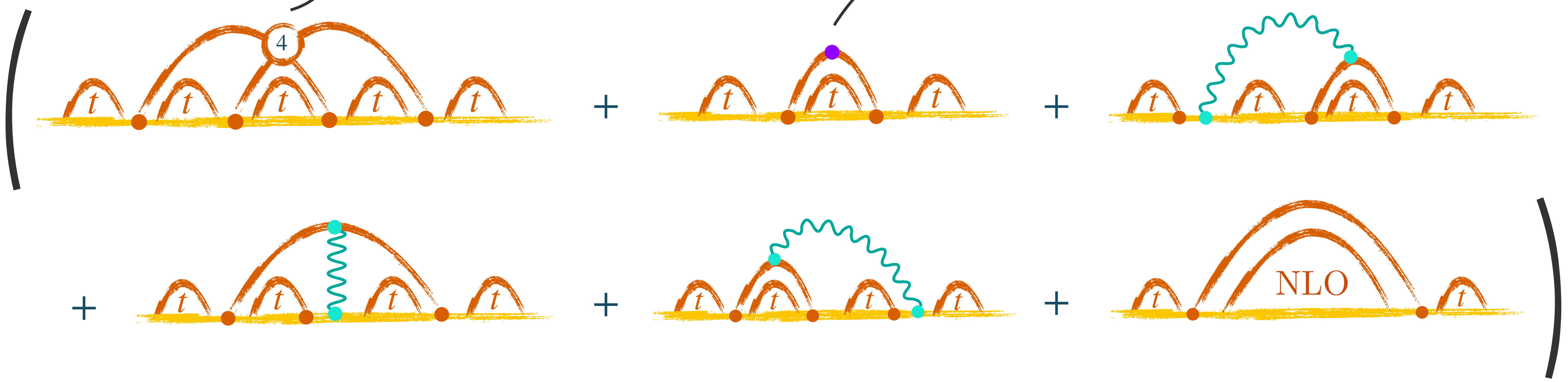
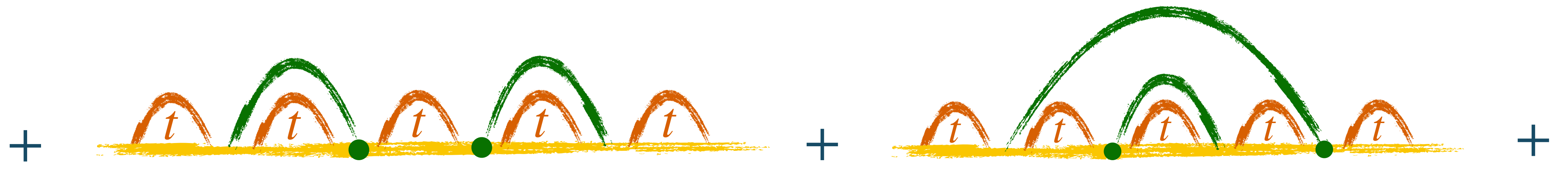


$$\langle \phi^{I_1} \dots \phi^{I_n} \rangle^{(1)} = \sum \Bigg($$

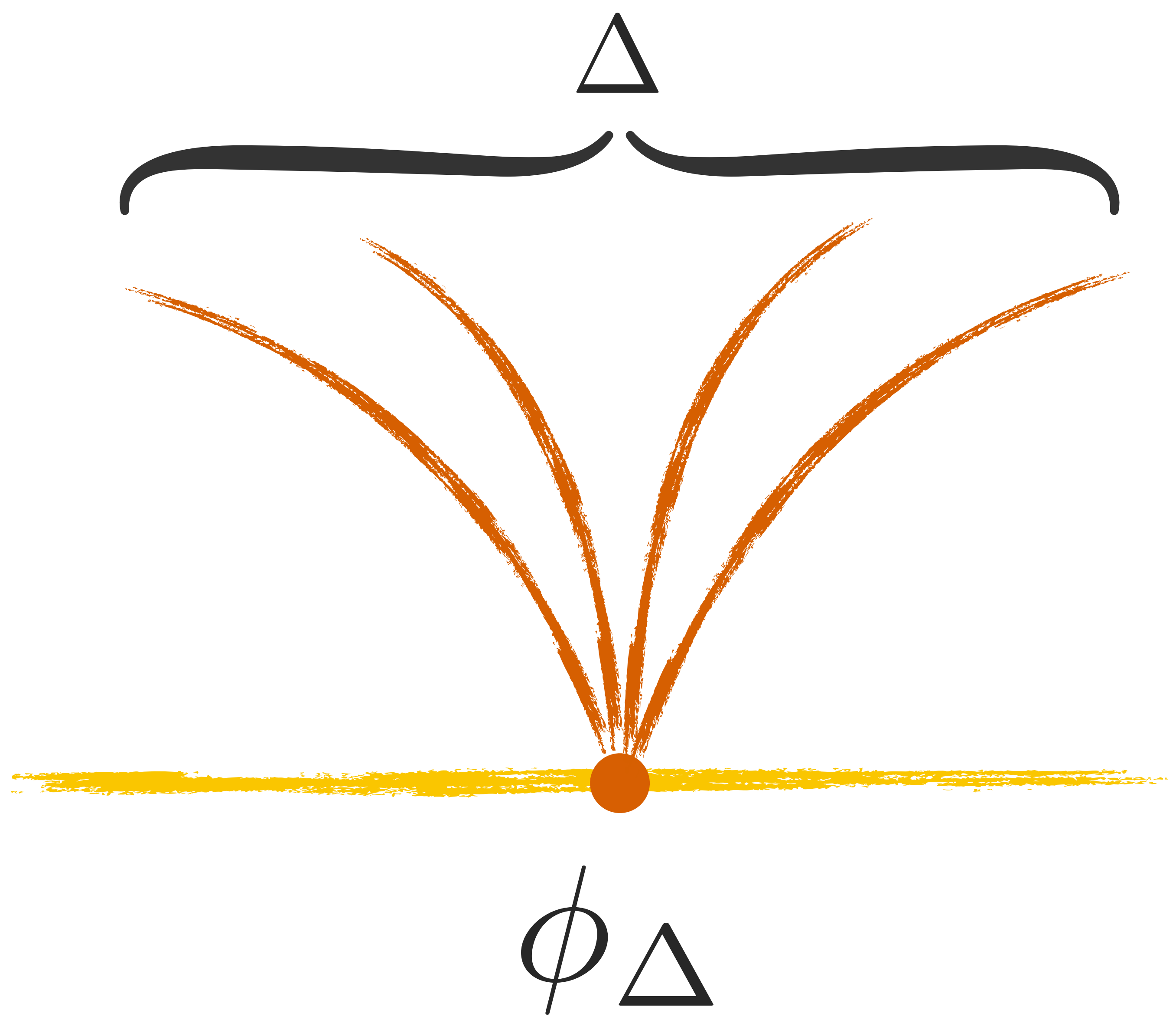




+ 13 more terms

EVEN

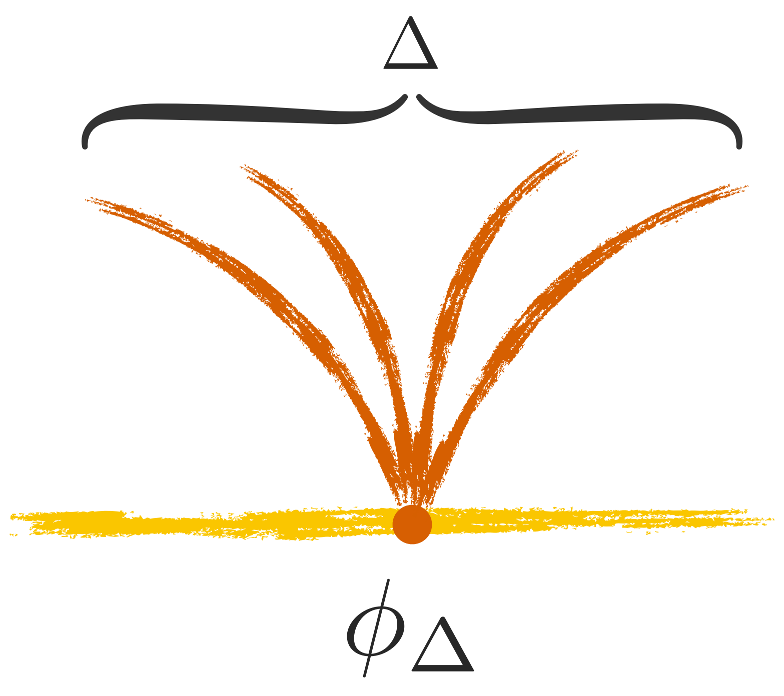
Results

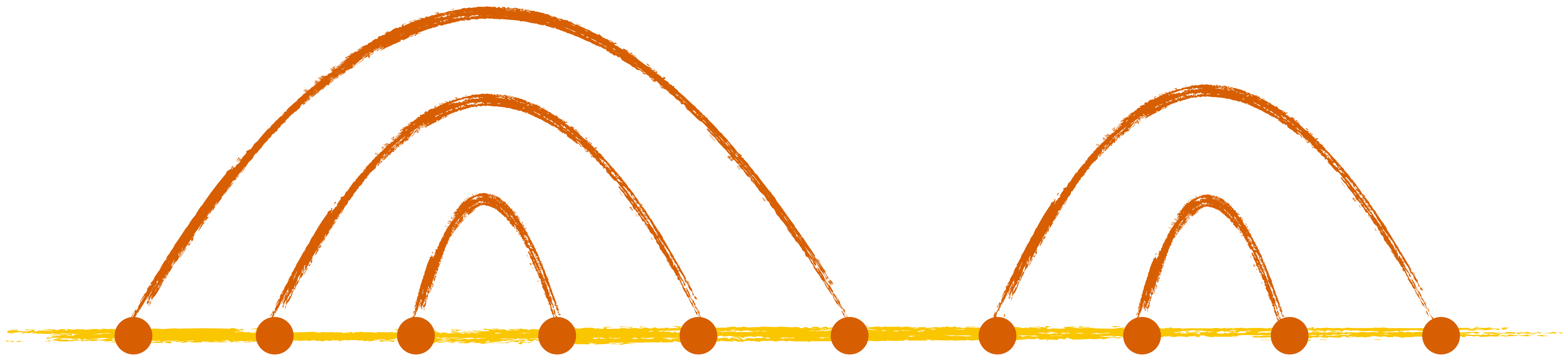


Results

From $\langle \phi_1^{I_1} \ldots \phi_1^{I_n} \rangle$ we can derive any $\langle \phi_{\Delta_1}^{I_1} \ldots \phi_{\Delta_n}^{I_n} \rangle$

Not the case in $\mathcal{N} = 4$ SYM !





$$\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \rangle$$

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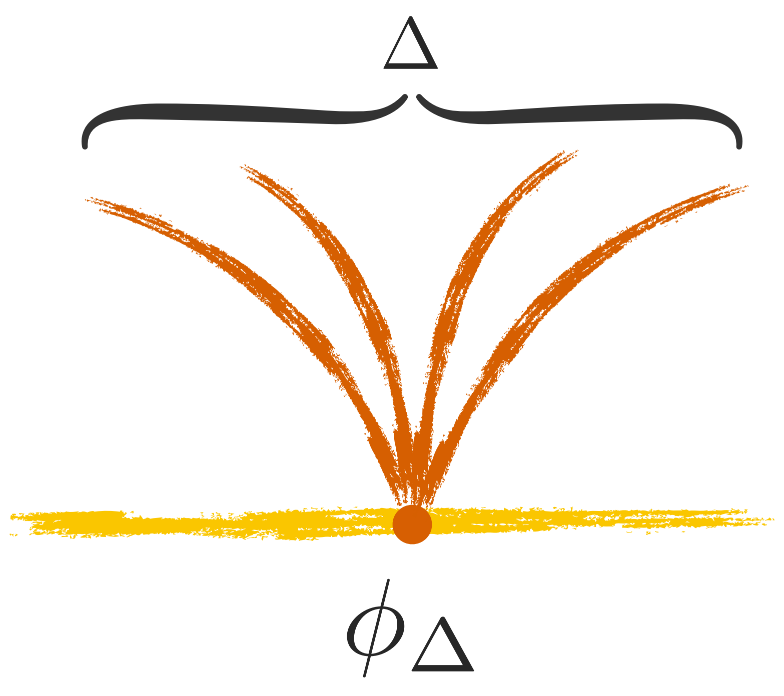

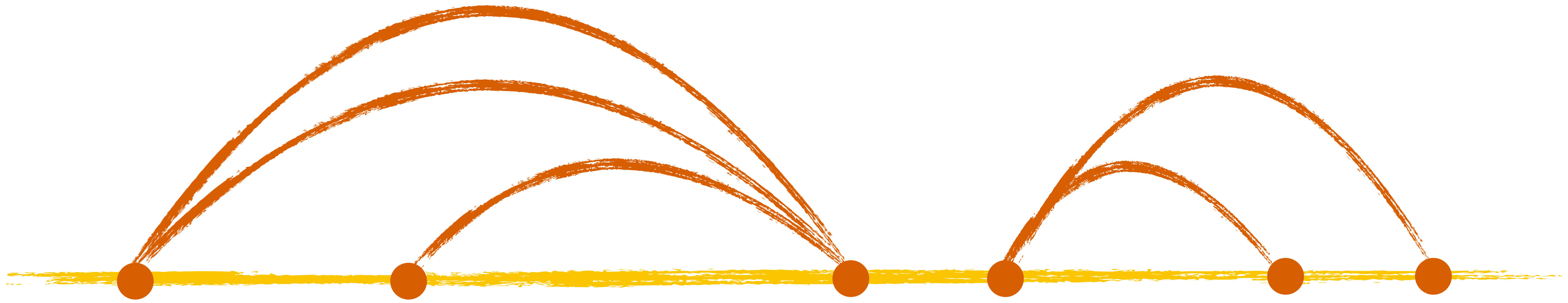

$$\langle \phi_2 \phi_1 \phi_3 \phi_2 \phi_1 \phi_1 \rangle$$

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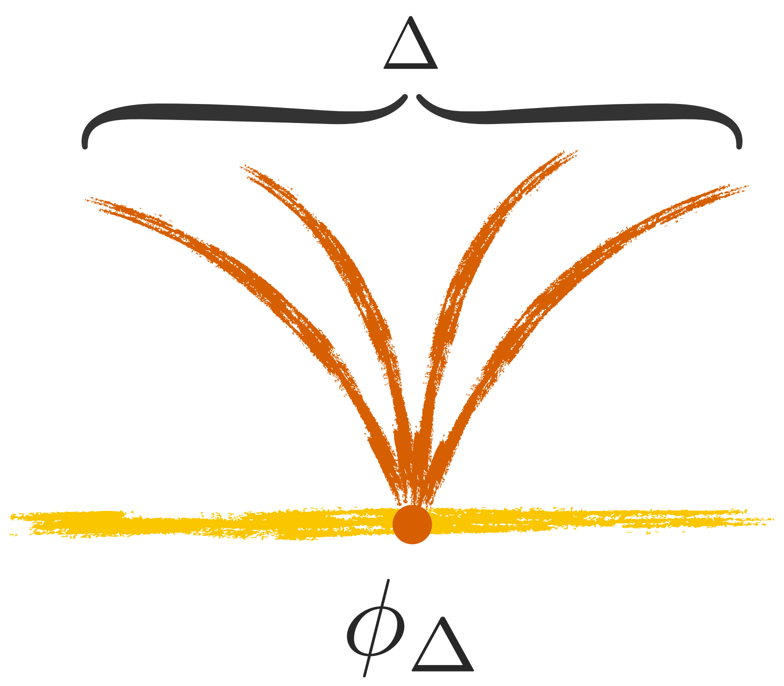

Not the case in $\mathcal{N} = 4$ SYM !

$\langle \phi_1 \phi_1 \phi_2 \phi_2 \rangle$

$\langle \phi_1 \phi_3 \phi_3 \phi_4 \rangle$

$\langle \phi_1 \phi_1 \phi_1 \phi_4 \phi_5 \rangle$

$\langle \phi^i \phi^j \phi^6 \phi^6 \phi^6 \rangle$

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_4 \rangle$

$\langle \phi^6 \phi^6 \phi^6 \phi^6 \phi^6 \rangle$

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_5 \rangle$

$\langle \phi_1 \phi_1 \phi_2 \phi_4 \phi_6 \rangle$

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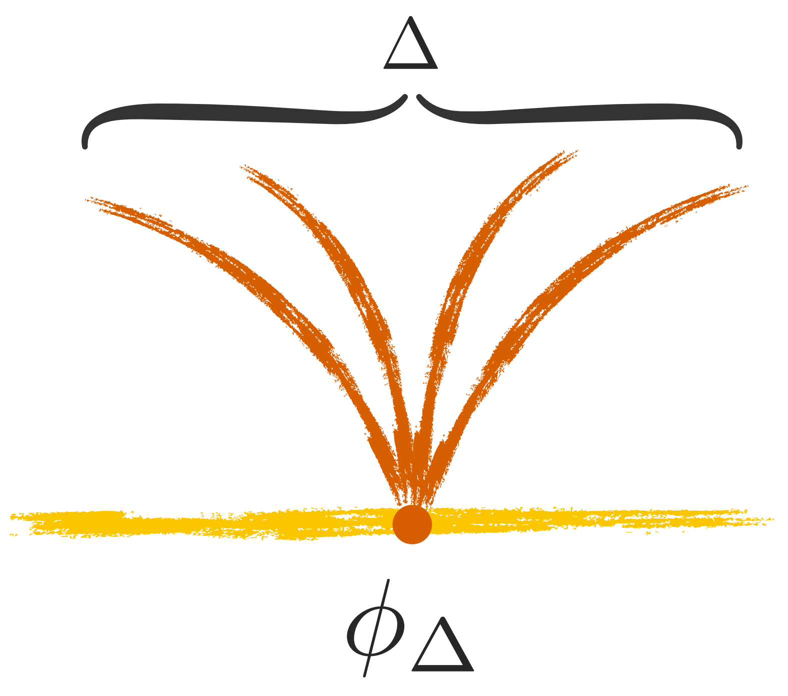
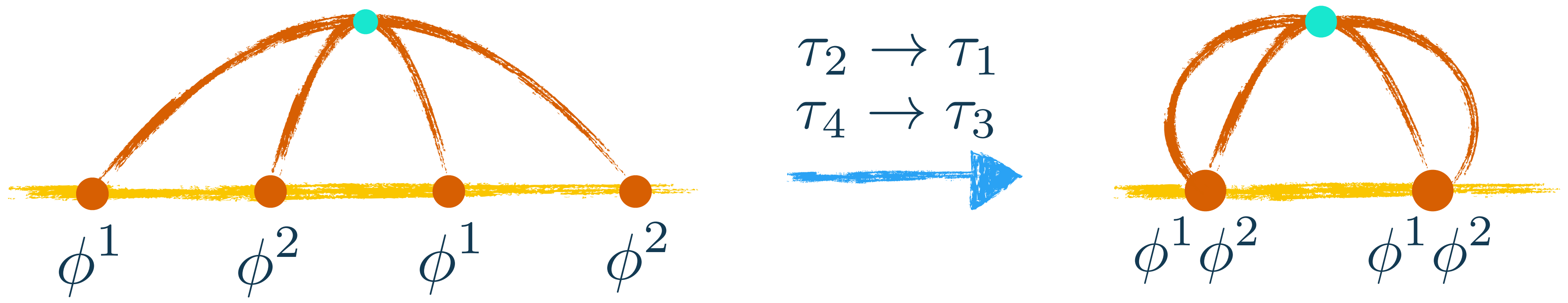
From $\langle \phi_1^{I_1} \dots \phi_1^{I_n} \rangle$ we can derive any $\langle \phi_{\Delta_1}^{I_1} \dots \phi_{\Delta_n}^{I_n} \rangle$



Not the case in $\mathcal{N} = 4$ SYM !

From $\langle \phi_1^{I_1} \dots \phi_1^{I_n} \rangle$ we can derive anomalous dimensions too!

$$O_A^{ij} := \phi^i \phi^j - \phi^j \phi^i$$
$$O_A^i := \phi^6 \phi^i - \phi^i \phi^6$$
$$O_S^i := \phi^6 \phi^i + \phi^i \phi^6$$
$$O_\pm := \delta^{ij} \phi^i \phi^j \pm \sqrt{5} \phi^6 \phi^6$$

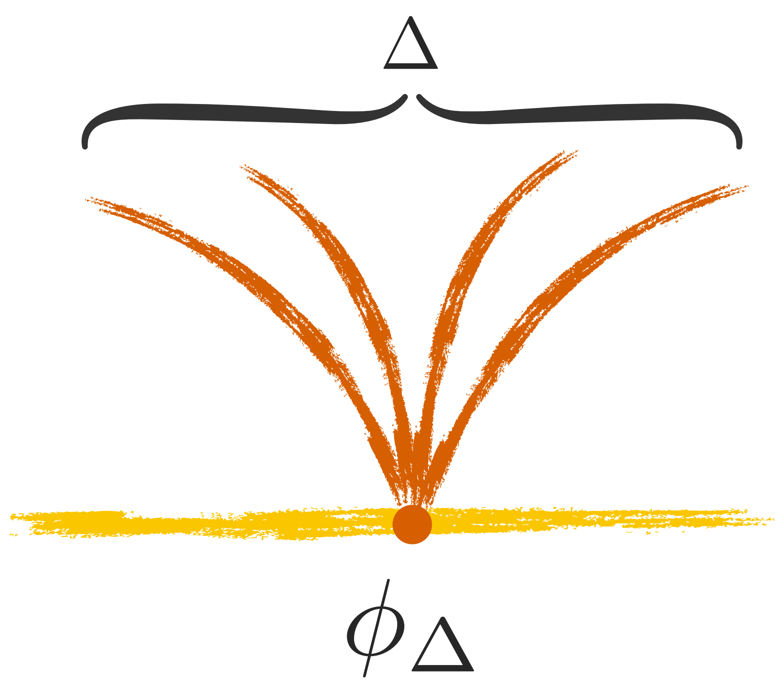


$$\left\langle O_A^{ij}(\tau_1) O_A^{ij}(\tau_2) \right\rangle \longrightarrow \Delta = 2 + \frac{\lambda}{4\pi^2} + O\left(\lambda^2\right)$$

[D. Correa, M. Leoni and S. Luque; 2018]

Results

From $\langle \phi_1^{I_1} \dots \phi_1^{I_n} \rangle$ we can derive any $\langle \phi_{\Delta_1}^{I_1} \dots \phi_{\Delta_n}^{I_n} \rangle$



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From $\langle \phi_1^{I_1} \dots \phi_1^{I_n} \rangle$ we can get correlation functions of composite operators!

$$\langle O\, O \dots \phi^I \phi^I \rangle \qquad \langle O\, O \dots O\, O \rangle$$

Ward Identities

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WI for $\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ were found in [Liendo, Meneghelli, Mitev; '18]

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Recent developments :

- [Ferrero, Meneghelli; '21] WI as constraint for NNNLO strong coupling result
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χ_i spacetime cross-ratios $\quad r_i, s_i, t_{ij}$ R-symmetry cross-ratios

$$\sum_{k=1}^{n-3} \left(\frac{1}{2} \partial_{\chi_k} + \alpha_k \partial_{r_k} - (1-\alpha_k)\, \partial_{s_k} \right) \mathcal{A}_{\Delta_1 \dots \Delta_n} \Bigg|_{\substack{r_k \to \alpha_k \\ s_k \to (1-\alpha_k)(1-\chi_k) \\ t_{ij} \to (\alpha_i - \alpha_j)(\chi_i - \chi_j)}} = 0$$

protected operators

Ward Identities

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protected operators

Perturbative computation leads to a non-perturbative constraint!

NNLO Correlator

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$\langle\phi_1\phi_1\phi_1\phi_1\rangle$ at NNLO

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$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right)\mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi,s=(1-\alpha)(1-\chi)} = 0$$

[Liendo, Meneghelli, Mitev; '18]

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right) \mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi,s=(1-\alpha)(1-\chi)} = 0 \quad \longrightarrow \quad \mathcal{A}(\chi;r,s) = \mathbb{F}\frac{\chi^2}{r} + \mathbb{D}f(\chi)$$

[Liendo, Meneghelli, Mitev; '18]

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right) \mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi, s=(1-\alpha)(1-\chi)} = 0 \longrightarrow \mathcal{A}(\chi; r, s) = \mathbb{F}\frac{\chi^2}{r} + \mathbb{D}f(\chi)$$

[Liendo, Meneghelli, Mitev; '18]

$$\mathcal{A}_{1111} := F_0(\chi) + \frac{\chi^2}{r}F_1(\chi) + \frac{s}{r}\frac{\chi^2}{(1-\chi)^2}F_2(\chi)$$

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right)\mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi, s=(1-\alpha)(1-\chi)} = 0 \longrightarrow \mathcal{A}(\chi; r, s) = \mathbb{F}\frac{\chi^2}{r} + \mathbb{D}f(\chi)$$

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$$\mathcal{A}_{1111} := F_0(\chi) + \frac{\chi^2}{r}F_1(\chi) + \frac{s}{r}\frac{\chi^2}{(1-\chi)^2}F_2(\chi)$$

$$F_0(\chi) = \left(\frac{2}{\chi} - 1\right)f(\chi) - (1-\chi)f'(\chi)$$

$$F_1(\chi) = \mathbb{F} - \frac{f(\chi)}{\chi^2} - \frac{1-\chi}{\chi}f'(\chi)$$

$$F_2(\chi) = \frac{(1-\chi)^2}{\chi^2}\left(f(\chi) - \chi f'(\chi)\right)$$

$\langle\phi_1\phi_1\phi_1\phi_1\rangle$ at NNLO

$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right)\mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi,s=(1-\alpha)(1-\chi)} = 0 \longrightarrow \mathcal{A}(\chi;r,s) = \mathbb{F}\frac{\chi^2}{r} + \mathbb{D}f(\chi)$$

[Liendo, Meneghelli, Mitev; '18]

$$\mathcal{A}_{1111} := F_0(\chi) + \frac{\chi^2}{r}F_1(\chi) + \frac{s}{r}\frac{\chi^2}{(1-\chi)^2}F_2(\chi)$$

$f(\chi)$

$$F_0(\chi) = \left(\frac{2}{\chi} - 1\right)f(\chi) - (1-\chi)f'(\chi)$$

$$F_1(\chi) = \mathbb{F} - \frac{f(\chi)}{\chi^2} - \frac{1-\chi}{\chi}f'(\chi)$$

$$F_2(\chi) = \frac{(1-\chi)^2}{\chi^2}\left(f(\chi) - \chi f'(\chi)\right)$$

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Known up to NNNLO at strong coupling

[Ferrero, Meneghelli; '21]

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Known up to NNNLO at strong coupling

[Ferrero, Meneghelli; '21]

$$f(\chi) := \frac{\chi}{1-\chi}h(\chi)$$

$$F_0(\chi) = \left(\frac{2}{\chi} - 1\right)f(\chi) - (1-\chi)f'(\chi)$$

$$F_1(\chi) = \mathbb{F} - \frac{f(\chi)}{\chi^2} - \frac{1-\chi}{\chi}f'(\chi)$$

$$F_2(\chi) = \frac{(1-\chi)^2}{\chi^2}\left(f(\chi) - \chi f'(\chi)\right)$$

$$h^{(0)}(\chi) = 1 - 2\chi$$

$$h^{(1)}(\chi) = -\frac{2\pi^2}{3}\chi - 2\left(H_{1,0} - H_{0,1}\right)$$

[Cavaglià, Gromov, Julius & Preti; '22]

$\langle\phi_1\phi_1\phi_1\phi_1\rangle$ at NNLO

$$\left(\frac{1}{2}\partial_\chi + \alpha\partial_r - (1-\alpha)\partial_s\right)\mathcal{A}_{\Delta_1\Delta_2\Delta_3\Delta_4}\Big|_{r=\alpha\chi,s=(1-\alpha)(1-\chi)} = 0 \quad \longrightarrow \quad \mathcal{A}(\chi;r,s) = \mathbb{F}\frac{\chi^2}{r} + \mathbb{D}f(\chi)$$

[Liendo, Meneghelli, Mitev; '18]

Known up to NNNLO at strong coupling

[Ferrero, Meneghelli; '21]

$$\mathcal{A}_{1111} := F_0(\chi) + \frac{\chi^2}{r}F_1(\chi) + \frac{s}{r}\frac{\chi^2}{(1-\chi)^2}F_2(\chi)$$

$$f(\chi) := \frac{\chi}{1-\chi}h(\chi)$$

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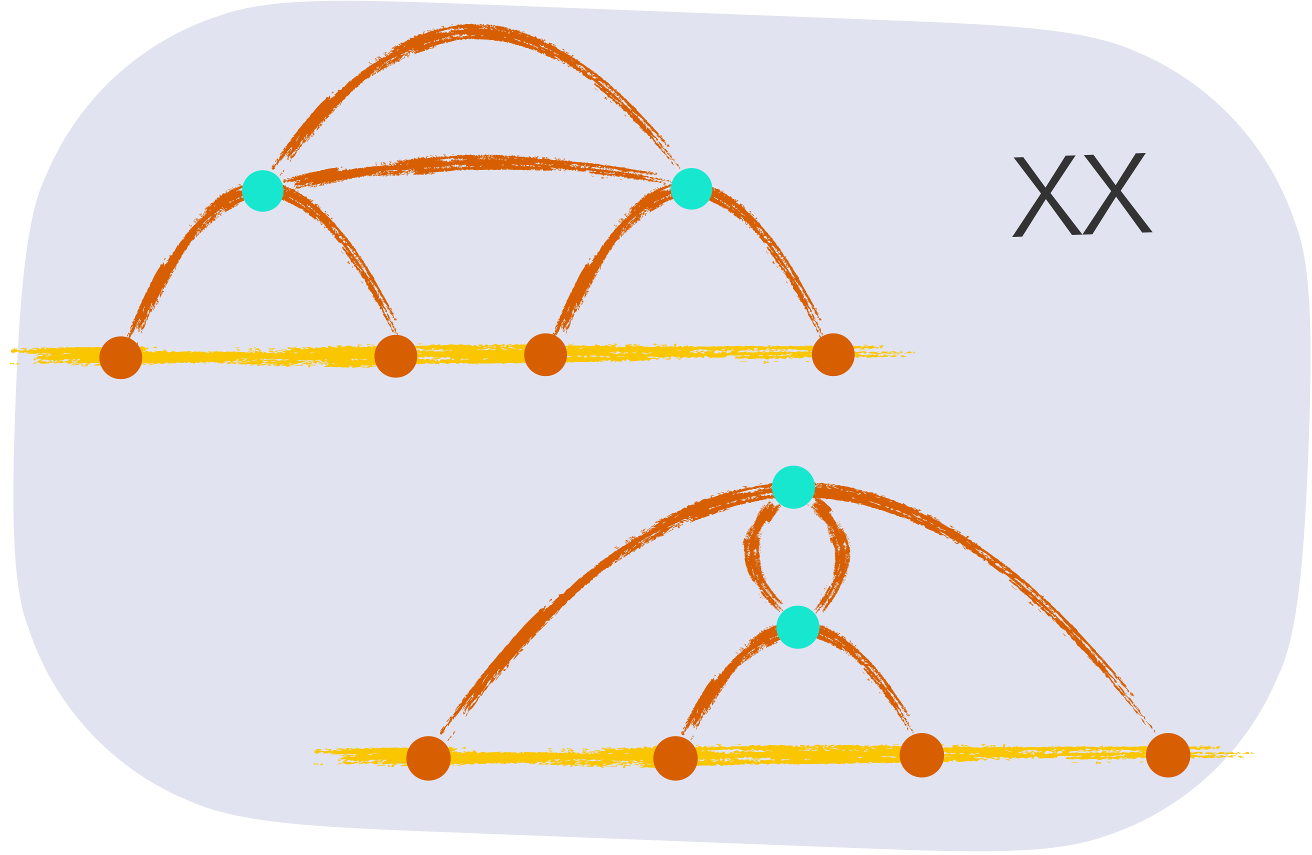
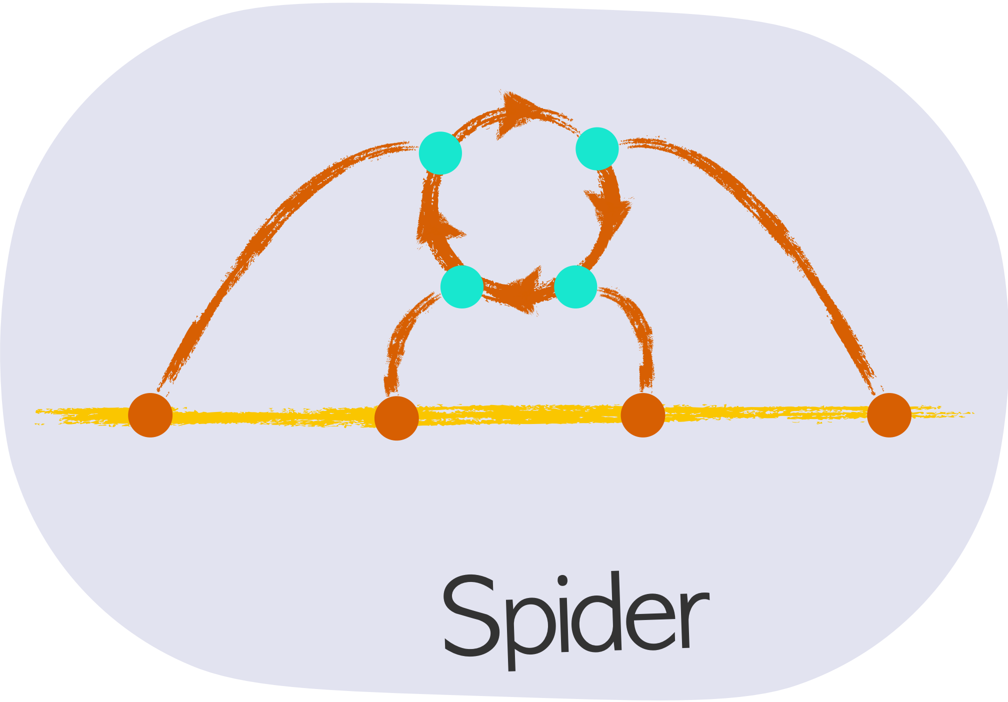
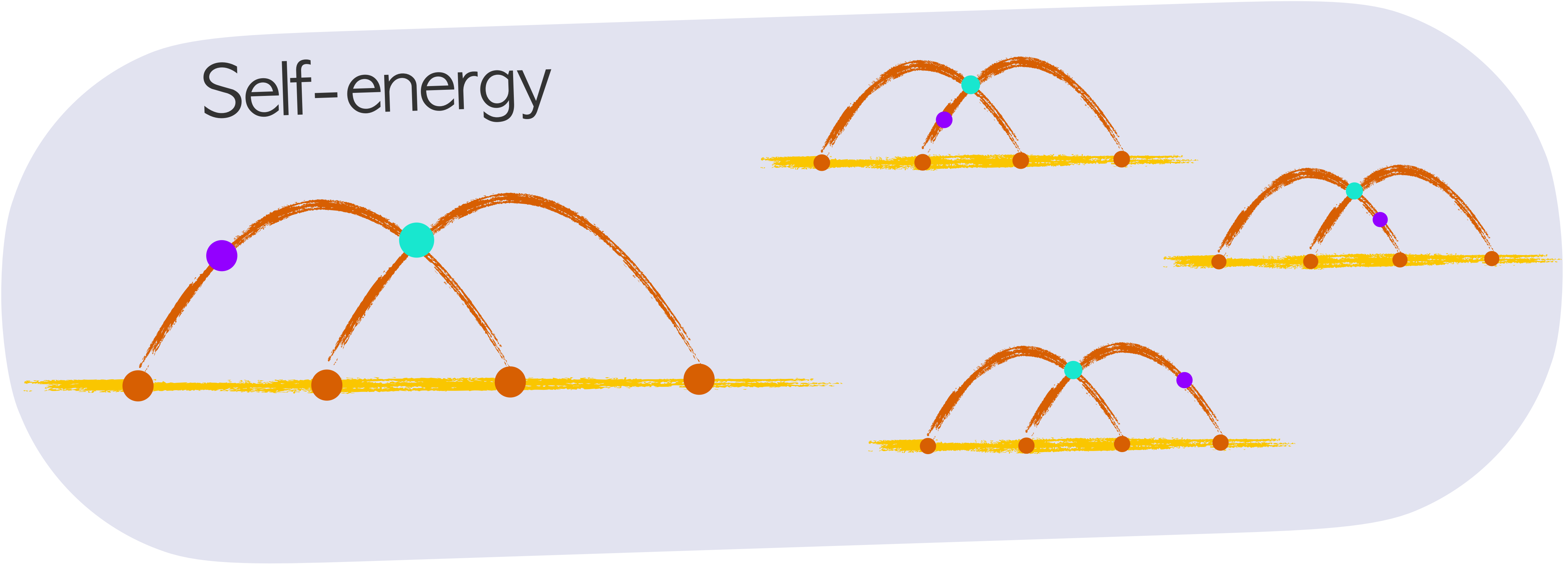
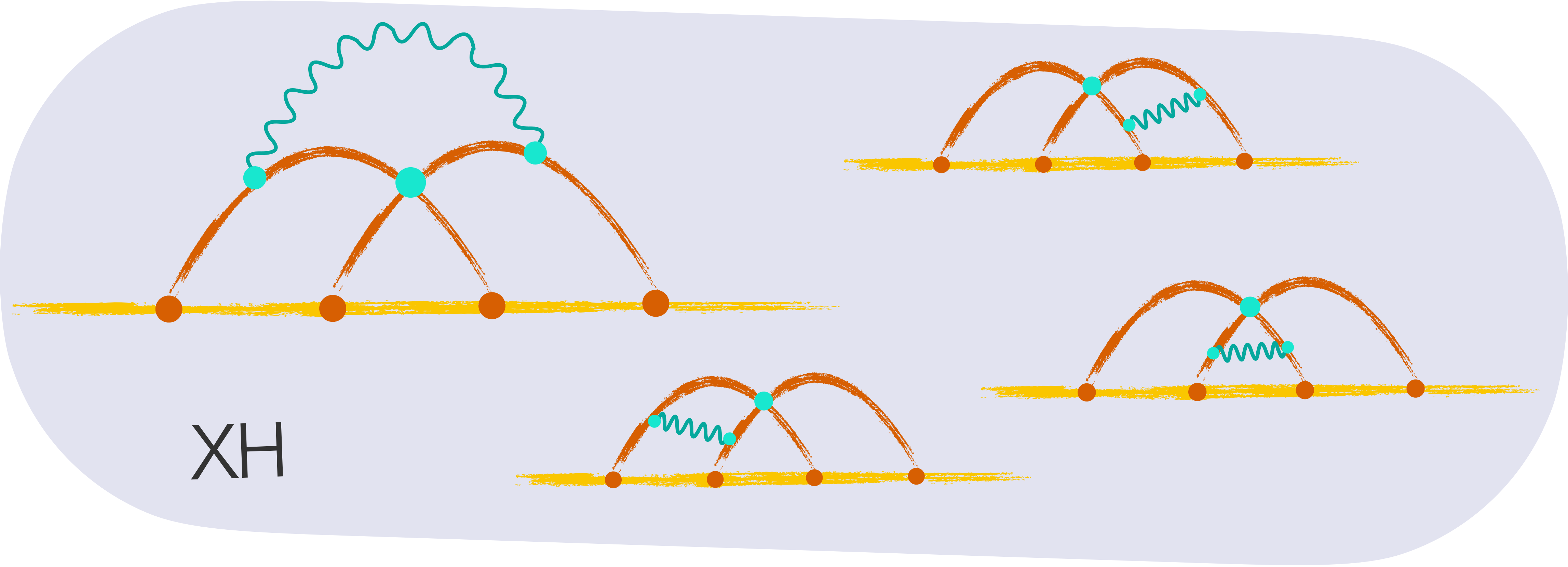
[Cavaglià, Gromov, Julius & Preti; '22]

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

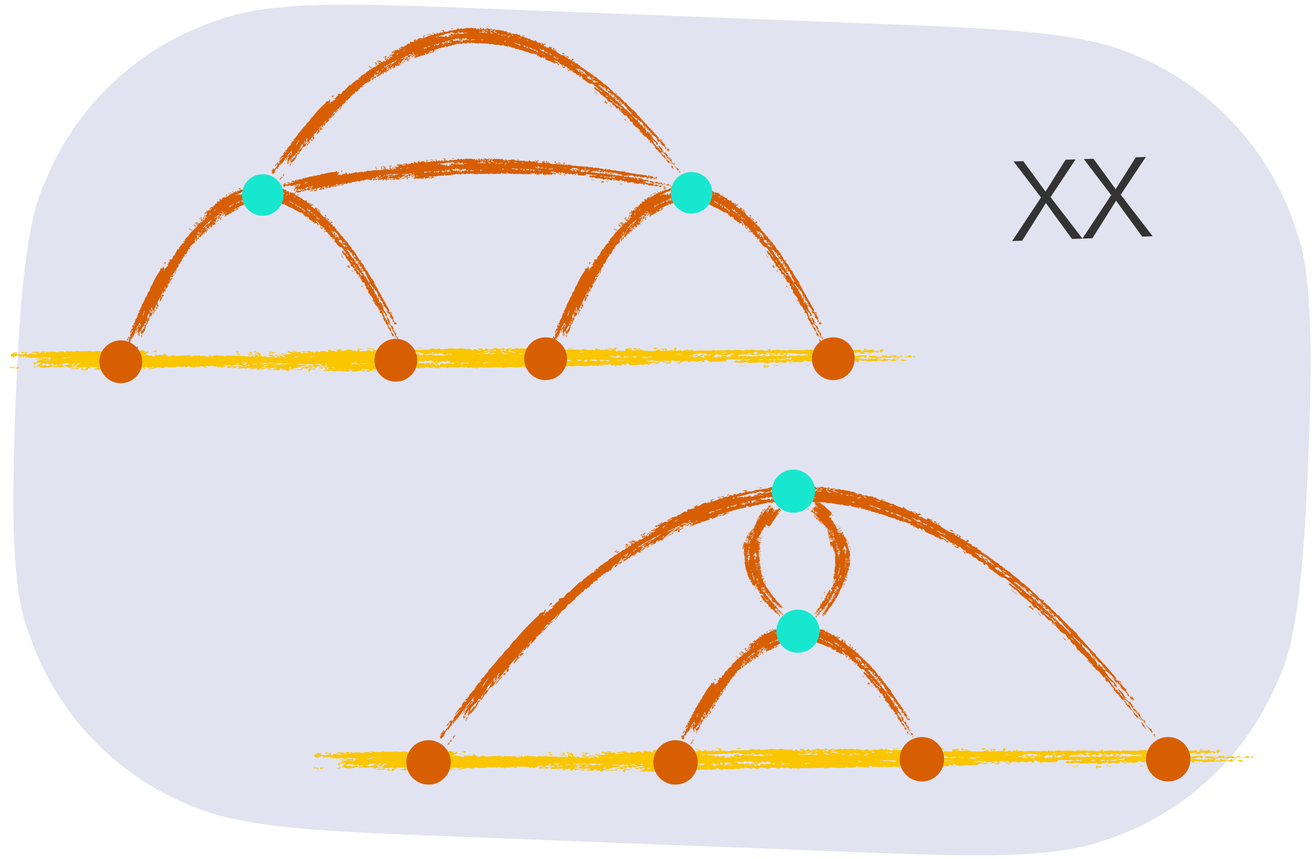


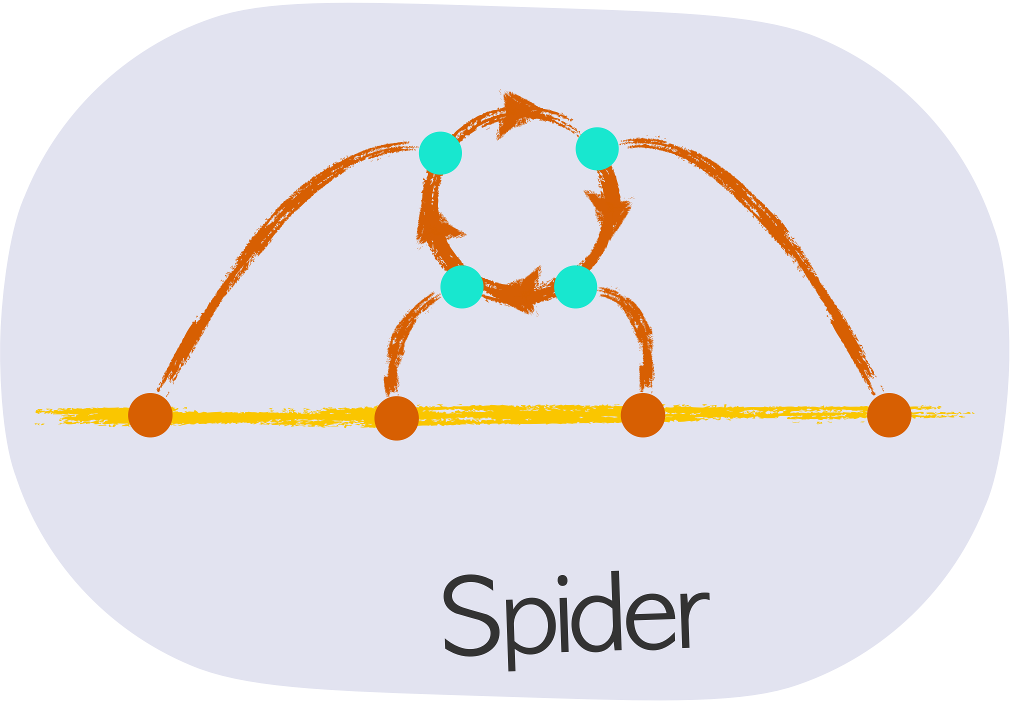


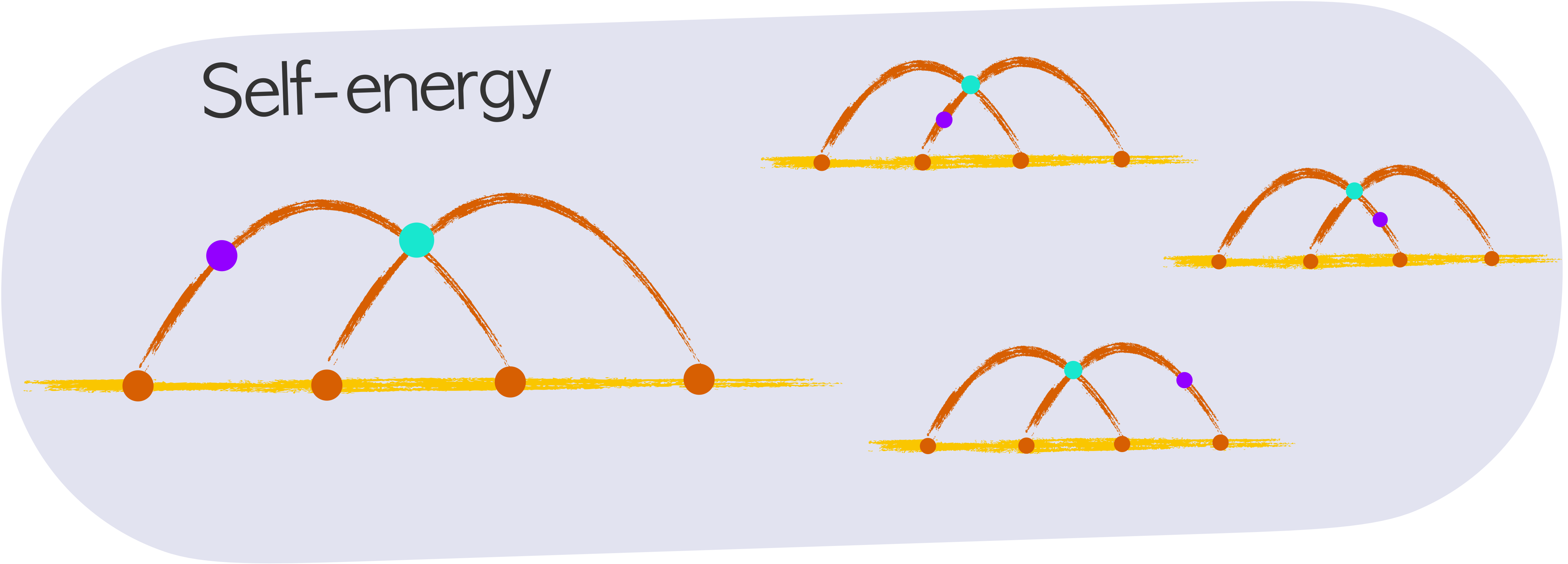


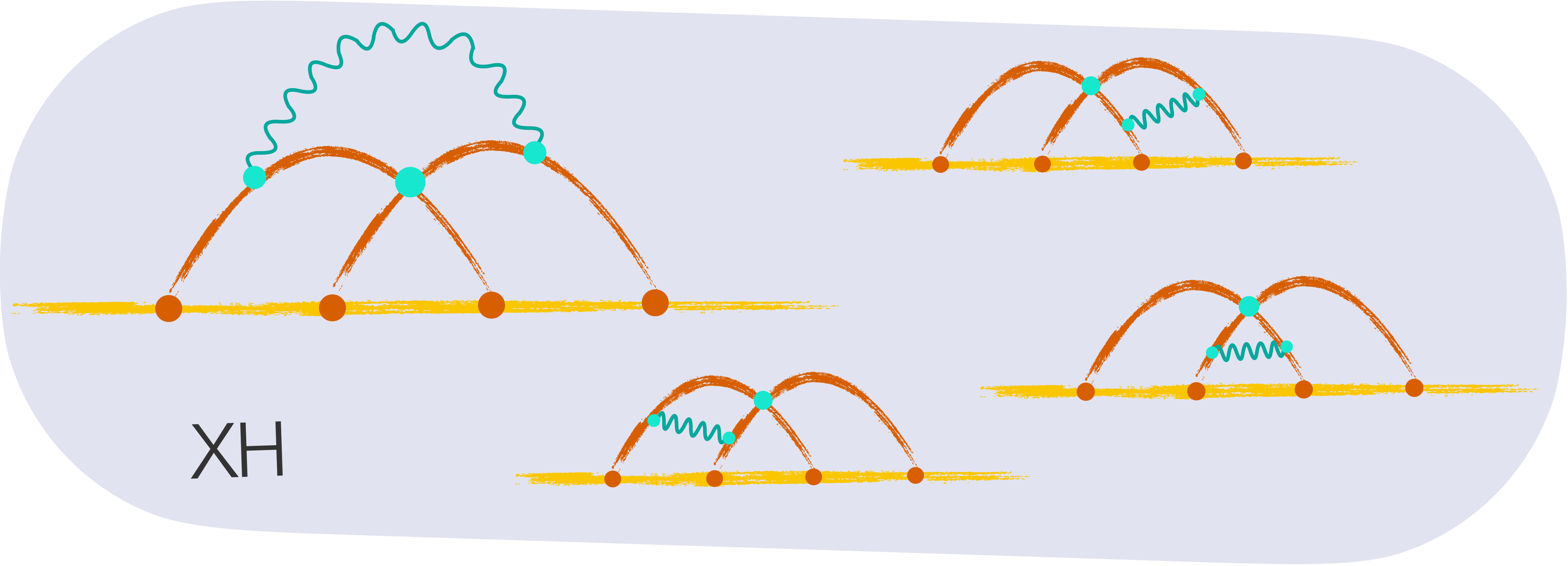


$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO





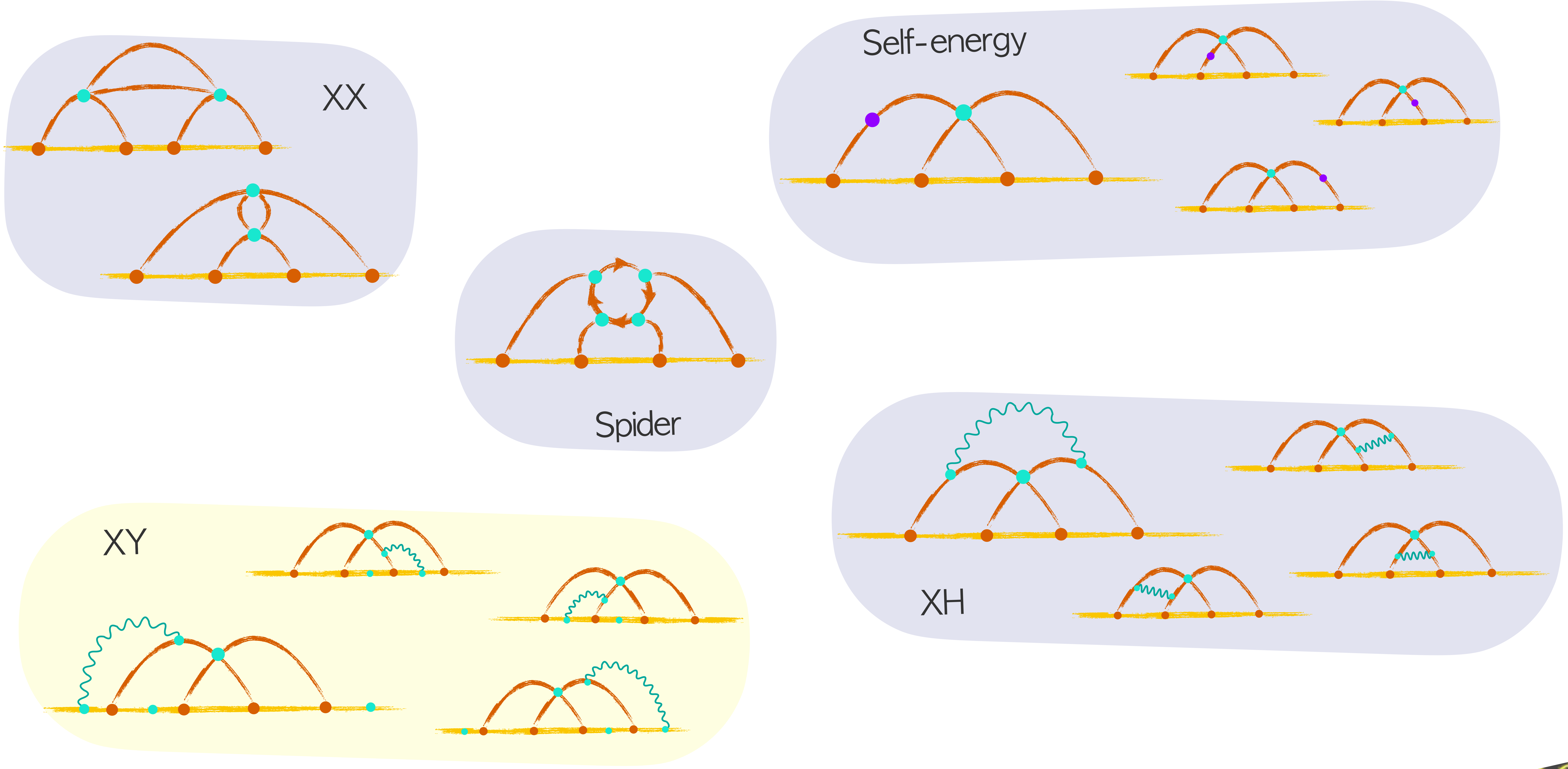




$-log(1-\chi)$ $\quad$ $Li_2(\chi)$

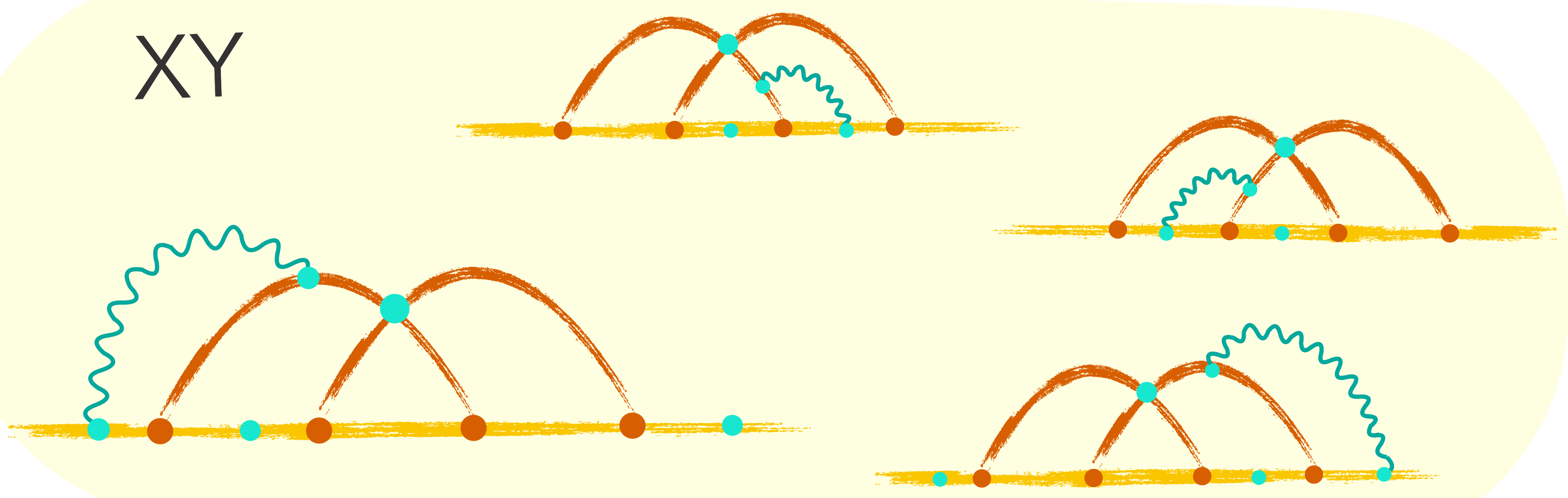
$$F_1^a = \frac{1}{8192\pi^8\chi(1-\chi)}\{-4H_1 + H_{1,0} + H_{0,1} - 2H_{1,1}$$
$$+ 3\left(H_{0,0,1} + H_{0,1,0} - 2H_{1,0,0}\right) - 2\left(H_{1,1,0} + H_{1,0,1} - 2H_{0,0,1}\right)$$
$$+ \chi\left(-3\zeta_3 - 4\left(H_0 + H_1\right) - 2\left(H_{0,0} - H_{1,1}\right)\right.$$
$$\left. - H_{0,0,1} + H_{0,1,0} - 2H_{1,0,0} - H_{1,0,1} + H_{1,1,0} - 2H_{0,1,1}\right)\}$$

$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO



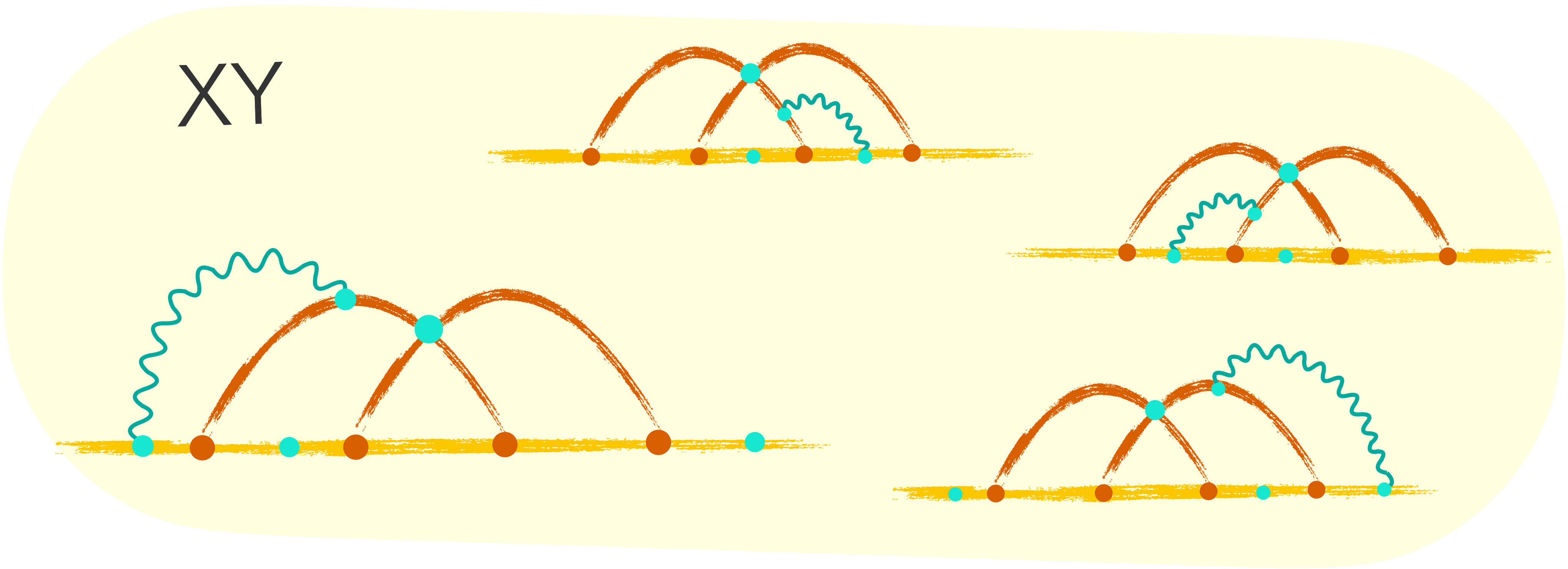
$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

XY



$\langle\phi_1\phi_1\phi_1\phi_1\rangle$ at NNLO

$$F_1^{XY,\mathrm{n}} = \frac{\lambda^4}{\chi(1-\chi)}\left(\sum_{\vec{a}} \alpha_{\vec{a}} H_{\vec{a}} + \chi \sum_{\vec{a}} \beta_{\vec{a}} H_{\vec{a}}\right)$$



$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO

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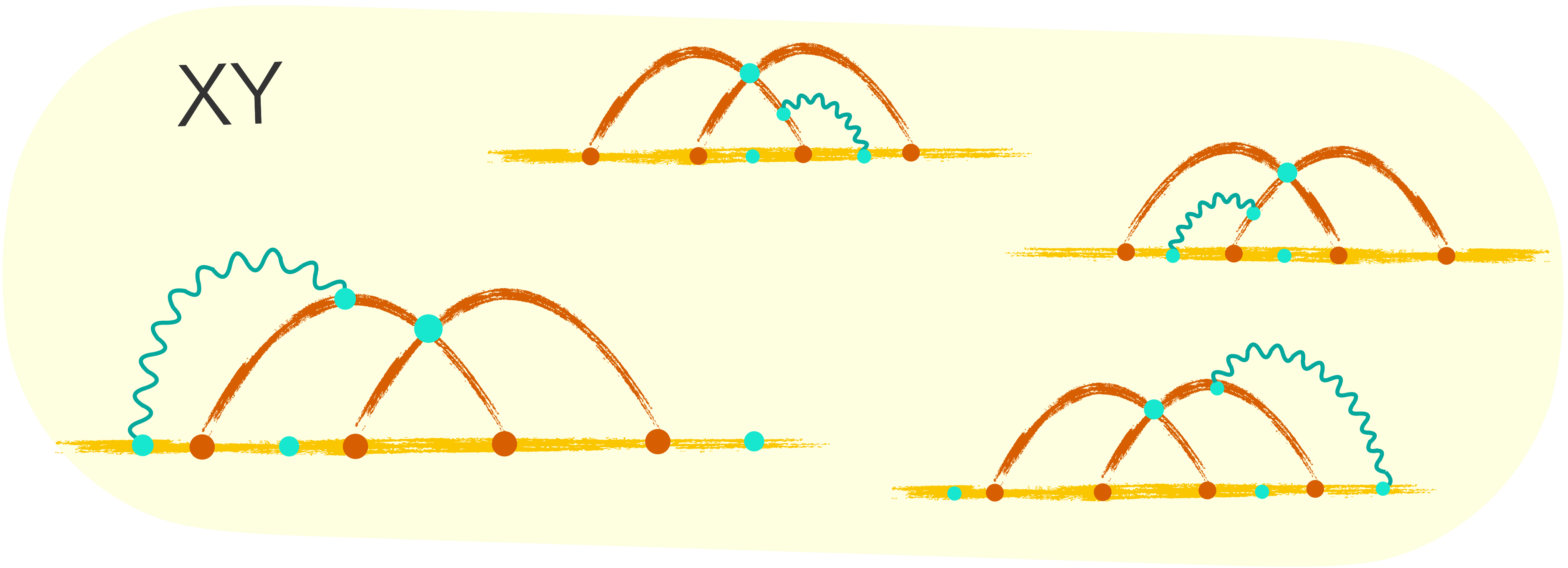
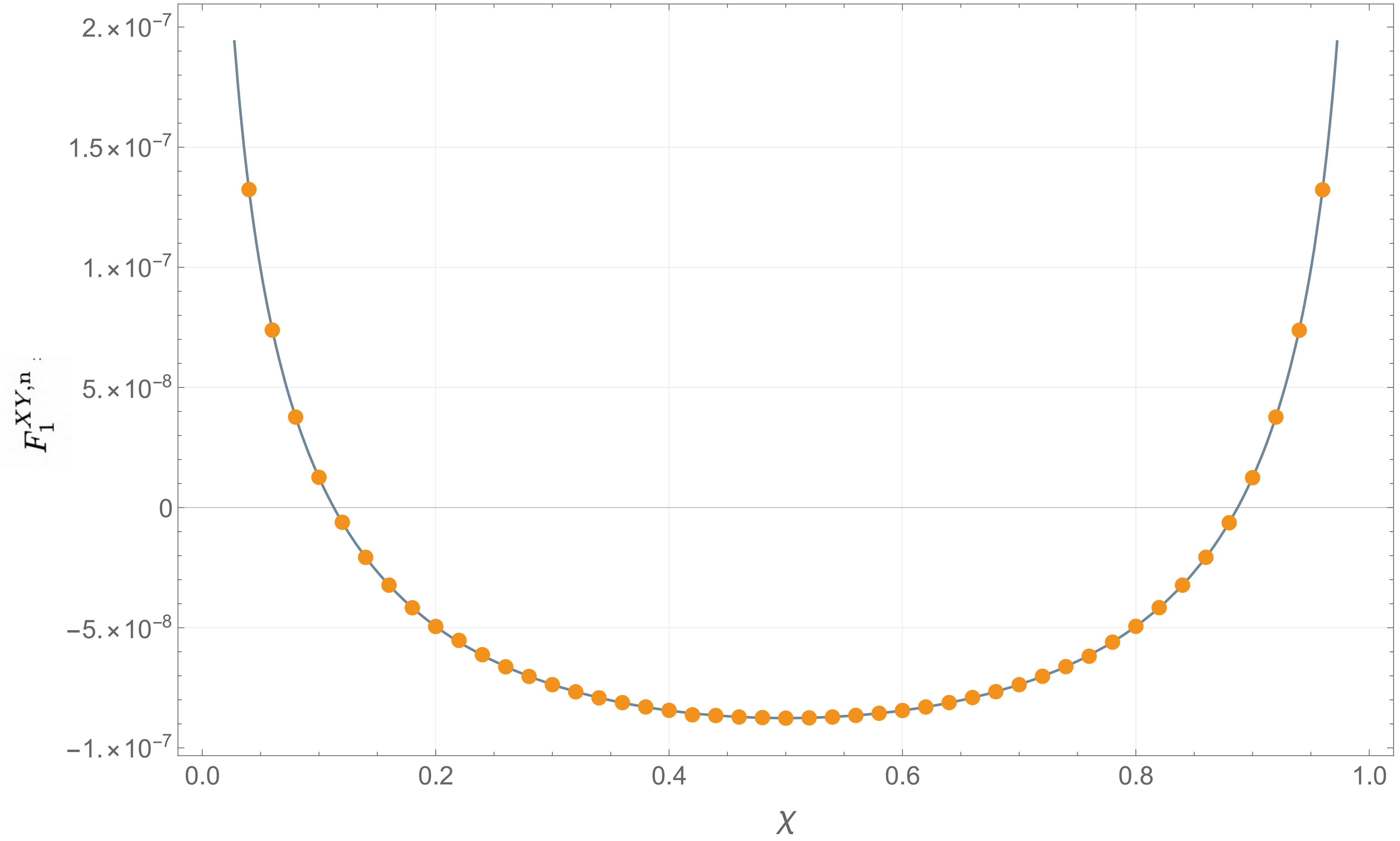
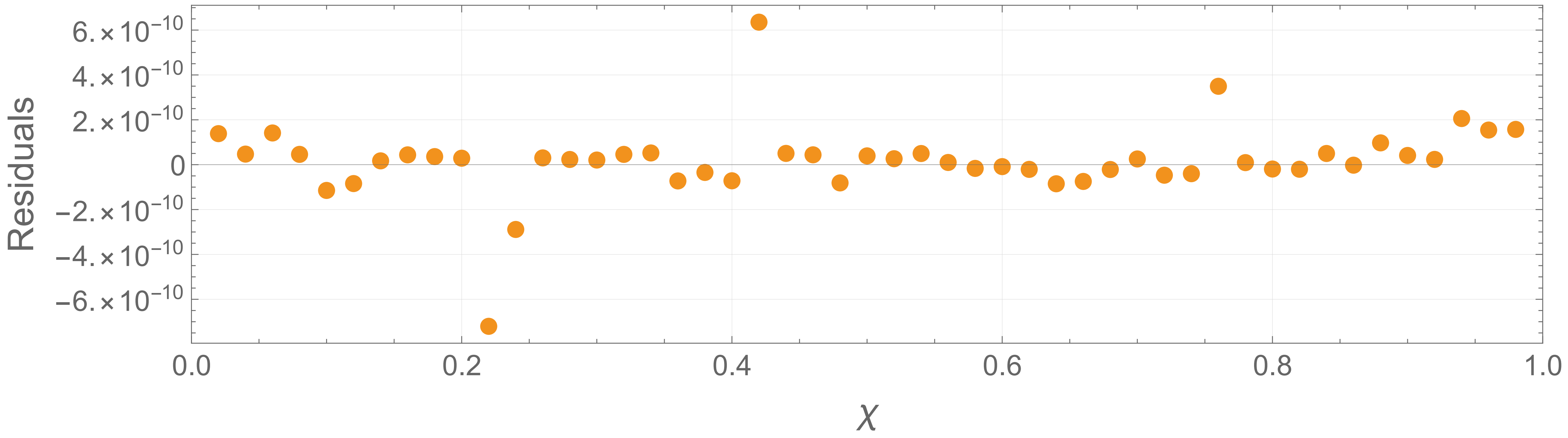
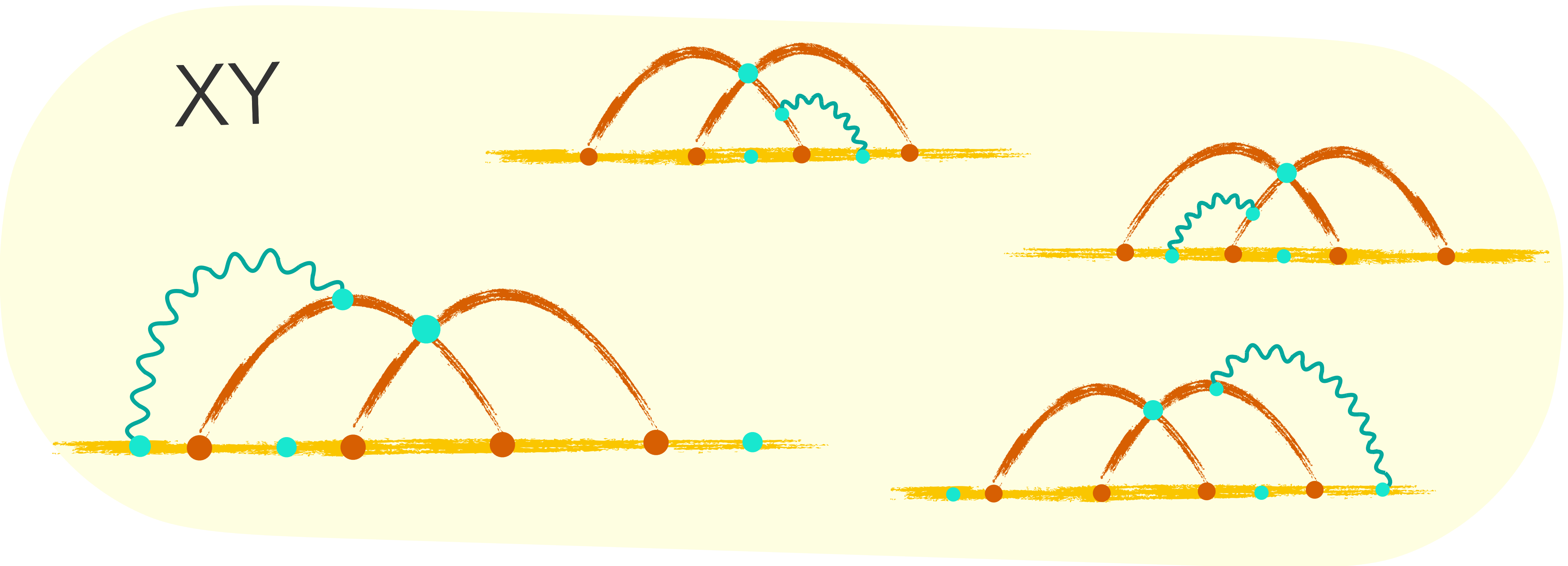


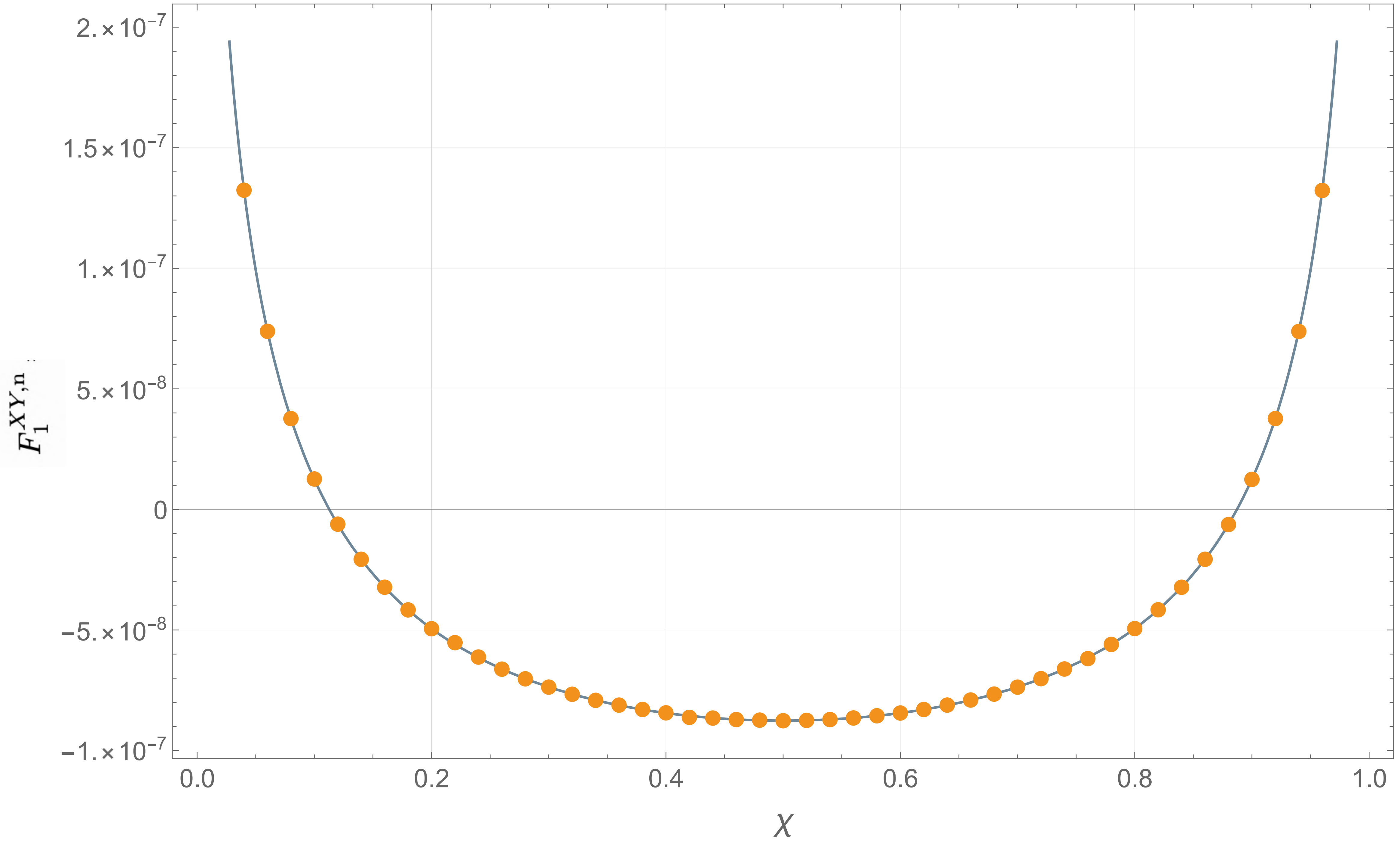


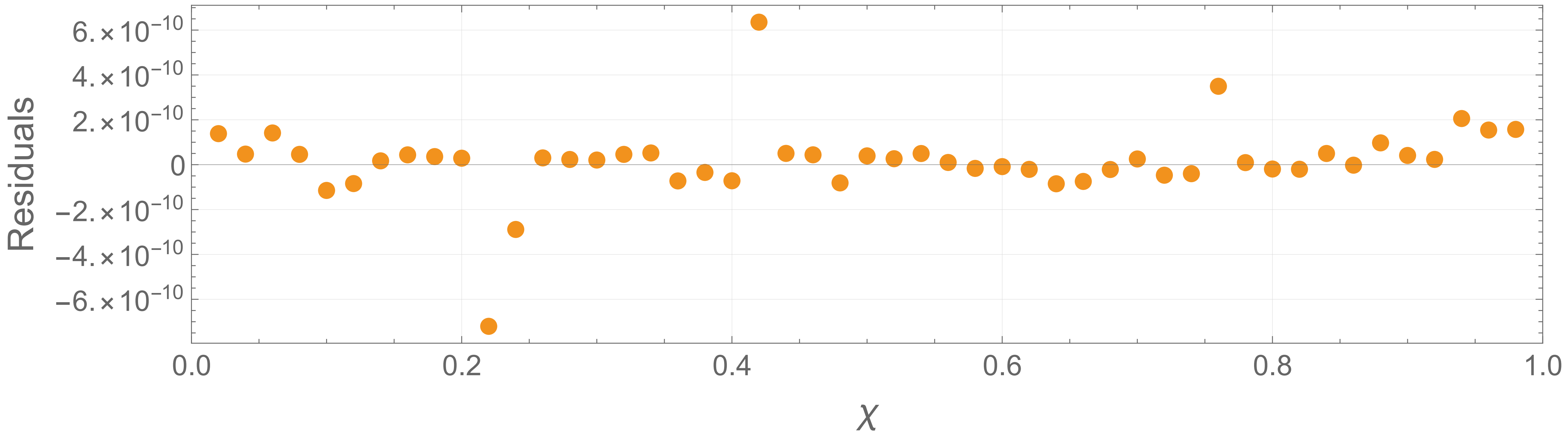


$\langle\phi_1\phi_1\phi_1\phi_1\rangle$ at NNLO

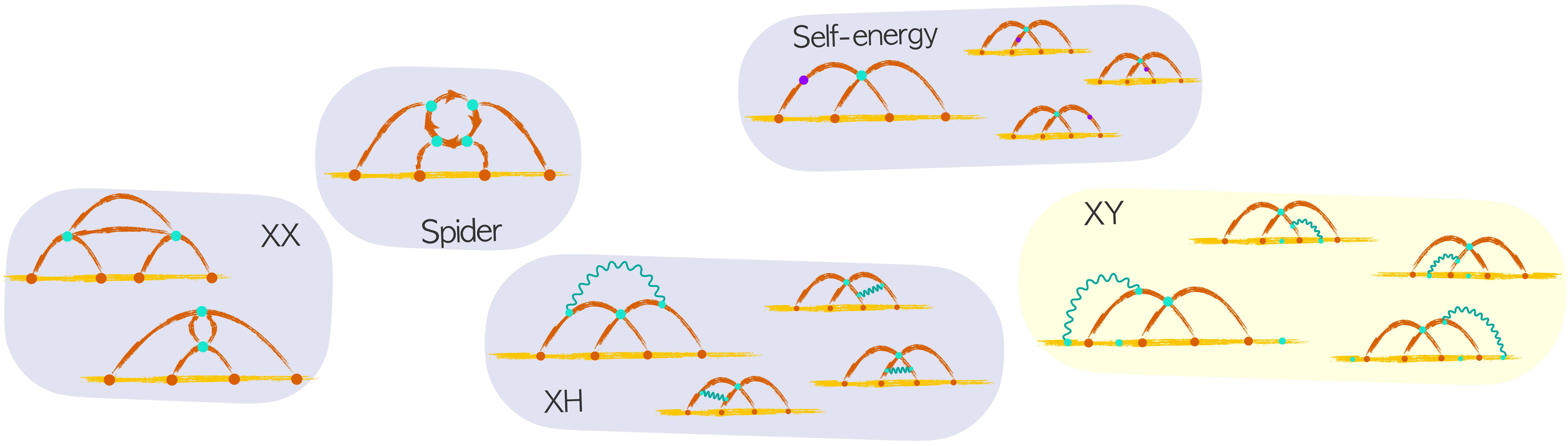
$$F_0^{XY,\mathrm{n}} = \frac{\lambda^4}{24576\pi^8}\frac{1}{\chi(1-\chi)}\big(2(6-\pi^2)H_1 + 3(H_{0,1}+H_{1,0}-2H_{1,1})
+ 3(3H_{0,0,1} - H_{0,1,0} - 2H_{1,0,0}) + \chi\big(2(\pi^2-6)H_0 + 2(\pi^2-3)H_1)
+ 6(H_{0,0}-H_{1,1}) + 3(3(H_{0,0,1}+H_{1,1,0}) - (H_{0,1,0}+H_{1,0,1})
- 2(H_{0,1,1}+H_{1,0,0})) - 9\zeta_3)\big)$$







$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$ at NNLO



$$F_1^{(2)} = \frac{1}{192\pi^4\chi(1-\chi)}\Big(\pi^2 H_1 - 3\left(H_{1,0,1} + H_{1,1,0} - 2\left(H_{0,1,0} + H_{0,1,1} - H_{1,0,0}\right)\right)$$
$$- 3\chi\left(\frac{\pi^2}{3}H_0 - \left(H_{0,0,1} + H_{1,1,0}\right) + H_{0,1,0} + H_{1,0,1} + 3\zeta_3\right)\Big)$$

[Cavaglià, Gromov, Julius & Preti; 2022]

5-point at strong coupling

23xx.xxxxx with J. Barrat, G. Bliard, P. Ferrero, C. Meneghelli

5-point at strong coupling

$$\langle\phi_1\phi_1\phi_1\phi_1\phi_1\phi_1\rangle \xrightarrow{6\to5} \langle\phi_1\phi_1\phi_1\phi_1\phi_2\rangle$$

5-point at strong coupling

$$\langle \phi_1\phi_1\phi_1\phi_1\phi_1\phi_1 \rangle \xrightarrow{6\to 5} \langle \phi_1\phi_1\phi_1\phi_1\phi_2 \rangle$$

$$\mathcal{A}_{11112} = F_0 + \frac{r_1}{\chi_1^2}F_1 + \frac{s_1}{(1-\chi_1)^2}F_2 + \frac{r_2}{\chi_2^2}F_3 + \frac{s_2}{(1-\chi_2)^2}F_4 + \frac{t}{\chi_{12}^2}F_5$$

5-point at strong coupling

$$\langle \phi_1\phi_1\phi_1\phi_1\phi_1\phi_1 \rangle \xrightarrow{6 \to 5} \langle \phi_1\phi_1\phi_1\phi_1\phi_2 \rangle$$

$$\mathcal{A}_{11112} = F_0 + \frac{r_1}{\chi_1^2} F_1 + \frac{s_1}{(1-\chi_1)^2} F_2 + \frac{r_2}{\chi_2^2} F_3 + \frac{s_2}{(1-\chi_2)^2} F_4 + \frac{t}{\chi_{12}^2} F_5$$

Ward identity

$$\left(\mathcal{D}_1 + \mathcal{D}_2\right) \mathcal{A}\Big|_{r_i \to \alpha_i \chi_i, s_i \to (1-\alpha_i)(1-\chi_i), t \to \alpha_{12}\chi_{12}} = 0$$

$$\mathcal{D}_i := \frac{1}{2}\partial_{\chi_i} + \alpha_i \partial_{r_i} - (1-\alpha_i)\,\partial_{s_i}$$

5-point at strong coupling

$$\langle \phi_1\phi_1\phi_1\phi_1\phi_1\phi_1 \rangle \xrightarrow{6 \to 5} \langle \phi_1\phi_1\phi_1\phi_1\phi_2 \rangle$$

$$\mathcal{A}_{11112} = F_0 + \frac{r_1}{\chi_1^2} F_1 + \frac{s_1}{(1-\chi_1)^2} F_2 + \frac{r_2}{\chi_2^2} F_3 + \frac{s_2}{(1-\chi_2)^2} F_4 + \frac{t}{\chi_{12}^2} F_5$$

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$$\left(\mathcal{D}_1 + \mathcal{D}_2\right) \mathcal{A}\Big|_{r_i \to \alpha_i \chi_i, s_i \to (1-\alpha_i)(1-\chi_i), t \to \alpha_{12}\chi_{12}} = 0$$

$$\mathcal{D}_i := \frac{1}{2}\partial_{\chi_i} + \alpha_i \partial_{r_i} - (1-\alpha_i)\,\partial_{s_i}$$

$$\longrightarrow \mathcal{A} = \mathbb{F} + \sum_{i=1,2,3} \mathbb{D}_i f_i\left(\chi_1, \chi_2\right)$$

5-point at strong coupling

- Witten diagrams

5-point at strong coupling

- Witten diagrams

5-point at strong coupling

- Witten diagrams

- Ansatz ⟵ 4pt strong coupling

5-point at strong coupling

- Witten diagrams

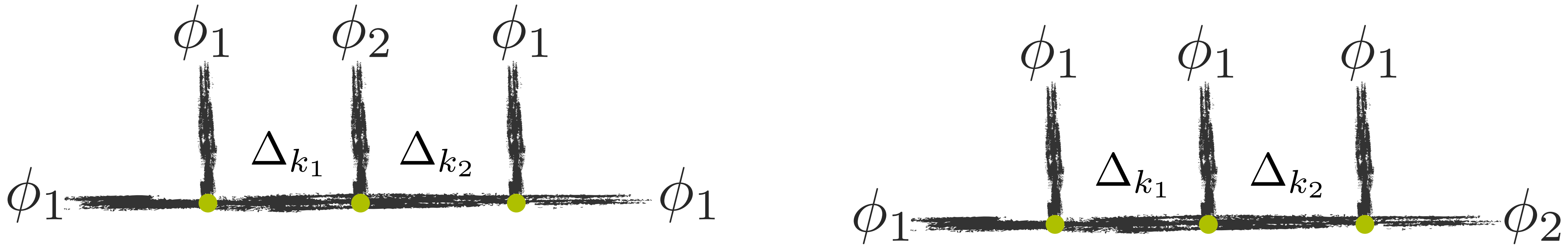
- Ansatz ← 4pt strong coupling

- Crossing symmetry $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$ + Braiding $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$

5-point at strong coupling

- Witten diagrams

- Ansatz ← 4pt strong coupling
- Crossing symmetry $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$ + Braiding $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$
- OPE + superconformal blocks



5-point at strong coupling

- Witten diagrams

- Ansatz ← 4pt strong coupling

- Crossing symmetry $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$ + Braiding $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$

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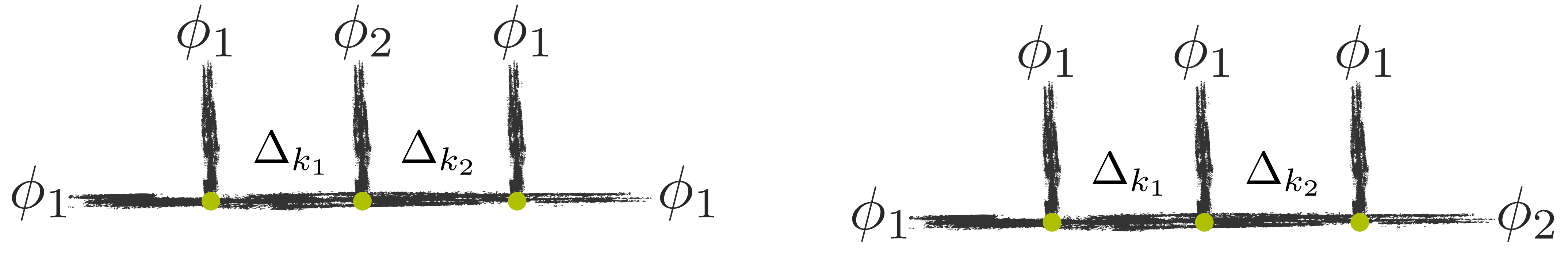
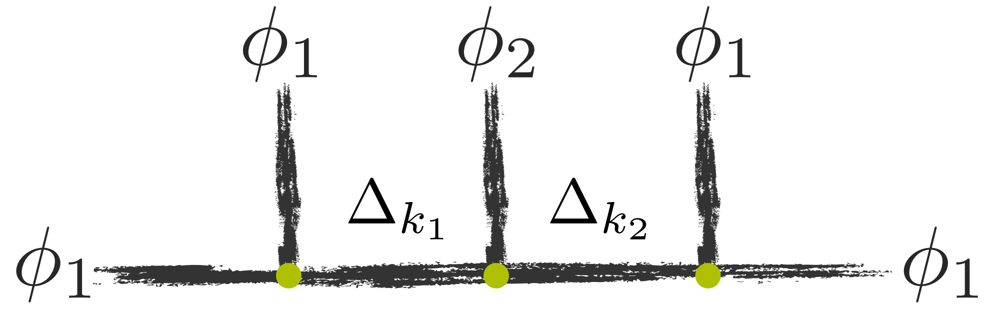


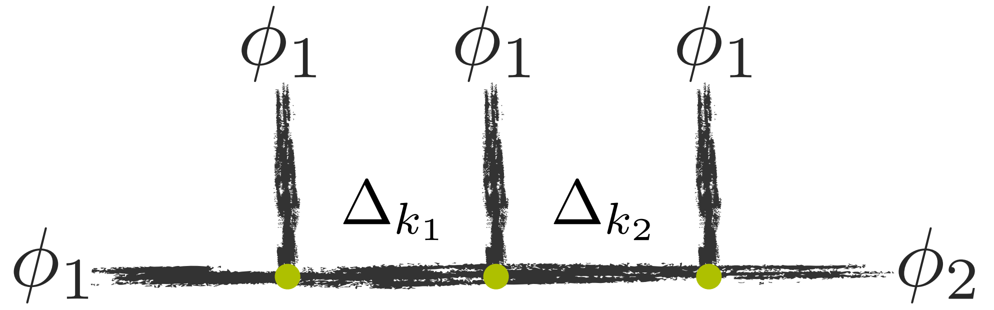
$$
\begin{aligned}
\mathcal{A} =& c_{112}\mathcal{G}_{\mathbb{I},\mathcal{D}_1} + \langle c_{112}c_{211}c_{112}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{D}_1} + \langle c_{112}c_{213}c_{312}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{D}_3} + \langle c_{112}c_{21\Delta}c_{\Delta 12}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{L}^{\Delta}_{0,[0,1]}} \\
&+ \langle c_{11\Delta}c_{\Delta 11}c_{112}\rangle \mathcal{G}_{\mathcal{L}^{\Delta}_{0,[0,0]},\mathcal{D}_1} + \langle c_{11\Delta}c_{\Delta 13}c_{312}\rangle \mathcal{G}_{\mathcal{L}^{\Delta}_{0,[0,0]},\mathcal{D}_3} + \langle c_{11\Delta_1}c_{\Delta_1 1\Delta_2}c_{\Delta_2 12}\rangle \mathcal{G}_{\mathcal{L}^{\Delta_1}_{0,[0,0]},\mathcal{L}^{\Delta}_{0,[0,1]}}
\end{aligned}
$$

5-point at strong coupling

- Witten diagrams

- Ansatz ← 4pt strong coupling
- Crossing symmetry $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$ + Braiding $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$
- OPE + superconformal blocks





$$\begin{aligned}\mathcal{A} =& c_{112}\mathcal{G}_{\mathbb{I},\mathcal{D}_1} + \langle c_{112}c_{211}c_{112}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{D}_1} + \langle c_{112}c_{213}c_{312}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{D}_3} + \langle c_{112}c_{21\Delta}c_{\Delta 12}\rangle \mathcal{G}_{\mathcal{D}_2,\mathcal{L}^{\Delta}_{0,[0,1]}} \\ &+ \langle c_{11\Delta}c_{\Delta 11}c_{112}\rangle \mathcal{G}_{\mathcal{L}^{\Delta}_{0,[0,0]},\mathcal{D}_1} + \langle c_{11\Delta}c_{\Delta 13}c_{312}\rangle \mathcal{G}_{\mathcal{L}^{\Delta}_{0,[0,0]},\mathcal{D}_3} + \langle c_{11\Delta_1}c_{\Delta_1 1\Delta_2}c_{\Delta_2 12}\rangle \mathcal{G}_{\mathcal{L}^{\Delta_1}_{0,[0,0]},\mathcal{L}^{\Delta}_{0,[0,1]}}\end{aligned}$$

$$\mathcal{G}_{\mathcal{X},\mathcal{Y}}(\chi_1,\chi_2;r_1,r_2,s_1,s_2,t) = \sum \alpha_k h_{[a,b],[c,d]}(\zeta_1,r_1,r_2,s_1,s_2,t)\, g_{h_1,h_2}(\chi_1,\chi_2)$$

Conclusion and outlook

- We develop an efficient algorithm to derive, up to NLO, multipoint correlation functions $\langle \phi^{I_1} \ldots \phi^{I_n} \rangle$ with an arbitrary number of fundamental scalar fields

- We use pinching to get operators of higher length and so in principle we can compute correlation functions of arbitrary operators containing fundamental scalars

→ anomalous dimension of operators of length 2 and we generate many correlators with non-protected scalars

- We obtain a lot of correlators of protected scalars up to n=8 and we observe that they are annihilated by a special class of differential operators, that we conjecture to be an extension of the Ward identities satisfied by the 4-pt

→ compute the NNLO of $\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle$

→ bootstrap $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_2 \rangle$

- Develop a similar recursive formula for fermionic fields (superfields)
- Prove the WI with superspace analysis
- Derived the WI for mixed setups
- Compute NNLO $\langle \phi_1 \phi_1 \phi_{\Delta_k} \phi_{\Delta_k} \rangle$
- Find recursive formula for NNLO ← Compute NNLO $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \rangle$
- Bootstrap other multipoint correlators at strong coupling (e.g. $\langle \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \phi_1 \rangle$)
- Apply this analysis to other defects e.g. fermonic Wilson line in ABJM

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- Prove the WI with superspace analysis
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- Apply this analysis to other defects e.g. fermonic Wilson line in ABJM

THANK YOU!

$$F_0^{(0)} = F_2^{(0)} = F_5^{(0)} = \frac{1}{512\pi^6}\,, \qquad F_j^{(0)} = 0 \text{ otherwise.}$$

$$\begin{aligned}
F_0^{(1)} =& -\frac{1}{12288\pi^6} + \frac{1}{4096\pi^8(\chi_2-\chi_1)}\left(\ell(\chi_1,\chi_2) + 2(\chi_2-\chi_1)\left(L_R\left(\frac{\chi_1-\chi_2}{\chi_1}\right) + \frac{i\pi}{2}\log\frac{\chi_1}{\chi_2}\right)\right)\,,\\
F_1^{(1)} =& 0\,,\\
F_2^{(1)} =& -\frac{\chi_2}{4096\pi^8\chi_1(\chi_2-\chi_1)}\ell(\chi_1,\chi_2)\,,\\
F_3^{(1)} =& \frac{\chi_2}{4096\pi^8\chi_1(\chi_2-\chi_1)}(\ell(1-\chi_1,1-\chi_2) + i\pi(\chi_2-\chi_1))\,,\\
F_4^{(1)} =& -\frac{1}{12288\pi^6} - \frac{1}{4096\pi^8(\chi_2-\chi_1)}\Bigg(\ell(\chi_1,\chi_2)\\
&\qquad -(\chi_2-\chi_1)\left(L_R\left(\frac{\chi_1-\chi_2}{1-\chi_2}\right) - i\pi\left(1+\log\frac{1-\chi_1}{1-\chi_2}\right)\right)\Bigg)\,,\\
F_5^{(1)} =& -\frac{5}{24576\pi^6} - \frac{1}{4096\pi^8\chi_1(1-\chi_2)}\Bigg(\chi_1\ell(1-\chi_1,1-\chi_2) - (1-\chi_2)\ell(\chi_1,\chi_2)\\
&\qquad +\chi_1(1-\chi_2)\left(\mathrm{Li}_2\left(\frac{1-\chi_1}{\chi_2-\chi_1}\right) - \mathrm{Li}_2\left(-\frac{1-\chi_2}{\chi_2-\chi_1}\right) - 2L_R\left(\frac{\chi_1}{\chi_1-\chi_2}\right)\right)\\
&\qquad +i\pi\chi_1\left((\chi_2-\chi_1) + (1-\chi_2)\log\left(-\frac{\chi_2(1-\chi_1)}{(\chi_1-\chi_2)^2}\right)\right)\Bigg)\,.
\end{aligned}$$

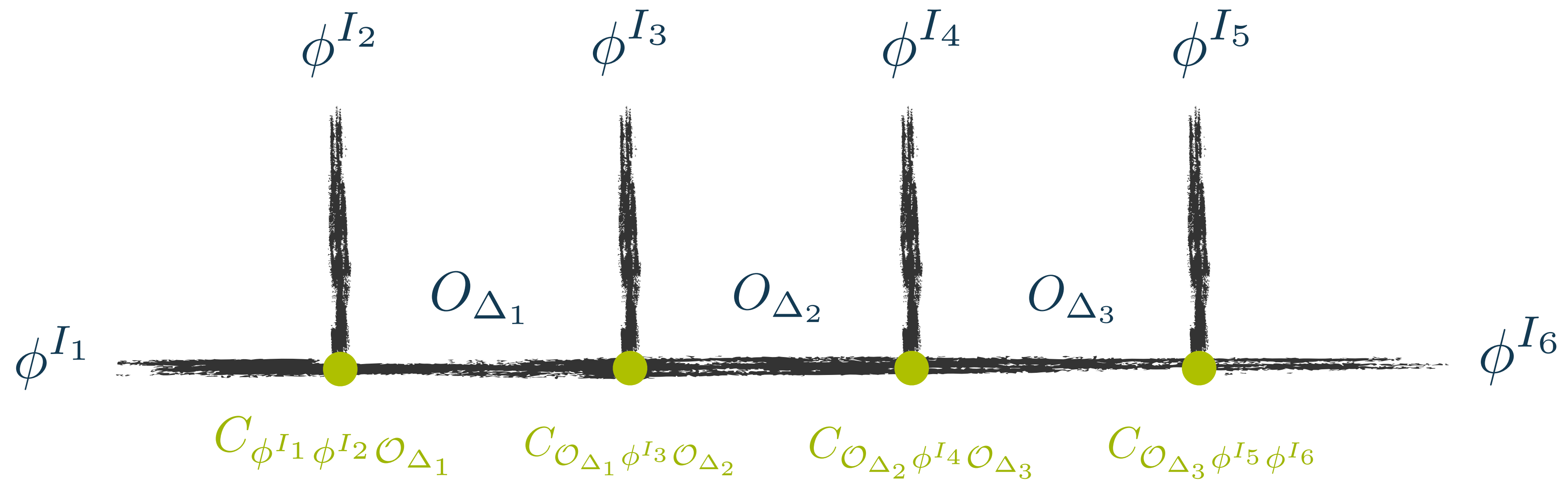
Block expansion

$$\mathcal{A}^{I_1 \dots I_6} = \sum_{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3} C_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3} \, g_{\Delta_1, \Delta_2, \Delta_3} (\chi_1, \chi_2, \chi_3)$$

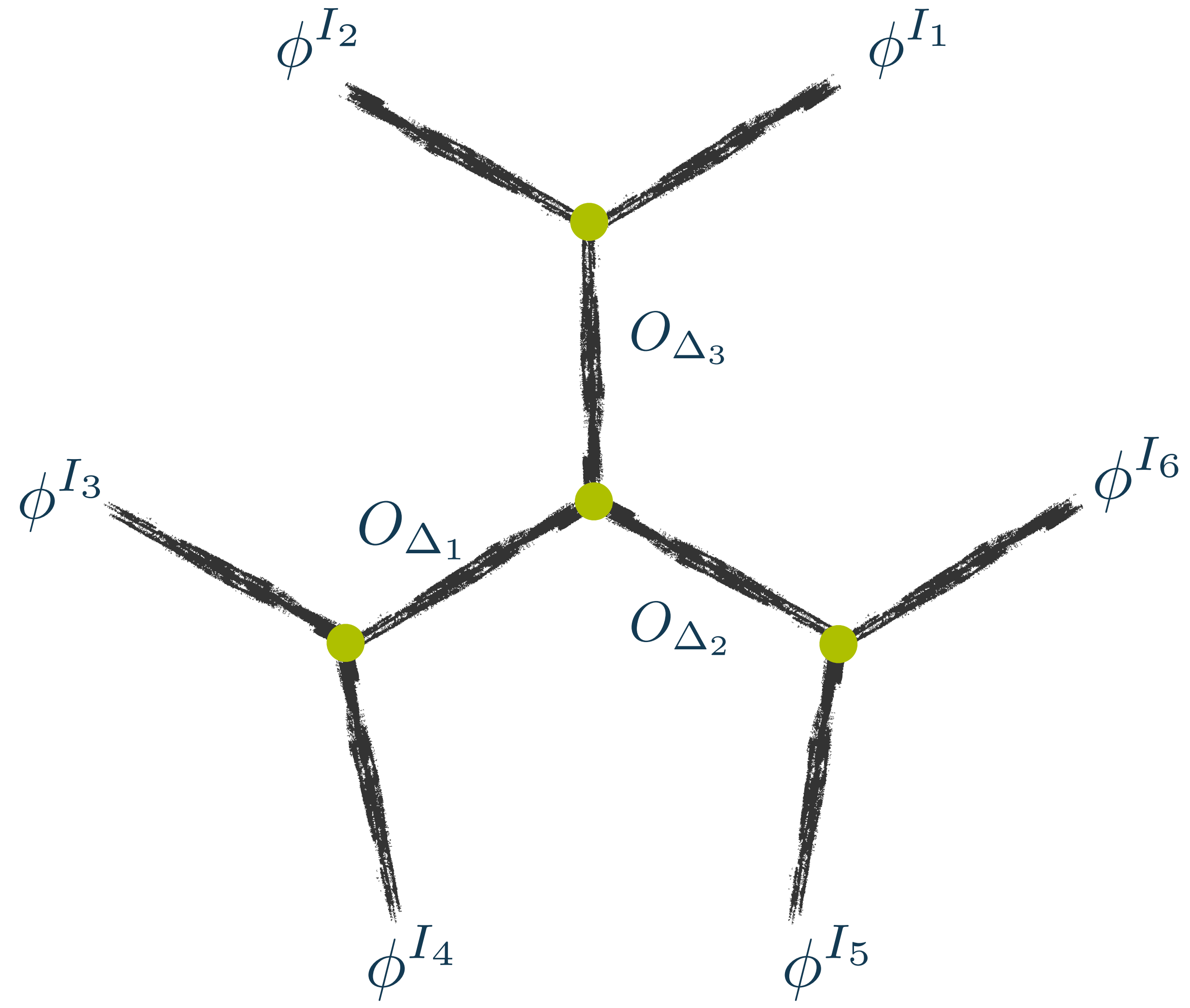
Block expansion

$$\mathcal{A}^{I_1\ldots I_6} = \sum_{\mathcal{O}_1,\mathcal{O}_2,\mathcal{O}_3} C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}\, g_{\Delta_1,\Delta_2,\Delta_3}(\chi_1,\chi_2,\chi_3)$$

Comb channel

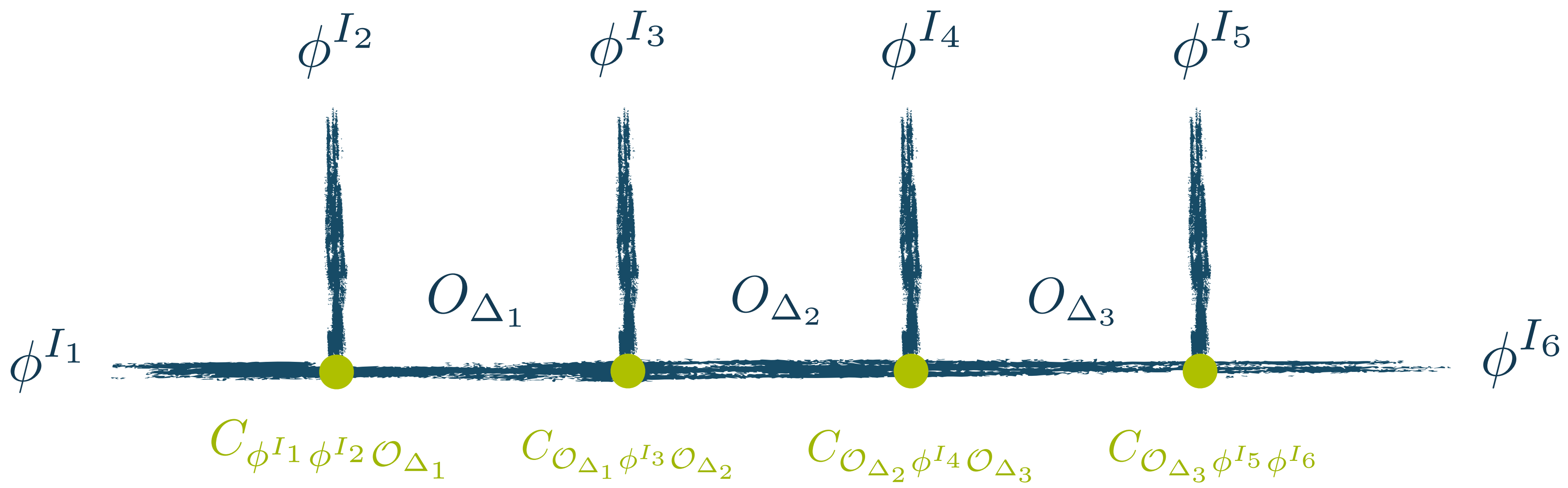


Snowflake channel



Block expansion

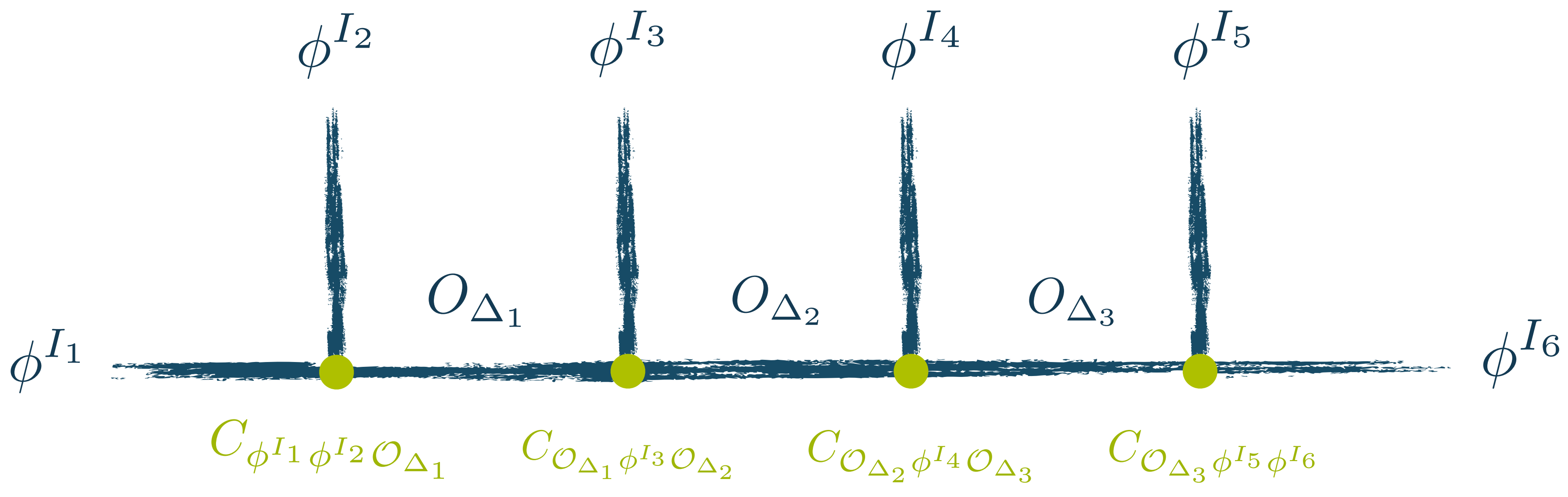
$$\mathcal{A}^{I_1 \ldots I_6} = \sum_{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3} C_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3}\, g_{\Delta_1, \Delta_2, \Delta_3}(\chi_1, \chi_2, \chi_3)$$



$$\langle \phi^6 \phi^6 \phi^6 \phi^6 \phi^6 \phi^6 \rangle$$

Block expansion

$$\mathcal{A}^{I_1 \dots I_6} = \sum_{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3} C_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3}\, g_{\Delta_1, \Delta_2, \Delta_3}\left(\chi_1, \chi_2, \chi_3\right)$$



$$\langle \phi^6 \phi^6 \phi^6 \phi^6 \phi^6 \phi^6 \rangle$$

$$C_{\phi^6 \phi^6 \mathcal{O}_{\Delta_1}} C_{\mathcal{O}_{\Delta_1} \phi^6 \mathcal{O}_{\Delta_2}} C_{\mathcal{O}_{\Delta_2} \phi^6 \mathcal{O}_{\Delta_3}} C_{\mathcal{O}_{\Delta_3} \phi^6 \phi^6}\Big|_{\mathcal{O}(\lambda^0)} = -\frac{64\pi^{3/2}}{4^{\Delta_1+\Delta_2+\Delta_3}} \frac{\Delta_1\left(\Delta_1-1\right)\Delta_{12}}{\left(2\Delta_1-1\right)\left(\Delta_1+\Delta_2-1\right)} \times \frac{\Gamma\left(\Delta_1+\Delta_2\right)^2}{\Gamma\left(\Delta_2\right)\Gamma\left(\Delta_1-1/2\right)^2\Gamma\left(\Delta_2-1/2\right)} \delta_{\Delta_1, \Delta_3}$$