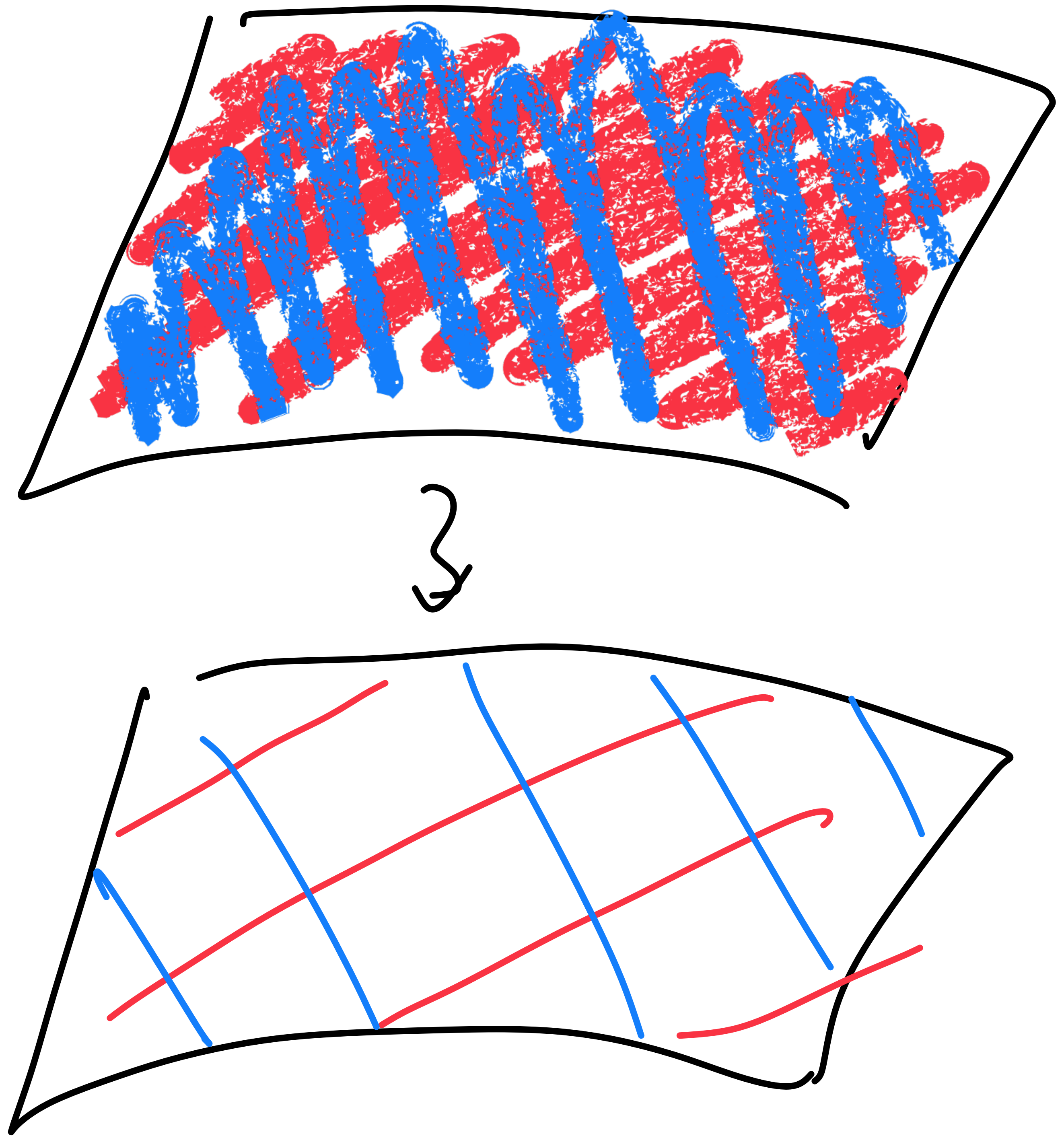
Discretizations of Integrable Field Theories From 4d Chern-Simons Theory



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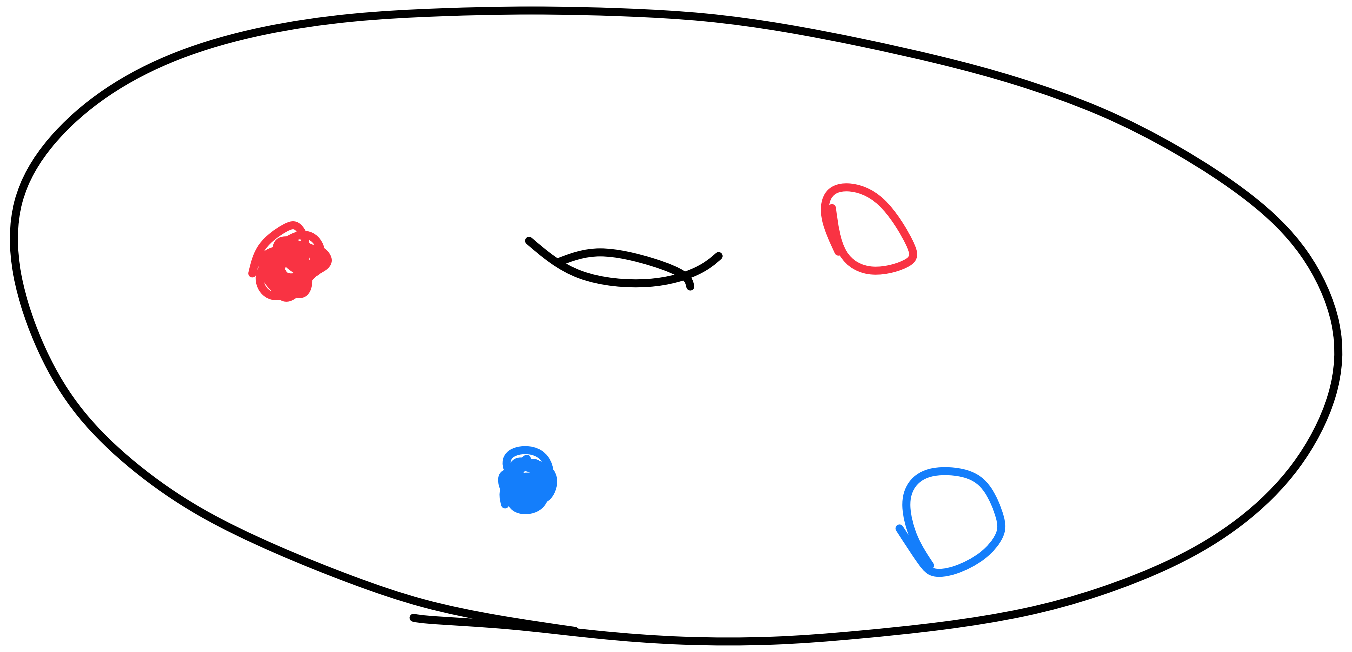

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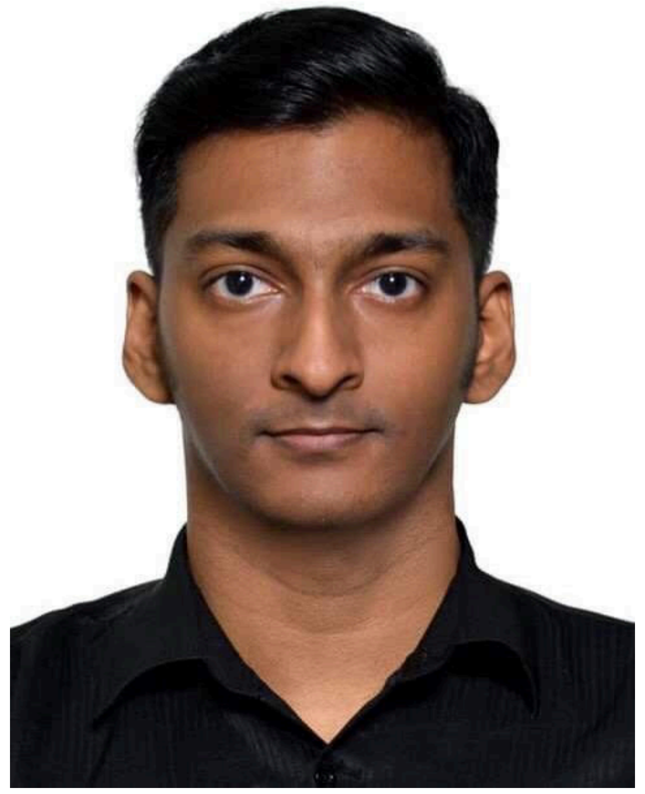
Work to appear (>100pp.)

in collaboration with



I'm on-site!

Meer Ashwin Kumar (IPMU → Bern)

Junichi Sakamoto (Torino → ?)

also

- Costello – MY ('19)
- Costello – Witten – MY ('17, '18)

Introduction

Revisiting Integrable Models via 4d Chern-Simons Theory

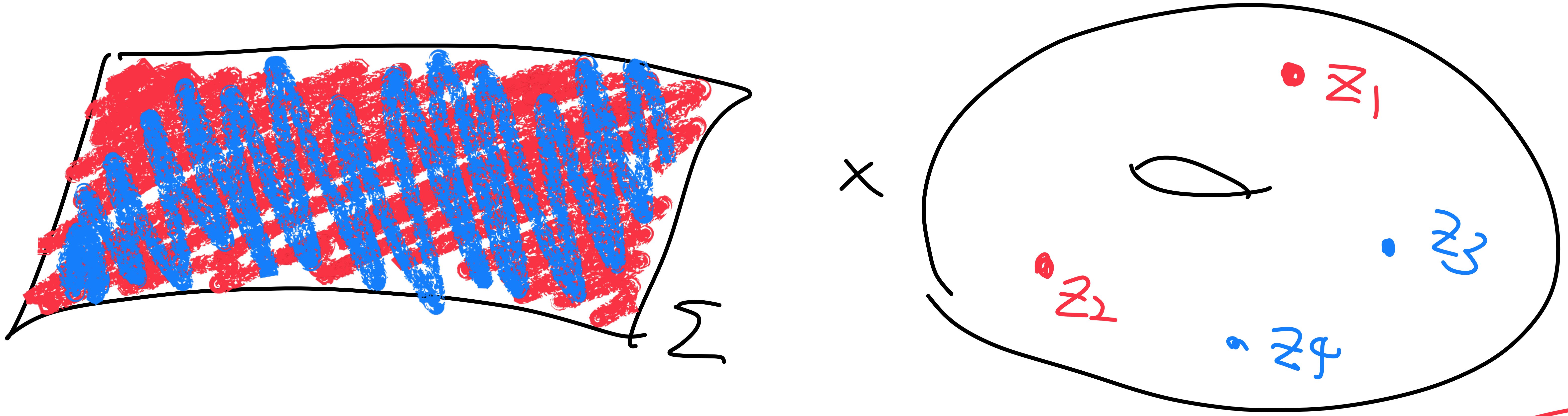
[Costello, Witten, MY; also "affine Gaudin model"; e.g. Vicedo, Delduc, Lacroix, Magro]

$$S = \frac{1}{2\pi\hbar} \int_{\underbrace{\Sigma_w \times C_z}_{C=\mathbb{C}}} \underbrace{\omega}_{\omega = dz} \wedge \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

$$A = A_w\, dw + A_{\bar{w}}\, d\bar{w} + A_{\bar{z}}\, d\bar{z}$$

[cf. knots from 3d CS theory; Witten]

4d CS + 2d surface defects



integrate along C

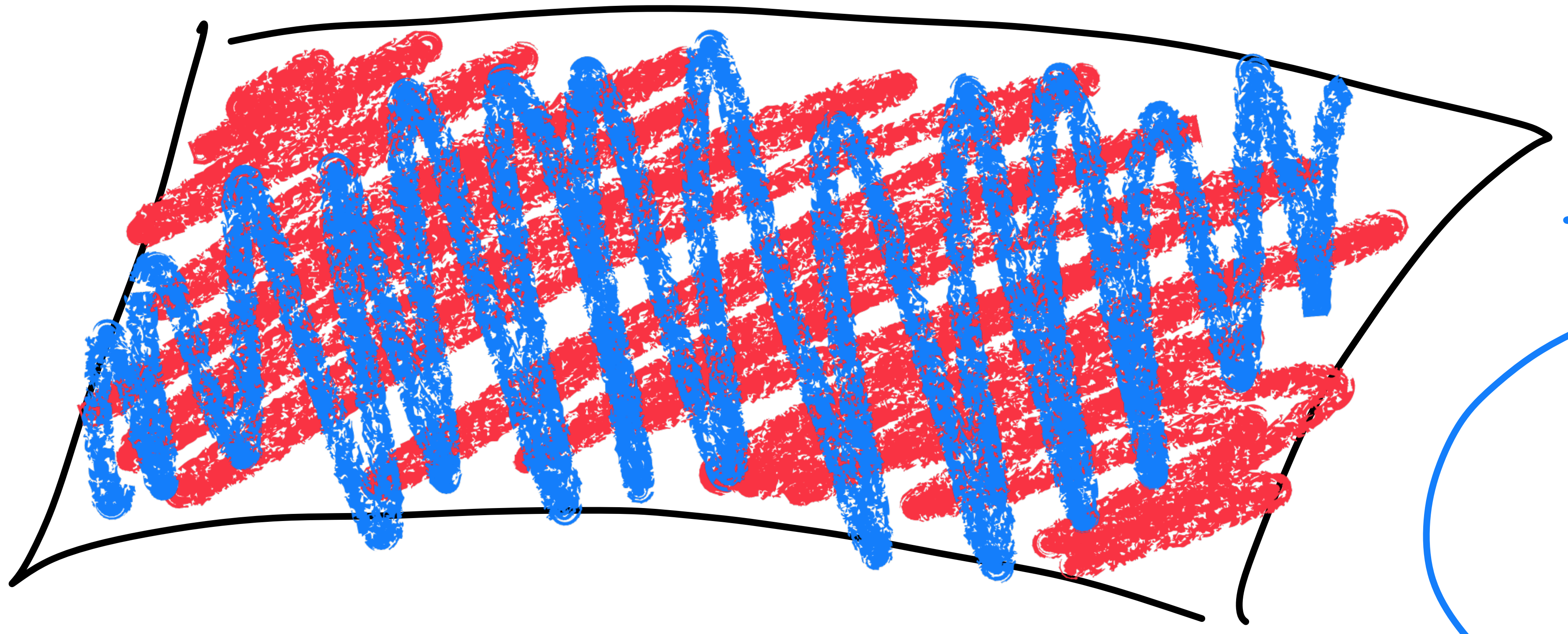
4d

e.o.m. (for $A_{\bar z} = 0$)

$$\partial_{\bar z} A_w = \partial_{\bar z} A_{\bar w} = 0$$

$$F_{w\bar w} = 0$$

2d Integrable QFTs

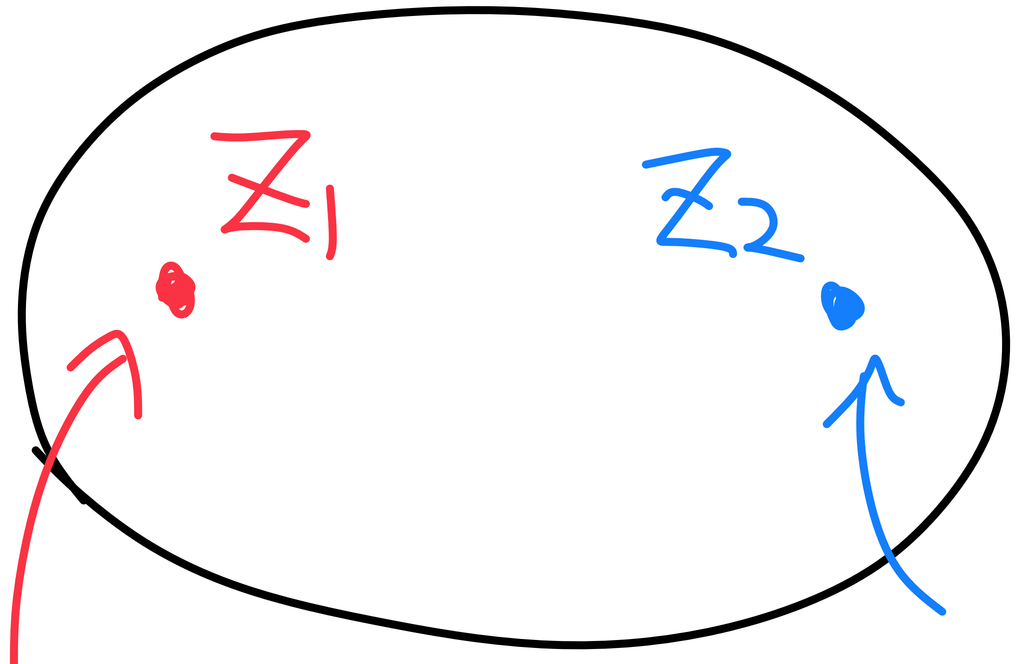


2d

flat connection

$$\mathcal{L} = A_w(z)\, dw + A_{\bar w}(z)\, d\bar w$$

e.g. chiral free fermion defects

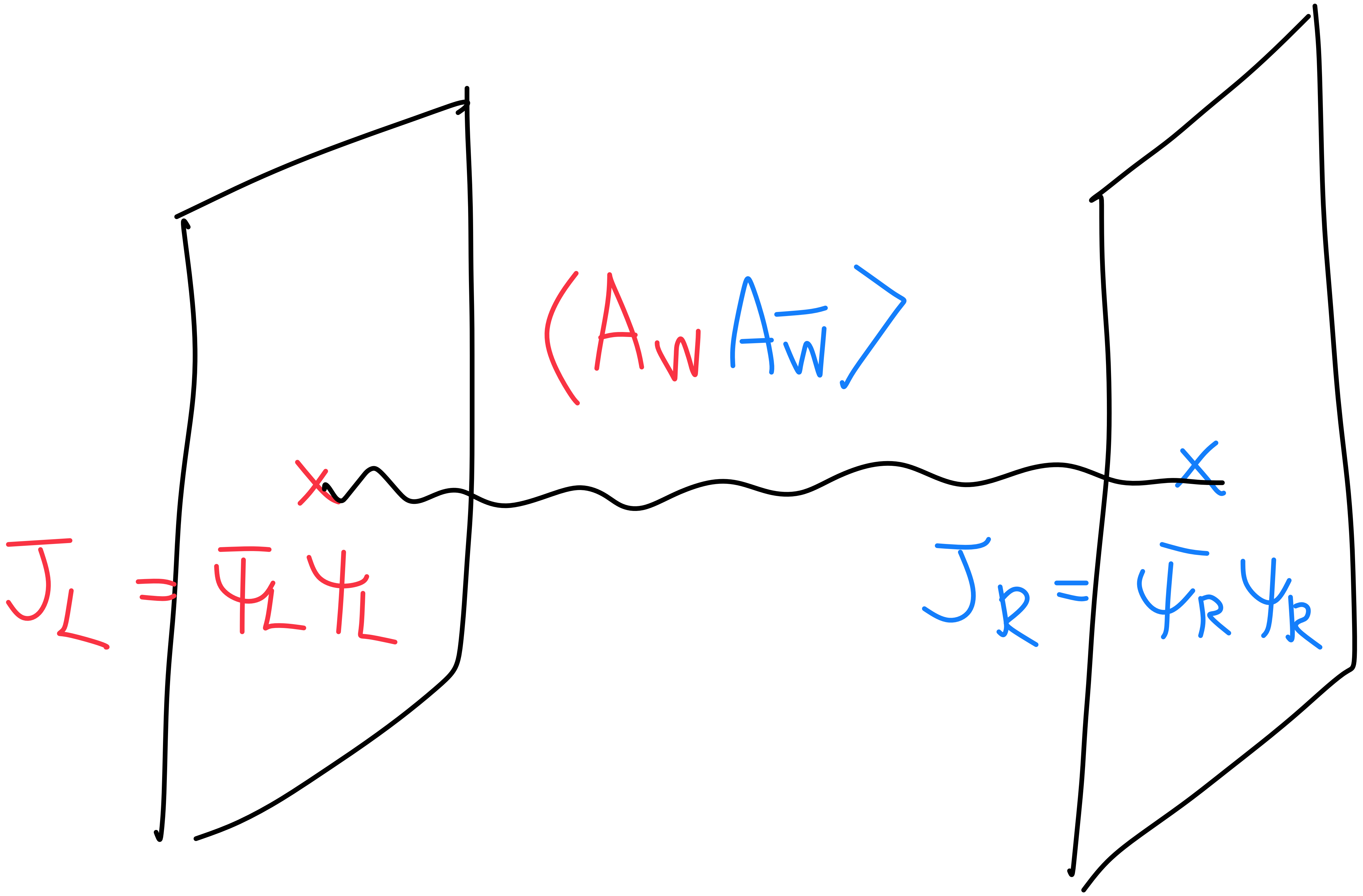


chiral fermion

$$\mathcal{L}_1 = \bar{\psi}_L (\not\partial + \not{A}_w) \psi_L$$

anti-chiral fermion

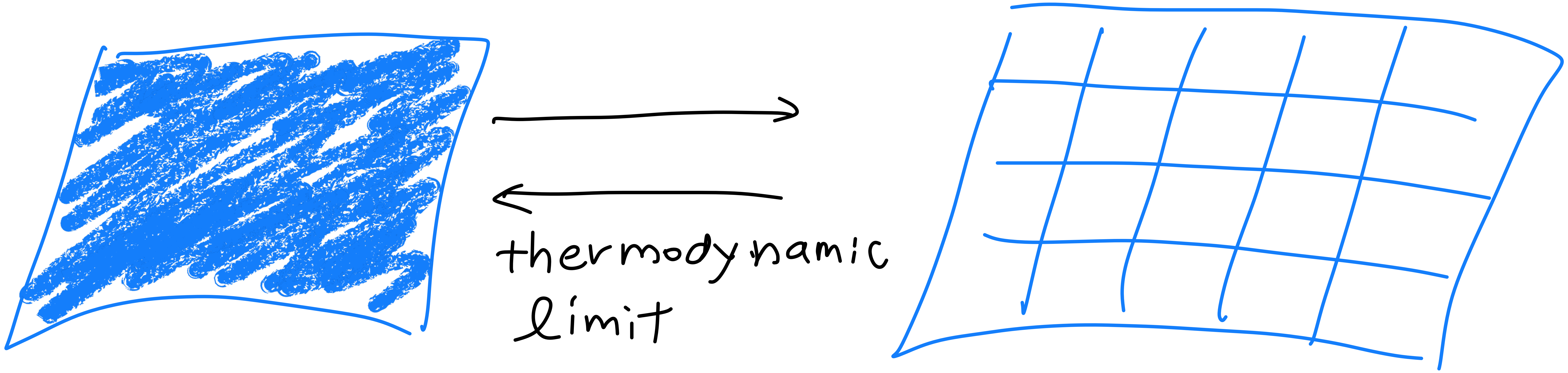
$$\mathcal{L}_2 = \bar{\psi}_R (\not{\bar\partial} + \not{A}_{\bar w}) \psi_R$$



$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \frac{1}{z_1 - z_2} \underbrace{(\bar{\psi}_L \psi_L)(\bar{\psi}_R \psi_R)}_{\text{4-fermi interaction}}$$

Question 1:

How to discretize integrable field theories?



Question 2:

How to describe dualities of IFTs?

e.g. non-Abelian bosonization

SU(N) Thirring $\longleftrightarrow$ SU(N) WZW

Basic Philosophy :

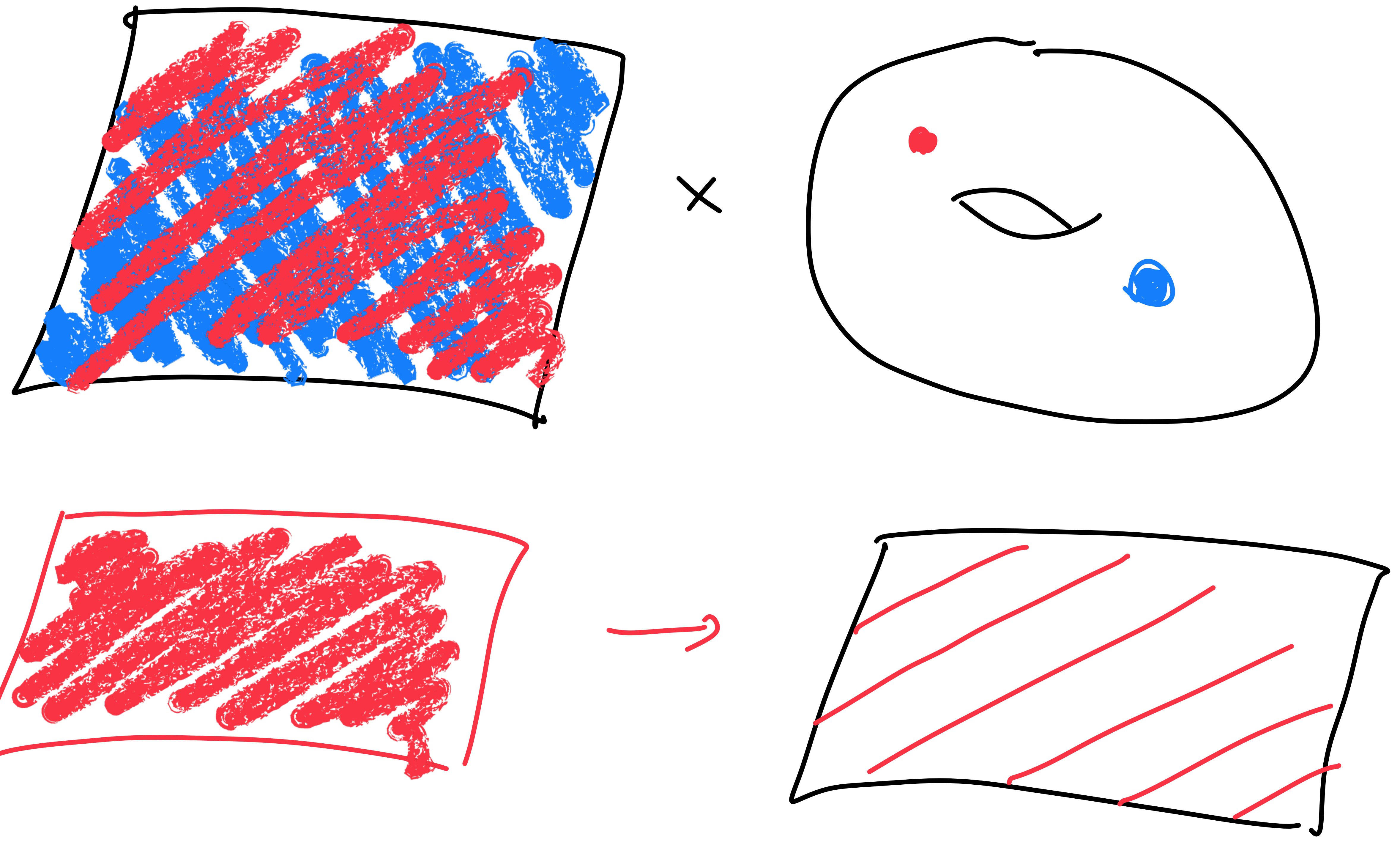
2d effective IFT

4d CS + 2d surface defects

simpler ☺ general ☺

Discretize/ Dualize each defect!

<u>Answer 1:</u> Discretize 2d defect into 1d defect



2d defect

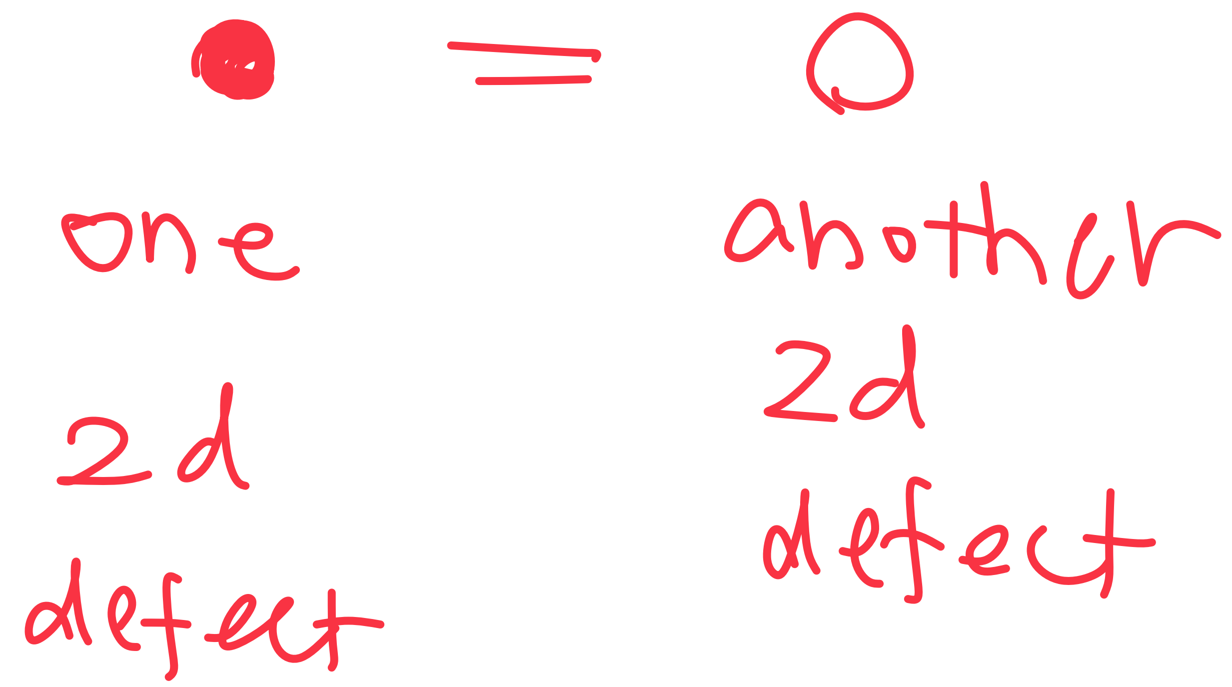
lattice of 1d defects

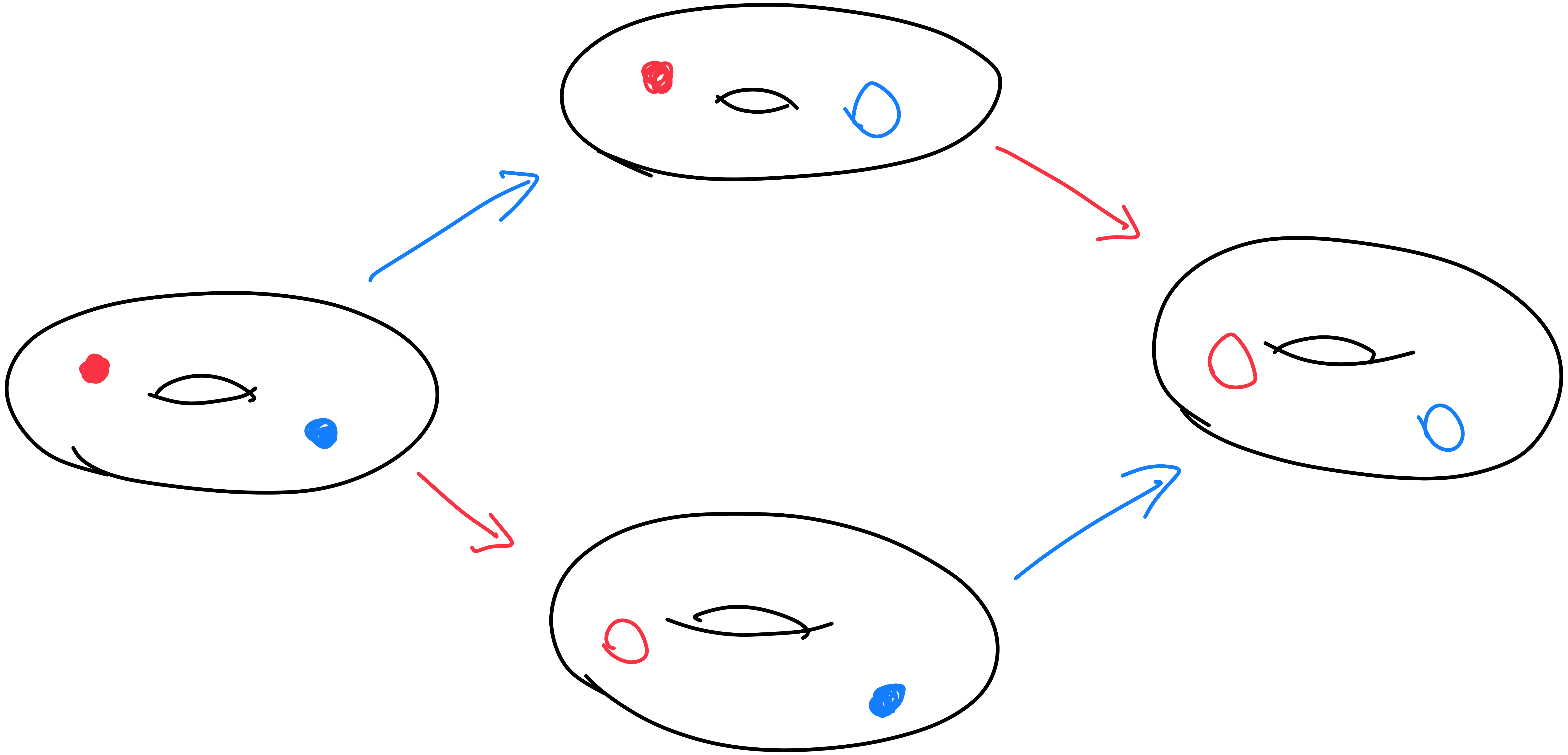
Standard spin chains

often converted to

Wilson lines

Answer 2: dualities among defects
(e.g. bosonization)





"duality web" for multiple defects

Discretization

We consider chiral/anti-chiral "order" surface defects

$$S^+_{4d-2d} \supset \int_{\Sigma \times \{z^+\}} A^a_w J^a$$

$$S^-_{4d-2d} \supset \int_{\Sigma \times \{z^-\}} A^a_{\bar{w}} \bar{J}^a$$

currents for global G-sym. of defect

$$S = \frac{1}{\hbar} \int dz \wedge CS(A) + S_{4d-2d} + S_{2d}$$

When integrated along C

n_+ chiral

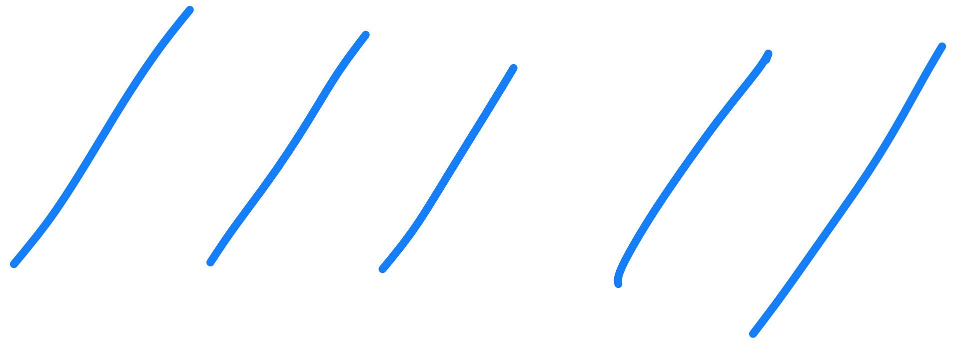
n_- anti-chiral

$$S^{\mathrm{eff}}_{2d} = \int_{\Sigma} \sum_{\alpha=1}^{n_+} \mathcal{L}_{\alpha}(\phi^{\alpha}) + \sum_{\bar\alpha=1}^{n_-} \mathcal{L}_{\bar\alpha}(\bar\phi^{\bar\alpha})$$

$$+ \sum_{\alpha=1}^{n_+} \sum_{\bar\alpha=1}^{n_-} \hbar \, \underbrace{r_{ab}(z_\alpha - z_{\bar\alpha})}_{\text{classical r-matrix}} \underbrace{J^a_\alpha \bar J^b_{\bar\alpha}}_{J\bar J\text{-deformation}}$$

Chiral defects

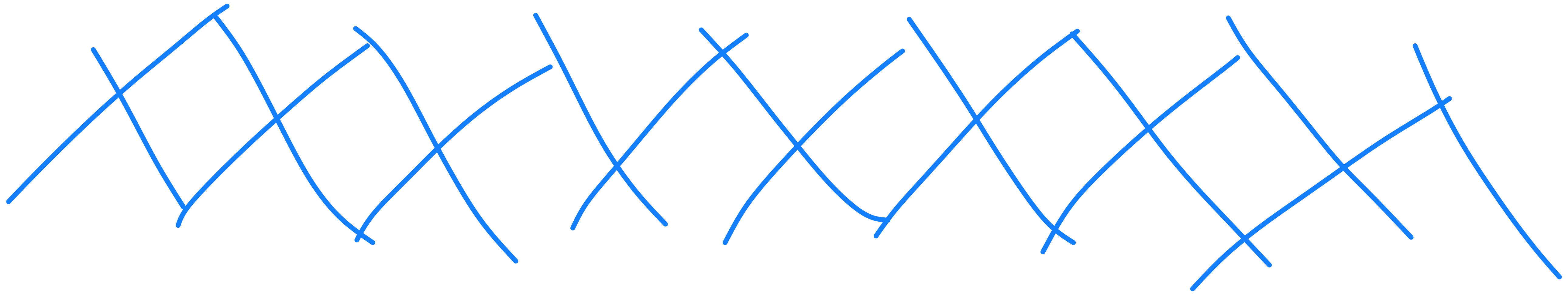
$$S = \int d^2\sigma \; \bar{\Psi} (\not{\partial}_w + \not{A}_w) \Psi \rightsquigarrow S = \sum_i \int d\sigma^- \; \bar{\Psi}_i (\not{\partial}_w + \not{A}_w) \Psi_i$$



anti-chiral defects

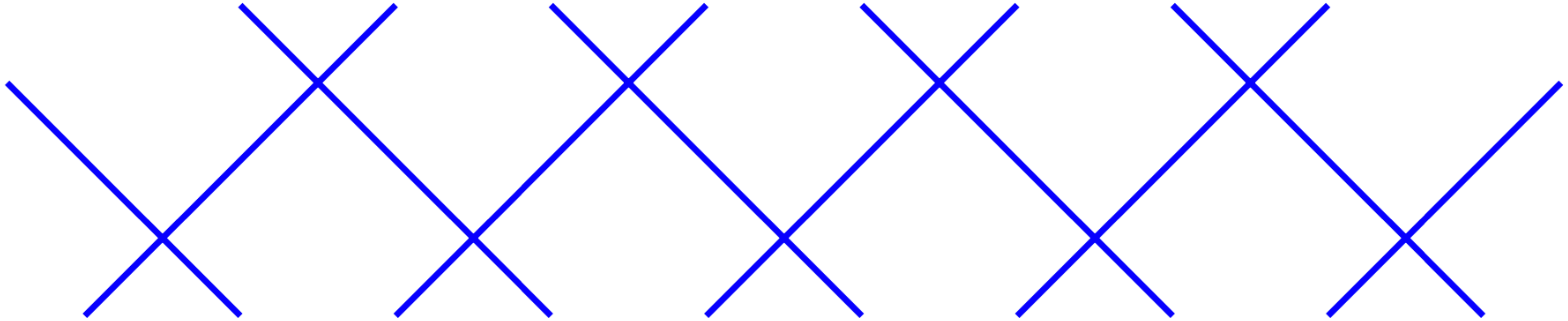
$$S = \int d^2\sigma \; \bar{\Psi} (\not{\partial}_{\bar{w}} + \not{A}_{\bar{w}}) \Psi \rightsquigarrow S = \sum_i \int d\sigma^+ \; \bar{\Psi}_i (\not{\partial}_{\bar{w}} + \not{A}_{\bar{w}}) \Psi_i$$

when combined, light-cone lattice of 1d defects



integrable light-cone discretization

[Many papers by Destri-de Vega
also Faddeev, Reshetikhin, Volkov, ...]

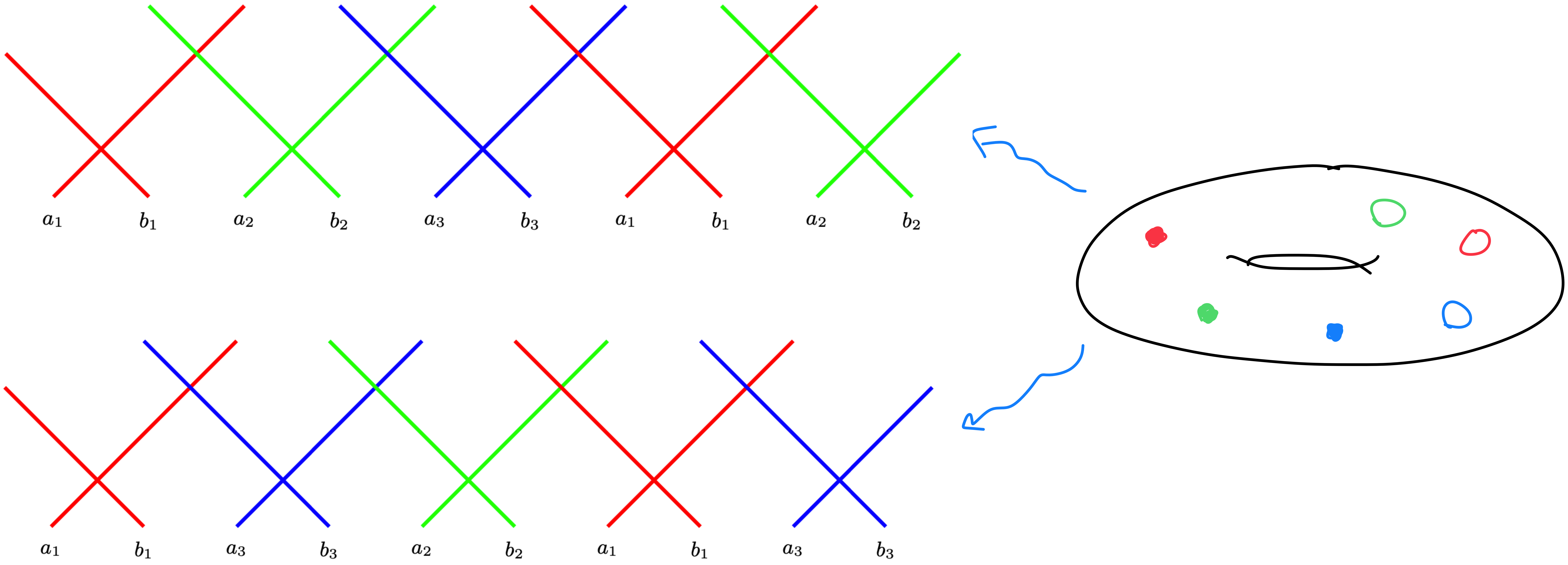


RLL relation $\rightsquigarrow$ Poisson bracket of IFT

recently revisited in quantum simulations/Floquet dynamics

[e.g. Vanicat-Zadnik-Prosen '18
Maruyoshi-Okuda-Pedersen-Suzuki-MY-Yoshida '22]

Multiple defects: inhomogeneous lattice



We can more generally consider

Vertex (operator) algebra A as defects

$\cup$

current algebra $\mathfrak{g}$ level k

or more generally a sheaf of VA

(e.g. curved $\beta\gamma$ - system with target)

2d defect

Vertex (Operator) Algebra $\supset$ current algebra $\mathfrak{g}_k$

$$\mathcal{O}(w) = \sum_n \frac{\mathcal{O}_n}{w^{n+1}}$$

+OPE

1 d defect

Zhu's algebra $\supset$ universal enveloping algebra $\mathcal{U}(\mathfrak{g})$

$$\{\mathcal{O}_0, *\}$$

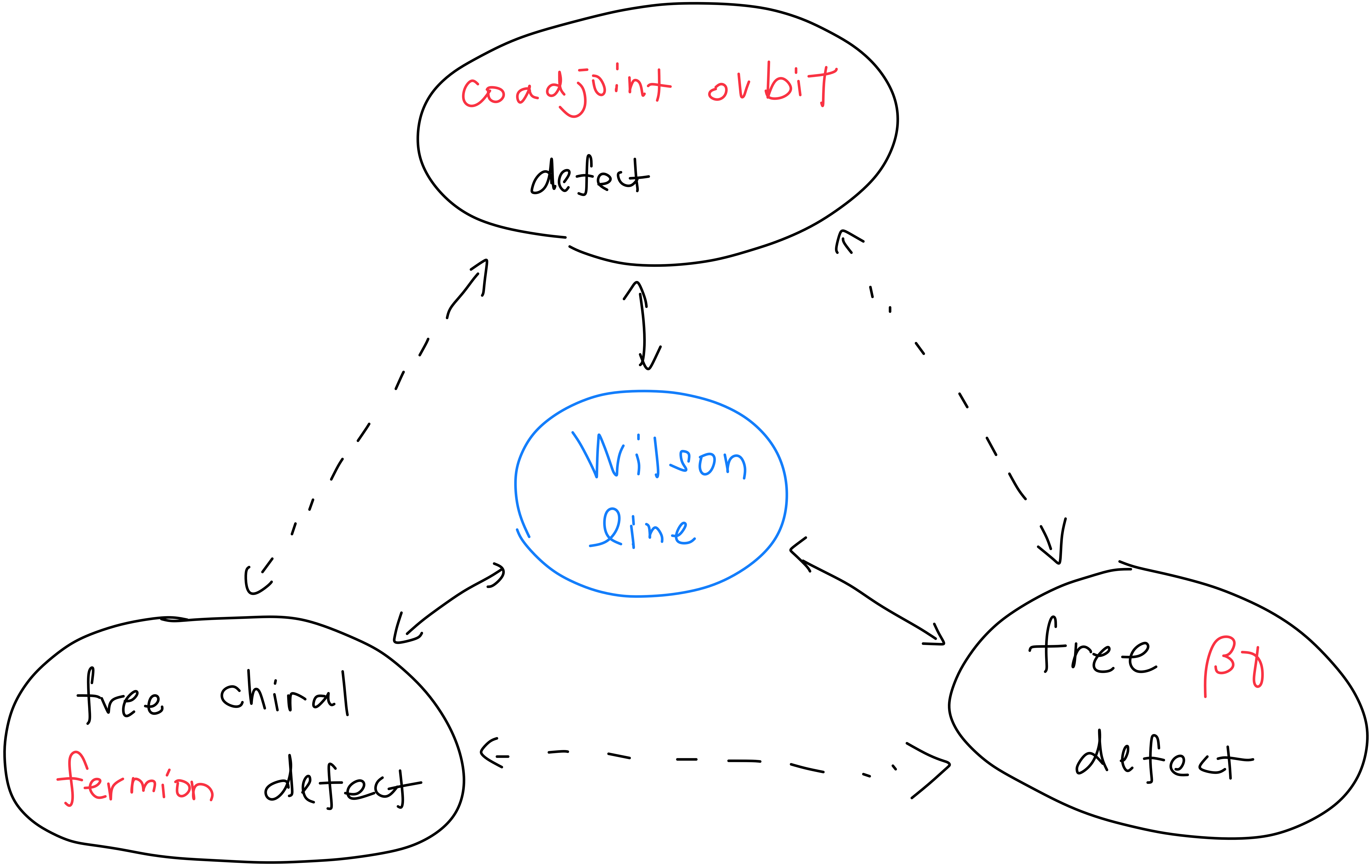
associative product from OPE

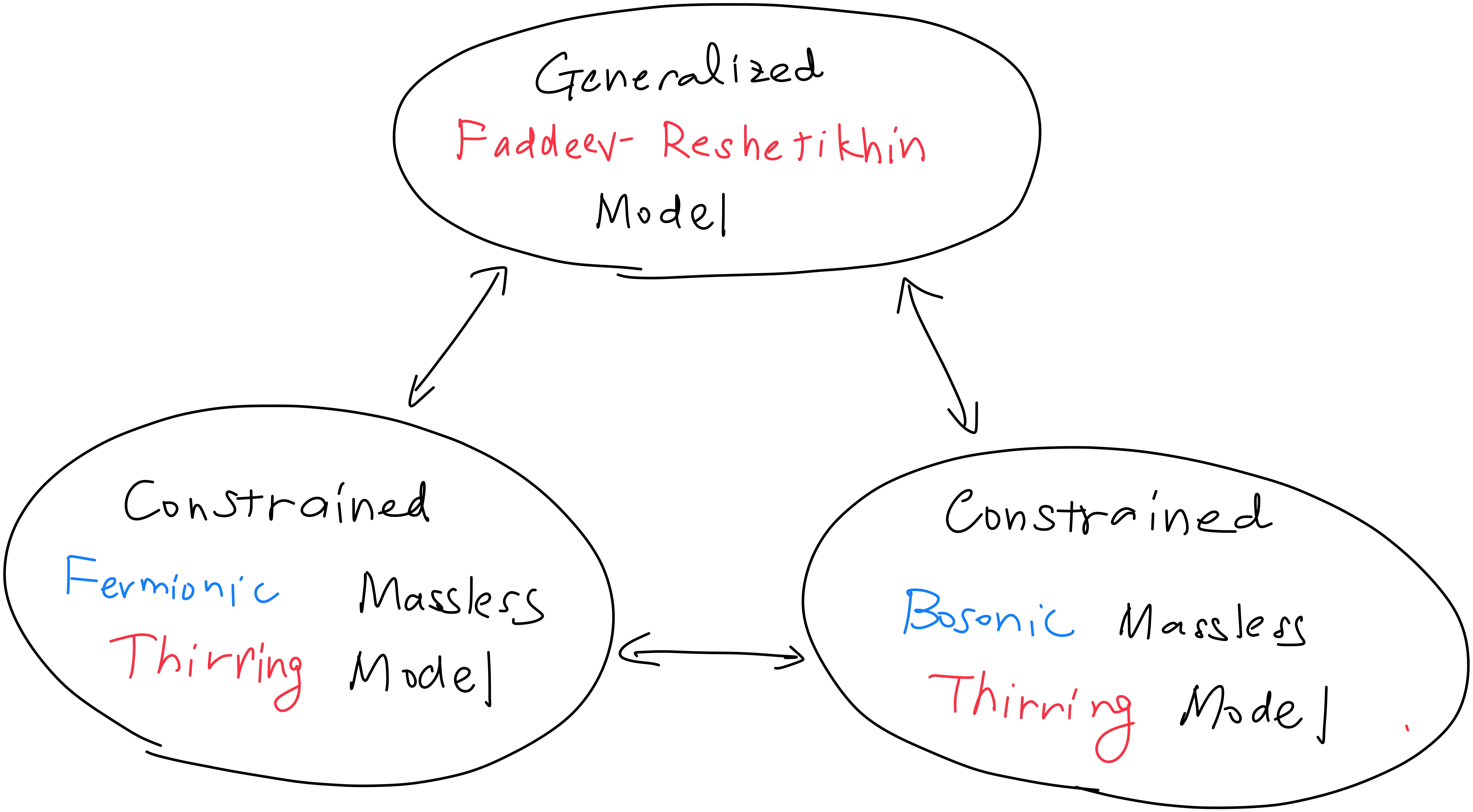
We can discuss discretizations of

- Free Chiral Fermion $\bar{\psi}(\not{\partial}+\not{A})\psi$
- Coadjoint Orbit Defect $\mathrm{Tr}\left(\Lambda g^{-1}(\partial+A)g\right)$
- Free $\beta\gamma$ defect $\beta(\partial+A)\gamma$
- Curved $\beta\gamma$ defect $\gamma : \mathbb{C}\to X \quad \beta\in\Omega^{1,0}(\mathbb{C},\gamma^{*}T^{*}X)$

all related to Wilson lines

[Borel-Weil-Bott, ...] $\rightsquigarrow$ standard spin chains


coadjoint orbit defect
Wilson line
free chiral fermion defect
free βγ defect



Generalized
Faddeev- Reshetikhin
Model
Constrained
Fermionic Massless
Thirring Model
Constrained
Bosonic Massless
Thirring Model

Dualities

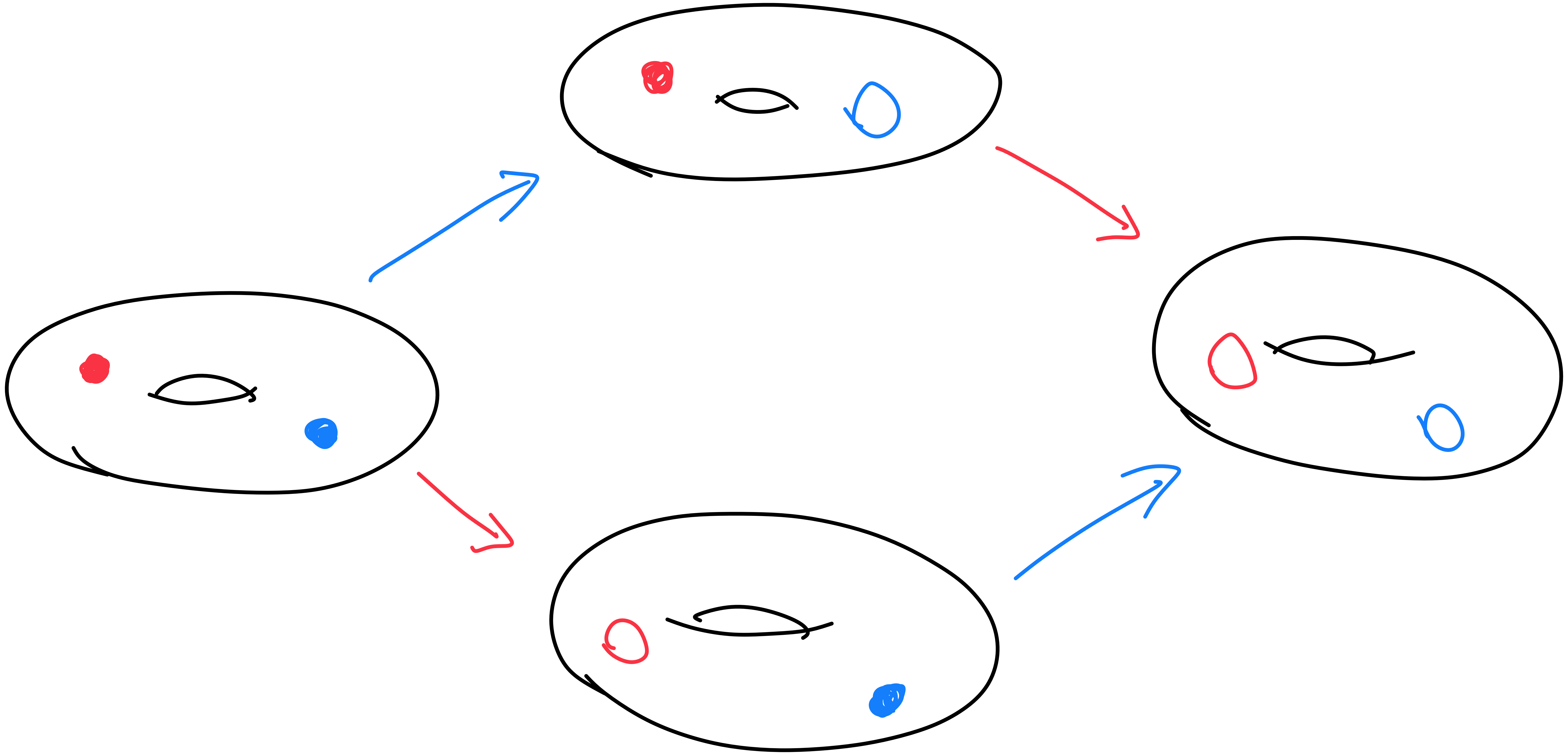
Idea: dualities among defects
(e.g. bosonization)

"Seed duality"

● = ○

one 2d defect

another 2d defect



"duality web" for multiple defects

"Seed duality" $\rightsquigarrow$ many new bosonizations

bosonization ('80s)

Free Fermion

$$\sum_{j=1}^{N_F} \bar{\Psi}_j \, i \underbrace{\not{D}_R}_{\parallel} \Psi_j$$

$$\not{\partial} + \not{A}\,\frac{1-\gamma_5}{2}$$

WZW model

$$N_F S_{WZW}[g_-]$$

$$+ \operatorname{Tr}[A_+ J_-]$$

$$J_- := \frac{N_F}{2\pi} g_-^{-1} \partial_- g_-$$

But chiral/anti-chiral defect has anomalies for 4d gauge sym?

Free Fermion

$$\sum_{j=1}^{N_F} \bar{\Psi}_j \, i \underbrace{D_R}_{\parallel} \Psi_j$$

$$\not{\partial} + \not{A} \frac{1-\gamma_5}{2}$$

WZW model

$$N_F S_{WZW}[g_-]$$

$$+ \mathrm{Tr}[A_+ J_-]$$

$$J_- := \frac{N_F}{2\pi} g_-^{-1} \partial_- g_-$$

anomalies at defects: $(\delta A = -D\varepsilon)$

$$\delta W \propto \underbrace{\sum_\alpha -\hbar k_+^\alpha \int_{\Sigma\times\{z_+^\alpha\}} \mathrm{Tr}(d\varepsilon\wedge A)}_{\text{chiral}} + \underbrace{\sum_{\bar\alpha} \hbar k_-^{\bar\alpha} \int_{\Sigma\times\{z_-^{\bar\alpha}\}} \mathrm{Tr}(d\varepsilon\wedge A)}_{\text{anti-chiral}}$$

anomaly inflow:

[Costello-MY, Gaiotto-Lee-Vicedo-Wu '20, ...]

$$\delta S_{cs} \propto \int_{\Sigma\times\mathbb{CP}^1} d\omega\wedge \mathrm{Tr}(d\varepsilon\wedge A)$$

$$\delta\omega \propto \hbar\left(\sum_\alpha \frac{k_+^\alpha}{z-z_+^\alpha} - \sum_{\bar\alpha}\frac{k_-^{\bar\alpha}}{z-z_-^{\bar\alpha}}\right)dz$$

poles to one-form

$$\mathcal{W}_{\mathrm{eff}} \equiv \overset{dz}{\widehat{\omega}} + \hbar \left(\sum_{\alpha} \frac{k_+^{\alpha}}{z - z_+^{\alpha}} - \sum_{\bar{\alpha}} \frac{k_-^{\bar{\alpha}}}{z - z_-^{\bar{\alpha}}} \right) dz$$

- absence of pole at ∞

$\rightsquigarrow$ anomaly concellation condition

$$\sum_{\alpha} k_+^{\alpha} = \sum_{\bar{\alpha}} k_-^{\bar{\alpha}}$$

- one-form $\mathcal{W}_{\mathrm{eff}}$ has a pole

$\rightsquigarrow$ A_w (or $A_{\bar{w}}$) has a zero

(Dirichlet defect)

We can start with the defects

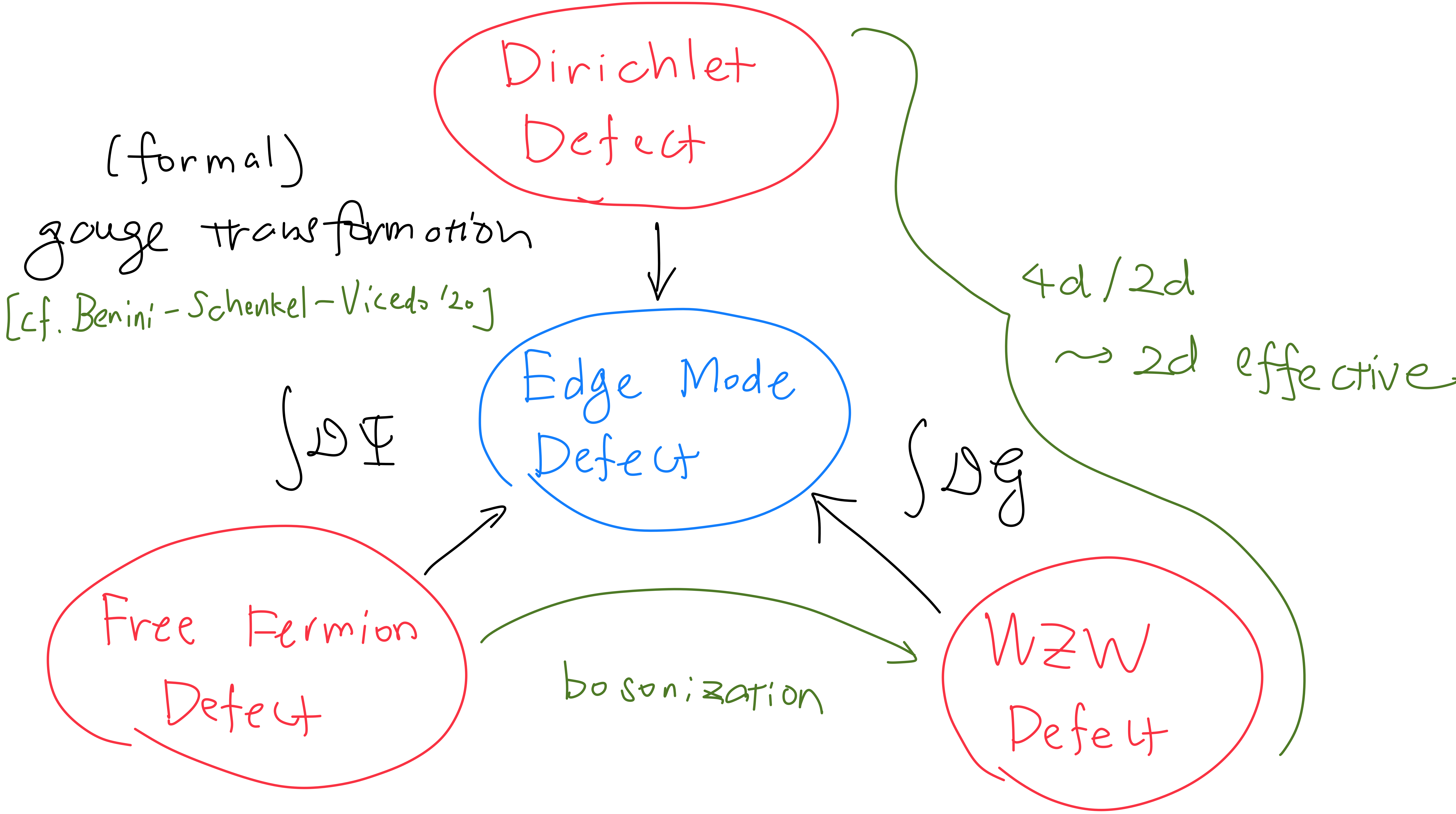
ω_{eff} $\underbrace{\text{pole}}_{\parallel}$ $\quad A_w$ (or $A_{\bar{w}}$) zero

double pole + $\underbrace{\text{single zero}}_{\text{disorder defect}}$

and derive 2d IFT [following Costello-MY '19]

... which coincides with bosonized theories

[also Delduc-Lacroix-Magro-Vicedo '19]


Dirichlet Defect
(formal) gauge transformation
[cf. Benini-Schenkel-Vicedo '20]
Edge Mode Defect
$\int \mathcal{D}\Phi$
$\int \mathcal{D}g$
4d/2d
⇝ 2d effective
Free Fermion Defect
bosonization
WZW Defect

String Theory

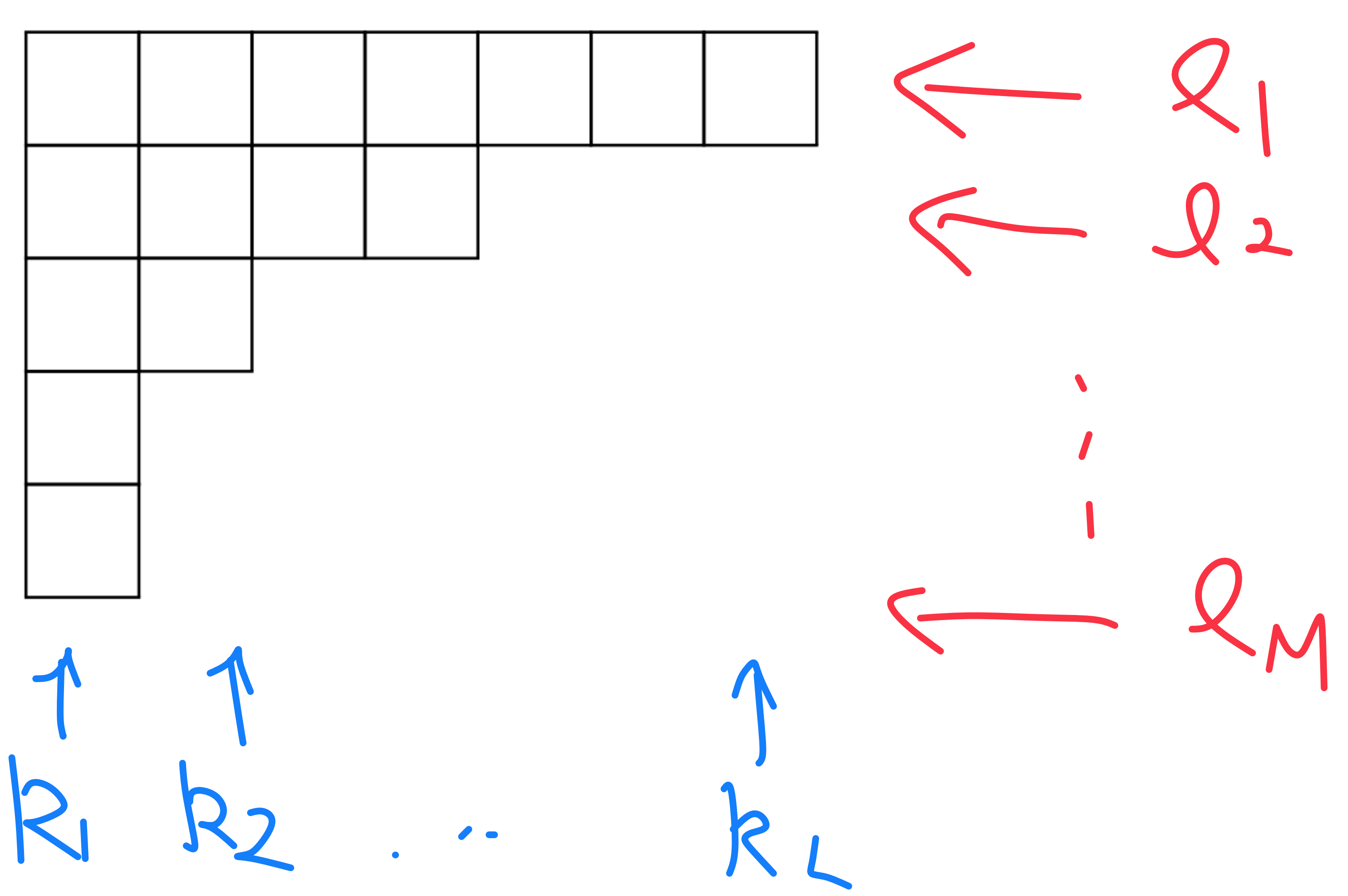
Realization of Wilson lines

	Σ		$\mathbb{R}^2_\hbar$		C		$N\Sigma\subset T^*\Sigma$		$\mathbb{R}^2_{-\hbar}$	
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D5	×	×	×	×	×	×				
$\mathbf{D3}_b^+$	×		×	×			×			
$\mathbf{D3}_b^-$		×	×	×				×		
$\mathbf{D3}_f^+$	×						×		×	×
$\mathbf{D3}_f^-$		×						×	×	×

[Costello - Yagi '18]

lattice model

spectral curve



[cf. Gomis - Passerini '06]

	Σ		$\mathbb{R}^2_{\hbar}$		C		$N\Sigma\subset T^*\Sigma$		$\mathbb{R}^2_{-\hbar}$	
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D5	×	×	×	×	×	×				
D3$_b^+$	×		×	×			×			
D3$_b^-$		×	×	×				×		
D3$_f^+$	×						×		×	×
D3$_f^-$		×						×	×	×

Lattice Model

thermodynamic limit

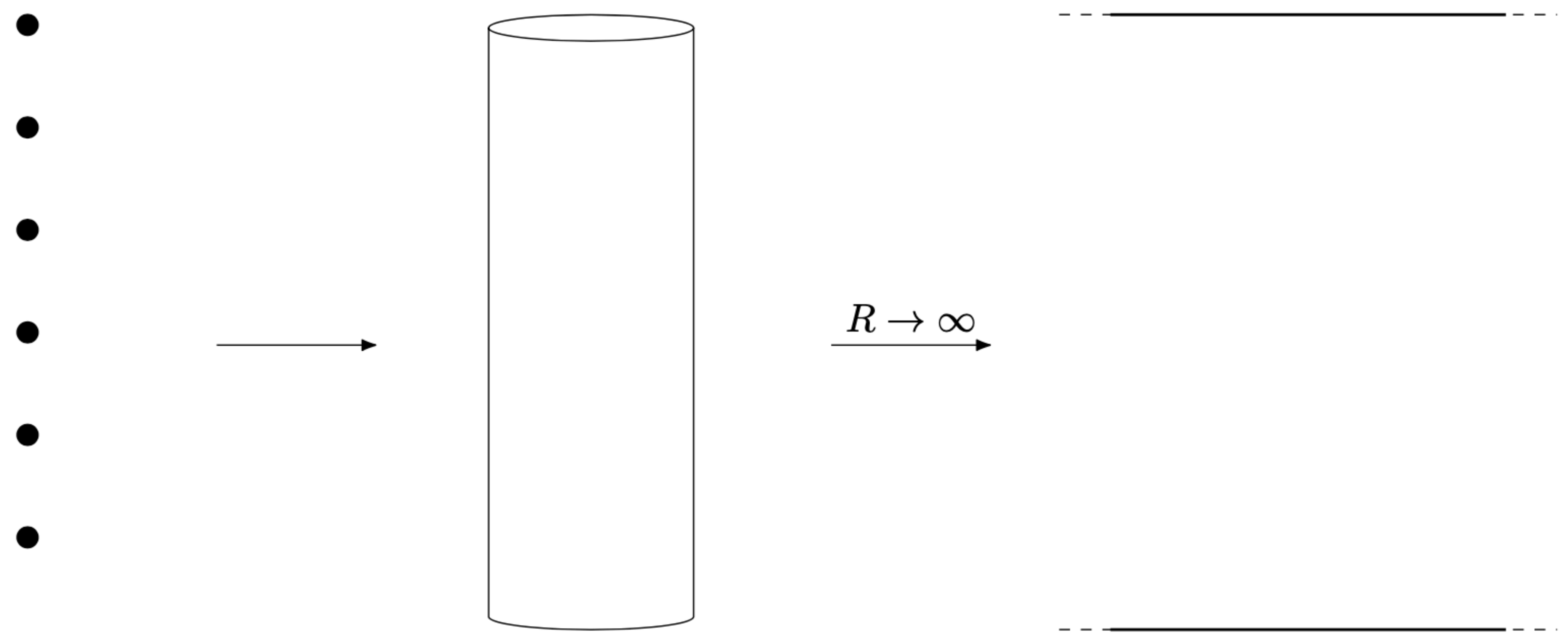
‖

Myers effect

Field Theory

	Σ		$\mathbb{R}^2_{\hbar}$		C		$N\Sigma\subset T^*\Sigma$		$\mathbb{R}^2_{-\hbar}$	
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D5	×	×	×	×	×	×				
D5$_b$	×	×	×	×			×	×		
D5$_f$	×	×					×	×	×	×

Supertube



[Mateos-Townsend '01]

Summary

- Discretization: 2d order defects

  $\longrightarrow$ 1d order line defects

  (often converted to Wilson lines)

  1d duality

- Duality: dualities among 2d defects

  $\longrightarrow$ a huge web of 2d dualities among IFTs

anomaly (cancellation)

play crucial roles