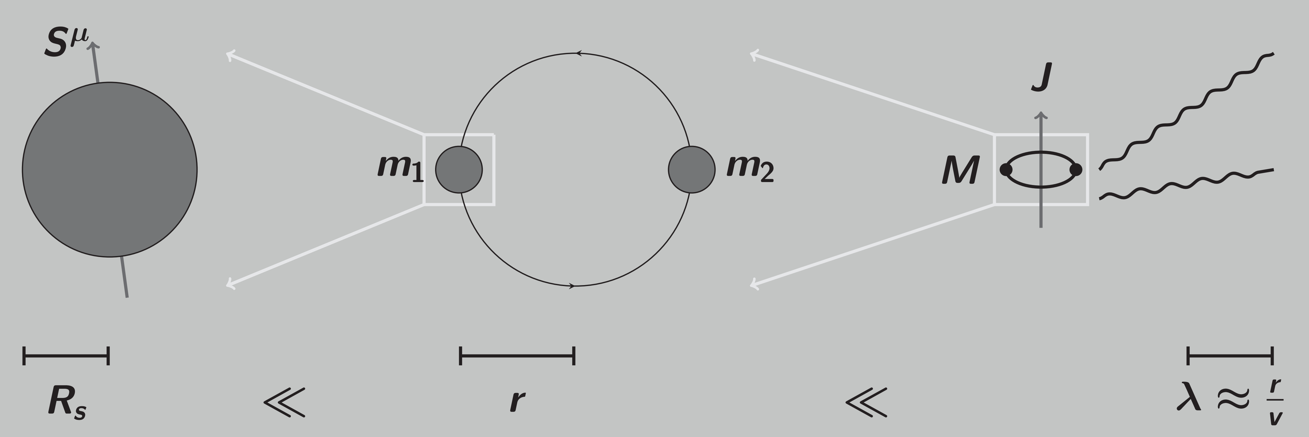
Spinning compact objects using EFTs

Raj Patil

MPI for Gravitational Physics (AEI) and Humboldt University

Post-Newtonian framework / separation of scales

In the inspiral phase of the binary coalescence of compact objects, the velocities are non-relativistic (slow velocity $v/c \ll 1$) and objects are far apart (weak field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$).



Hence, we have a hierarchy of scale and EFTs exactly fo the job of explaining the simplest framework that captures the essential physics at these scales R_s, r and λ.

Effective action for compact objects

A spinning compact object could be approximated by a point particle with internal structure at large scales as compared to R_S, using the action [1],

$$S_{\mathrm{pp}(a)} = \sum_{a=1,2} \int d\tau \left(-m_{(a)} c \sqrt{u_{(a)}^2} - \frac{1}{2} S_{(a)\mu\nu} \Omega_{(a)}^{\mu\nu} - \frac{S_{(a)\mu\nu} u_{(a)}^\nu}{u_{(a)}^2} \frac{du_{(a)}^\mu}{d\tau} + \mathcal{L}_{(a)}^{(R)} + \mathcal{L}_{(a)}^{(R^2)} + \dots \right)$$

The non-minimal couplings relevant upto 5PN and upto quadratic in spin are,

$$\mathcal{L}_{(a)}^{(R)} = -\frac{1}{2 m_{(a)} c} C_{\mathrm{ES}^2(a)} \frac{E_{\mu\nu}}{u_{(a)}} \left[S_{(a)}^\mu S_{(a)}^\nu \right]_{\mathrm{STF}} + \dots$$

$$\mathcal{L}_{(a)}^{(R^2,S^0)} = \frac{1}{2} C_{\mathrm{E}^2(a)} \frac{G_N^4 m_{(a)}^5}{c^7} \frac{E_{\alpha\beta} E^{\alpha\beta}}{u_{(a)}^3} + \dots$$

$$\mathcal{L}_{(a)}^{(R^2,S^2)} = \frac{1}{2} C_{\mathrm{E}^2\mathrm{S}^2(a)} \frac{G_N^2 m_{(a)}}{c^5} \frac{E_{\mu\alpha} E_\nu{}^\alpha}{u_{(a)}^3} \left[S_{(a)}^\mu S_{(a)}^\nu \right]_{\mathrm{STF}} + \dots$$

where the willson coefficients $C_{(a)}$ are function of the constants of the conservative system $m_{(a)}$ and $S_{(a)}$. We can extract the explicity dependence on the spin by expanding the coefficients as follows

$$C_{(a)} = C_{(a)}^{(0)} + C_{(a)}^{(2)} \left(\frac{S_{(a)}^2 c^2}{G_N^2 m_{(a)}^4} \right) + C_{(a)}^{(4)} \left(\frac{S_{(a)}^2 c^2}{G_N^2 m_{(a)}^4} \right)^2 + \dots$$

The coefficient $C_{\mathrm{ES}^2}^{(0)}$ starts contributing from 2PN (LO $\mathbf{S}^2$ sector), whereas the other two coefficients $C_{\mathrm{E}^2}^{(2)}$ and $C_{\mathrm{E}^2\mathrm{S}^2}^{(0)}$ contribute for the first time at 5PN. Out of them, $C_{\mathrm{ES}^2}^{(0)}$ is related to the **spin-induced quadruple moment** of the compact object: for Kerr black holes, it is known to be 1 and for neutron stars its value ranges within 2-8. The other two coefficients, $C_{\mathrm{E}^2}^{(2)}$ and $C_{\mathrm{E}^2\mathrm{S}^2}^{(0)}$, encode **quadrupolar deformations, due to an external field and spin-square effects**, and are unknown for particular compact objects.

Diagramatic approach for bound state dynamics

Method of regions: Gravitational interactions can be seperated into potential gravitons ($H_{\mu\nu}$) and radiation gravitons ($\bar{h}_{\mu\nu}$) which scale as

$$H_{\mu\nu} \to (k_0, \mathbf{k}) \equiv (1/r, v/r) \qquad \bar{h}_{\mu\nu} \to (k_0, \mathbf{k}) \equiv (v/r, v/r)$$

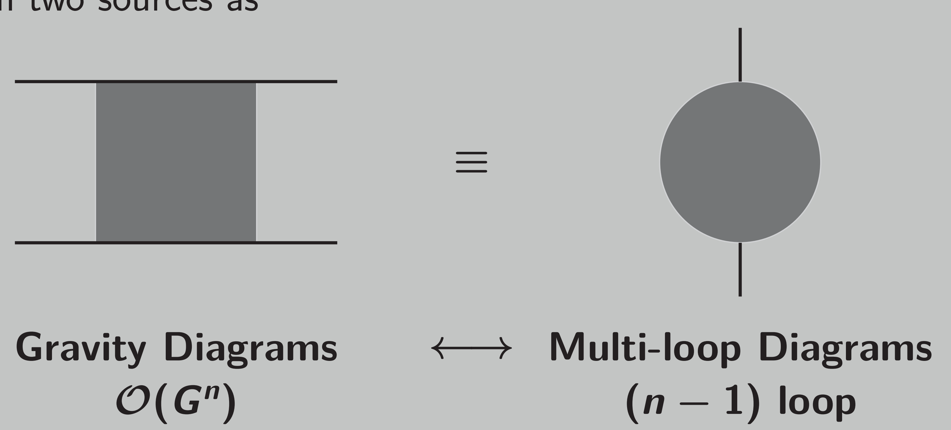
The conservative effective potential for the point particles can then be computed by integrating out the potential gravitons (with $\bar{h}_{\mu\nu} = 0$) for the action,

$$S_{\mathrm{eff}} = S_{\mathrm{EH}} + S_{\mathrm{pp}(1)} + S_{\mathrm{pp}(2)}$$

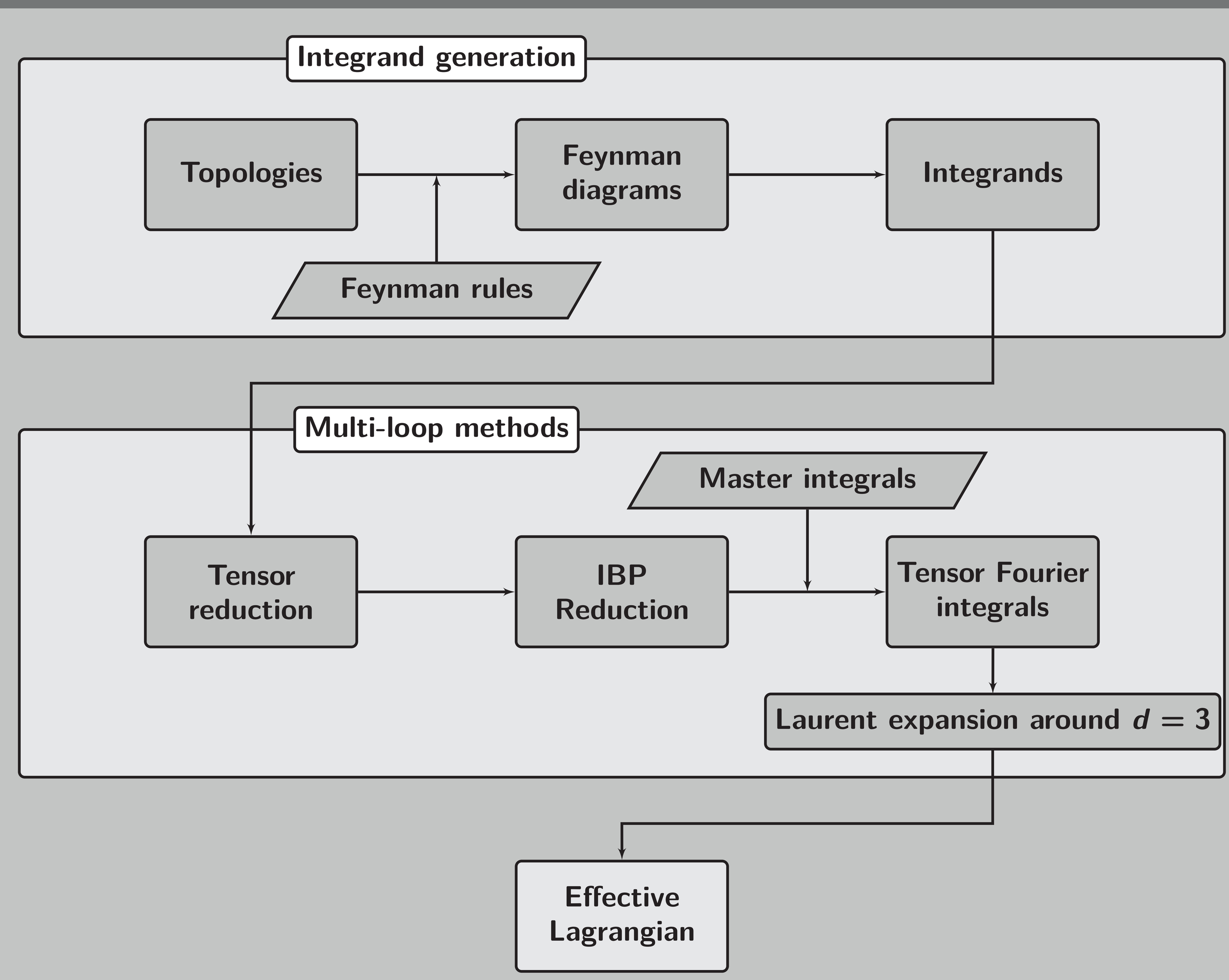
where,

$$S_{\mathrm{EH}} = -\frac{c^4}{16\pi G_N} \int d^4x \sqrt{g}\, R[g_{\mu\nu}] + \frac{c^4}{32\pi G_N} \int d^4x \sqrt{g}\, g_{\mu\nu} \Gamma^\mu \Gamma^\nu$$

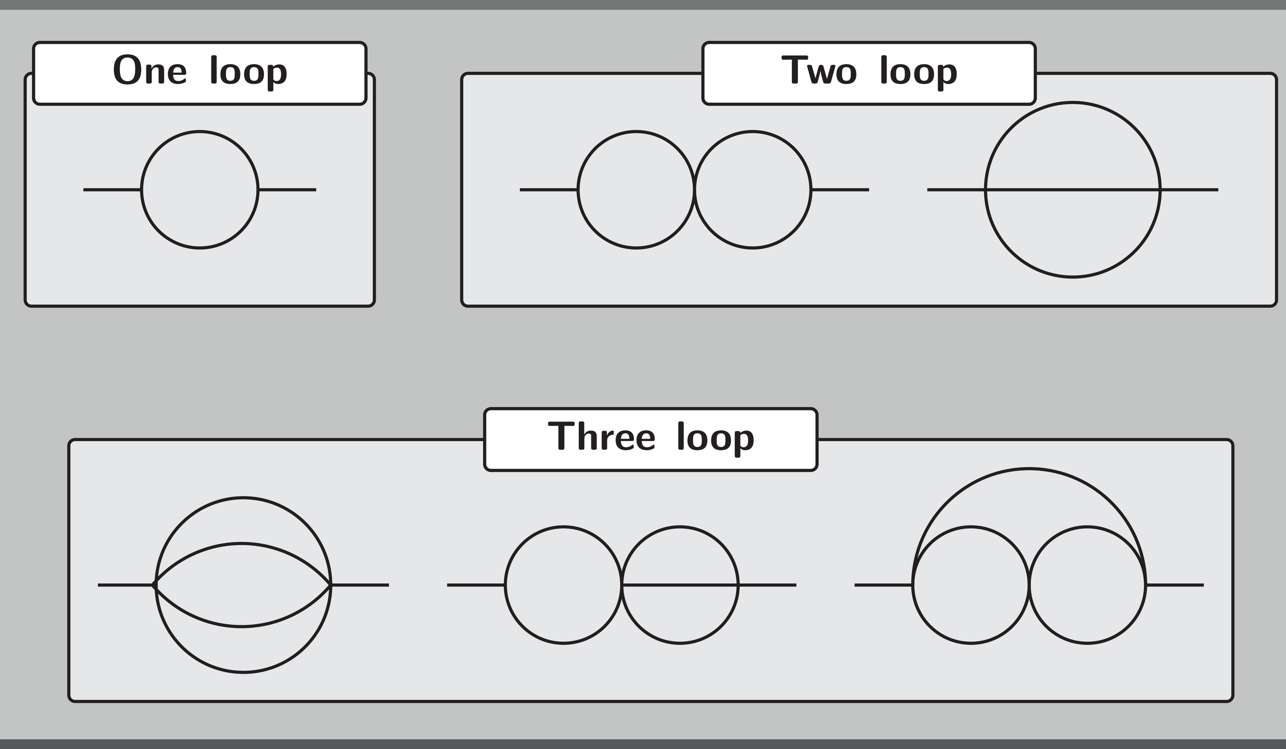
The generated Feynman diagrams can be understood as two-point multi-loop Feynman diagrams with all internal lines mass-less and the external momentum, identified with the momentum transferred between two sources as



Computational Algorithm



Master Integrals



State-of-the-art results at NNNLO

Spin-orbit coupling at 4.5PN [2, 3]: analogous to the fine structure correction of hydrogen atom

$$\left(\mathbf{S}_{(a)} . \mathbf{L} \right)$$

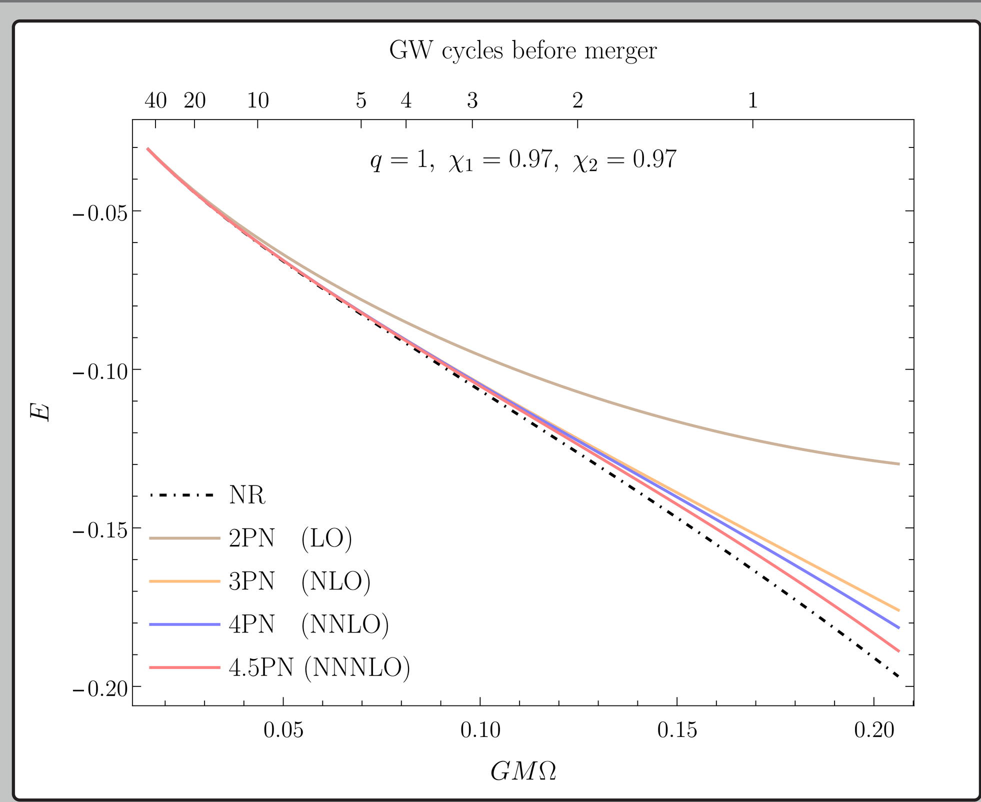
Spin-orbit
arXiv:2209.00611

Quadratic in spin couplings at 5PN [4, 5]: analogous to the hyperfine structure corrections to the hydrogen atom

$$\left(\mathbf{S}_{(1)}^2 \right), \left(\mathbf{S}_{(1)} . \mathbf{S}_{(2)} \right) \text{ and } \left(\mathbf{S}_{(2)}^2 \right)$$

Quadratic-in-spin
arXiv:2210.09176

Observables - Binding Energy



Binding energy vs orbital frequency: Here we plot the gauge invariant binding energy for binary black hole systems as a function of orbital frequency for different orders in PN approximation.

- point-particle, spin-orbit and quadratic in spin contributions upto 2PN (brown), 3PN (orange) and 4PN (blue)
- point-particle and quadratic in spin contributions upto 4PN and spin-orbit contributions upto 4.5PN [2] (Red)
- exact solution using numerical relativity [6] (dashed black line).

References

[1] Michele Levi and Jan Steinhoff. Spinning gravitating objects in the effective field theory in the post-Newtonian scheme. *JHEP*, 09:219, 2015.

[2] Manoj K. Mandal, Pierpaolo Mastrolia, Raj Patil, and Jan Steinhoff. Gravitational Spin-Orbit Hamiltonian at NNNLO in the post-Newtonian framework. 9 2022.

[3] Jung-Wook Kim, Michèle Levi, and Zhewei Yin. N^{3}LO Spin-Orbit Interaction via the EFT of Spinning Gravitating Objects. 8 2022.

[4] Manoj K. Mandal, Pierpaolo Mastrolia, Raj Patil, and Jan Steinhoff. Gravitational Quadratic-in-Spin Hamiltonian at NNNLO in the post-Newtonian framework. 10 2022.

[5] Jung-Wook Kim, Michèle Levi, and Zhewei Yin. N^{3}LO Quadratic-in-Spin Interactions for Generic Compact Binaries. 9 2022.

[6] Serguei Ossokine, Tim Dietrich, Evan Foley, Reza Katebi, and Geoffrey Lovelace. Assessing the Energetics of Spinning Binary Black Hole Systems. *Phys. Rev. D*, 98(10):104057, 2018.

raj.patil@aei.mpg.de