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THE HEBREW UNIVERSITY OF JERUSALEM

Kite and Triangle diagrams through Symmetries of Feynman Integrals

Barak Kol and Subhajit Mazumdar

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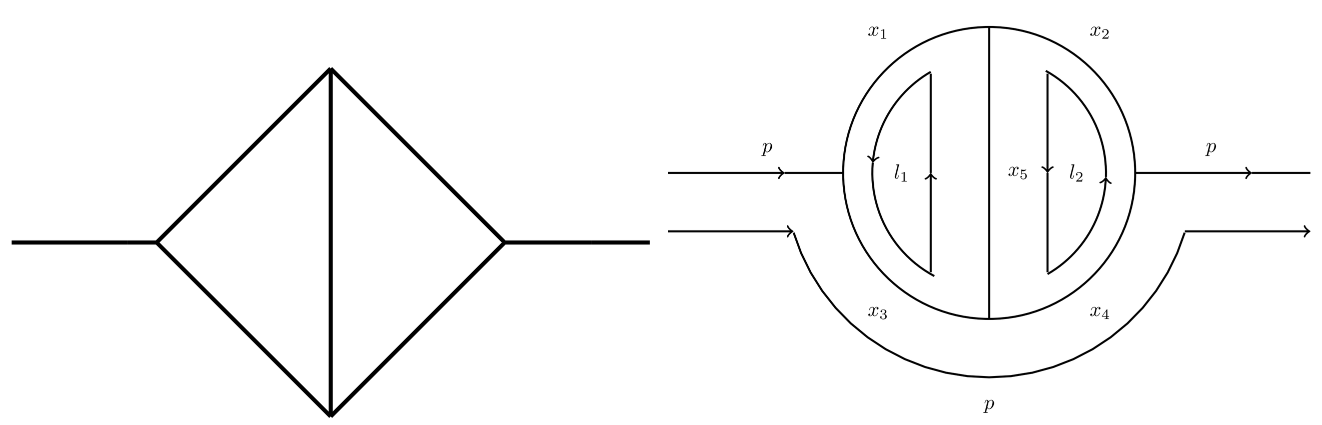
서 울 대 학 교
SEOUL NATIONAL UNIVERSITY

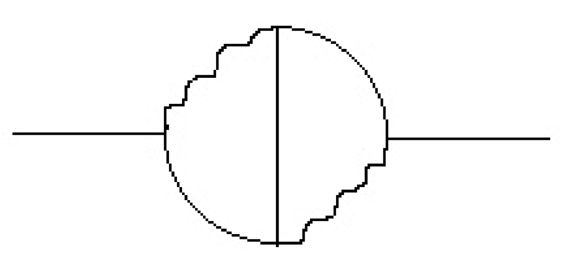
Kite Feynman Diagram



Associated Feynman Integral

$$I\ (p^2; x_1, x_2, x_3, x_4, x_5) = = \int \frac{d^d l_1\, d^d l_2}{(l_1^2 - x_1)(l_2^2 - x_2)((l_1+p)^2 - x_3)((p+l_2)^2 - x_4)((l_1-l_2)^2 - x_5)} .$$

Applications: e.g. e-Field Strength Renormalization in QED.



SFI Equation Set

SFI is Symmetries of Feynman Integrals. The Feynman Integral is shown to satisfy the following set of partial differential equations

$$c^a\, I + T x^a_j\, \partial^j\, I + J^a = 0$$

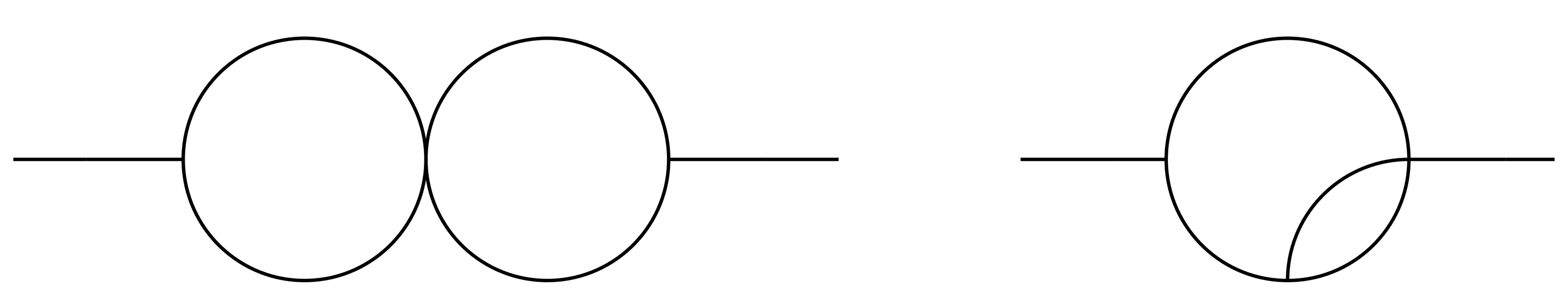
where

$$X = (x_1, ..., x_6) = (m_1^2, ..., m_5^2, p^2)$$

and

$$a = 1, ..., 7 .$$

The sources J^a depends on simpler diagrams, namely,



Idea is to solve system of partial differential equation set instead of direct evaluation of I.

Kite Results

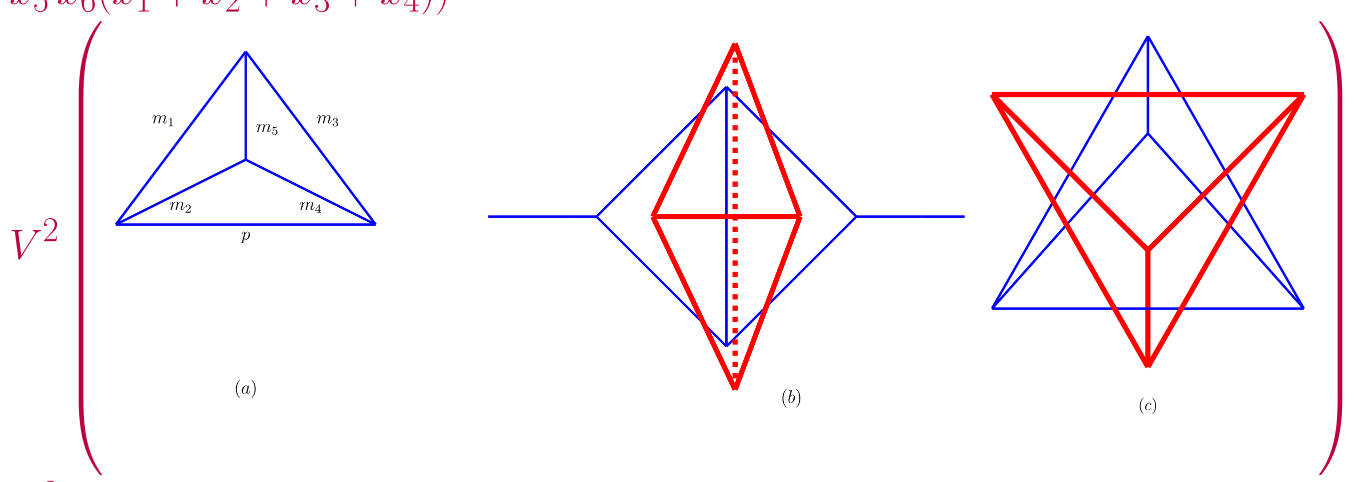
SFI group acting on X

$$G = T_{2,1} \equiv \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & * \end{pmatrix} .$$

Singular Solution. At

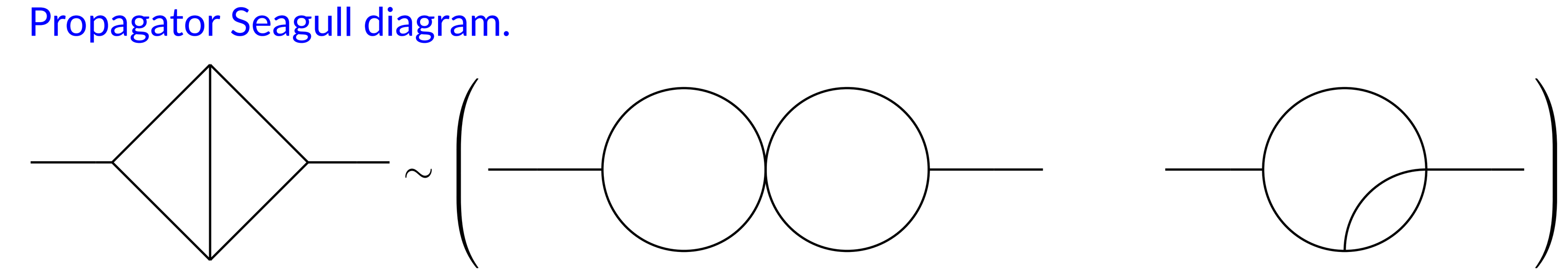
$$\begin{aligned} 0 = B_3 \ &= x_1 x_4(x_1+x_4) + x_2 x_3(x_2+x_3) + x_5 x_6(x_5+x_6) + \\ &+ x_1 x_2 x_5 + x_1 x_3 x_6 + x_2 x_4 x_6 + x_3 x_4 x_5 + \\ &- (x_1 x_4(x_2+x_3+x_5+x_6) + x_2 x_3(x_1+x_4+x_5+x_6) \\ &+ x_5 x_6(x_1+x_2+x_3+x_4)) \end{aligned}$$

$= V^2$



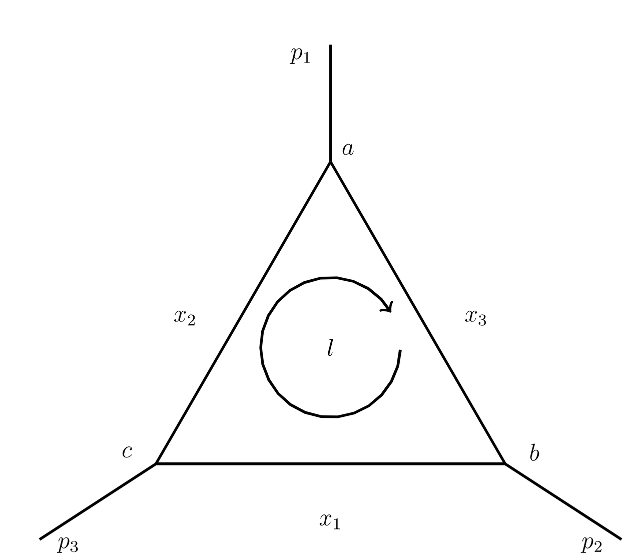
$= V^2$ (dual tetrahedron) .

At this locus of parameters the kite is given by linear combination of Figure 8 and Propagator Seagull diagram.



This method generalizes the massless case K. G. Chetyrkin and F. V. Tkachov (1981)
And also "Diamond Rule" B. Ruijl et. all (2015).

Triangle Feynman Diagram



Associated Feynman Integral

$$I = \int \frac{d^d l}{\prod_{i=1}^{3} (k_i{}^2 - m_i{}^2)}$$

where,

$$k_i = l + (p_{i+1} - p_{i-1})/3 \ \ , i = 1, 2, 3; \qquad X = (x_1, ..., x_6) = (m_1^2, m_2^2, m_3^2, p_1^2, p_2^2, p_3^2) .$$

Triangle Results

SFI Equation Set (of 7 equations) is obtained. SFI Group - Upper Triangular Group $T_{1,2}$.

Two important quantities

$$\lambda\,(x, y, z) := x^2 + y^2 + z^2 - 2xy - 2xz - 2yz; \qquad \lambda_\infty := \lambda\,(x_4, x_5, x_6)$$

and

$$\begin{aligned} B_3 \ &= x_1{}^2 x_4 + x_1 x_4{}^2 + x_2{}^2 x_5 + x_2 x_5{}^2 + x_3{}^2 x_6 + x_3 x_6{}^2 \\ &+ x_1 x_2 x_6 + x_1 x_3 x_5 + x_2 x_3 x_4 + x_4 x_5 x_6 \\ &- (x_2 x_5(x_1+x_3+x_4+x_6) + x_3 x_6(x_1+x_2+x_4+x_5) + x_1 x_4(x_2+x_3+x_5+x_6)) . \end{aligned}$$

Novel Derivation of Triangle Feynman Integral.

$$I = \frac{c_\Delta}{\sqrt{|\lambda_\infty|/4}} \left[F(h^2, c_1{}^2, a_2{}^2) + F(h^2, c_1{}^2, a_3{}^2) + cyc. \right]$$

where,

$$c_\Delta := -i\pi^{\frac{d}{2}} \Gamma\left(\frac{6-d}{2}\right) ;$$

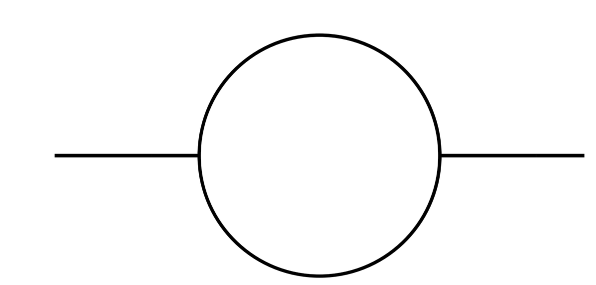
$$F(h^2,\, c^2,\, a^2) := \int_{\Delta_{a,c}} d^2 q\ \left(h^2 + q^2\right)^{\frac{d-6}{2}}$$

$$\int_{\Delta_{a,c}} d^2 q := \int_0^{|a|} dq_y \int_0^{\frac{|b|}{|a|} q_y} dq_x$$

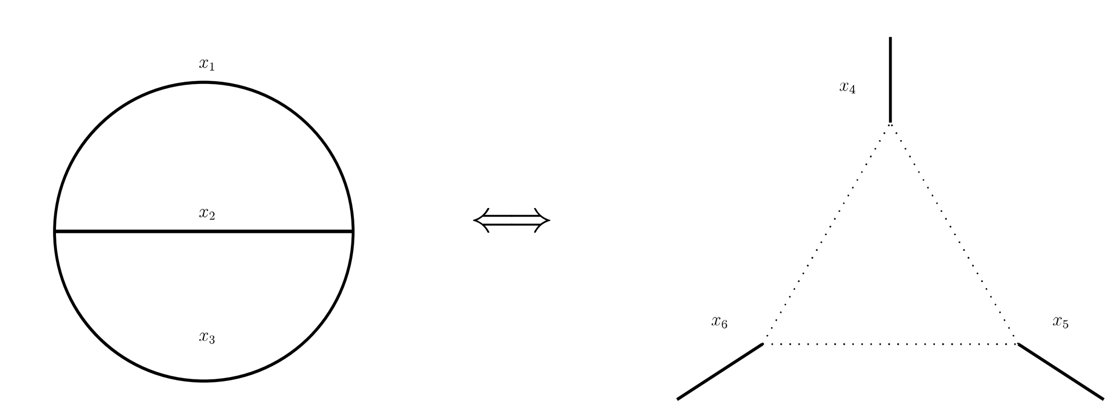
and

$$h^2 = \frac{B_3}{\lambda_\infty}; \qquad c_1{}^2 = x_1 - \frac{B_3}{\lambda_\infty}; \qquad a_1{}^2 = -\frac{(\partial_1 B_3)^2}{4 x_4 \lambda_\infty} = -\frac{\lambda_a}{4 x_4} - \frac{B_3}{\lambda_\infty} .$$

The singular locus is identified and the diagram's value on the locus's two components ($\lambda_\infty = 0$ and $B_3 = 0$) is expressed as a linear combination of descendant bubble diagrams.



Massless Triangle and the associated Magic Connection are revisited.



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