Proportionality of gravitational and electromagnetic radiation by an electron in an intense plane wave

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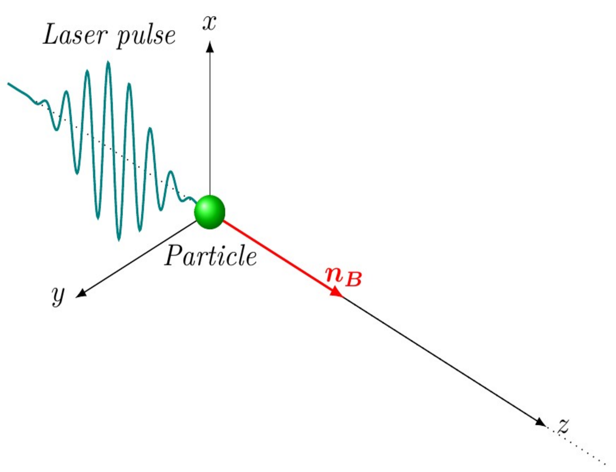
Dressed electrons in a plane wave, strong-field QED



- When an electron is driven by a *strong field* the effects of the latter on the dynamics cannot be trated perturbatively
- In this case the Furry picture [1] is adopted and the Dirac equation in presence of the background field has to be solved

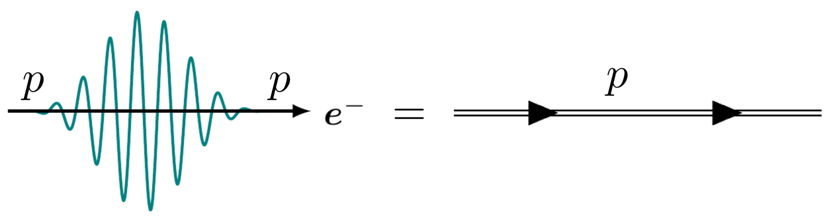
$$(i\gamma_\mu\partial^\mu - e\gamma_\mu A^\mu_B - m)\,\psi = 0$$

- Analytical solutions are known only for highly symmetric fields: *plane waves* belong to this class

- *Laser pulses* can be modeled as strong modulated plane waves

$$A^\mu_B(\phi) \quad , \quad \phi = n_B \cdot x$$

- n^μ_B is the null wave vector direction

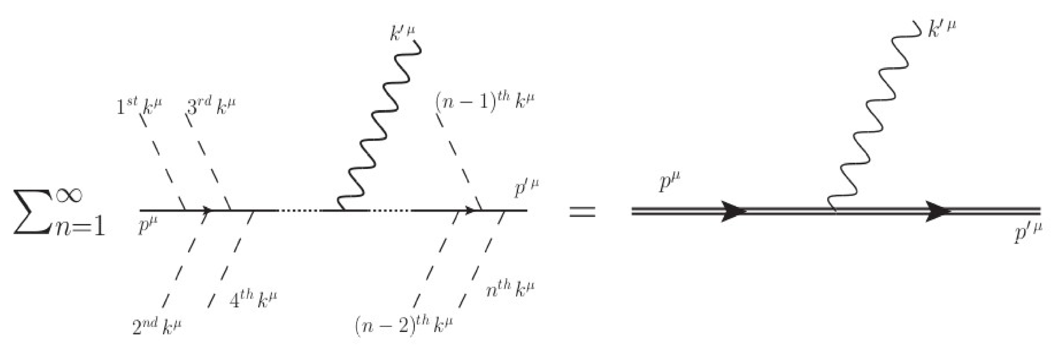


- Solving the equation corresponds to resumming the contributions of an infinite number of collinear photons attached to a fermion line
- The resulting wavefunctions are known as *Volkov states* [2]

$$\psi_p(x) = e^{iS_p(x)}\left[1 + \frac{e\gamma_\mu\gamma_\nu n^\mu_B A^\nu_B(\phi)}{2n_B\cdot p}\right]u_p$$

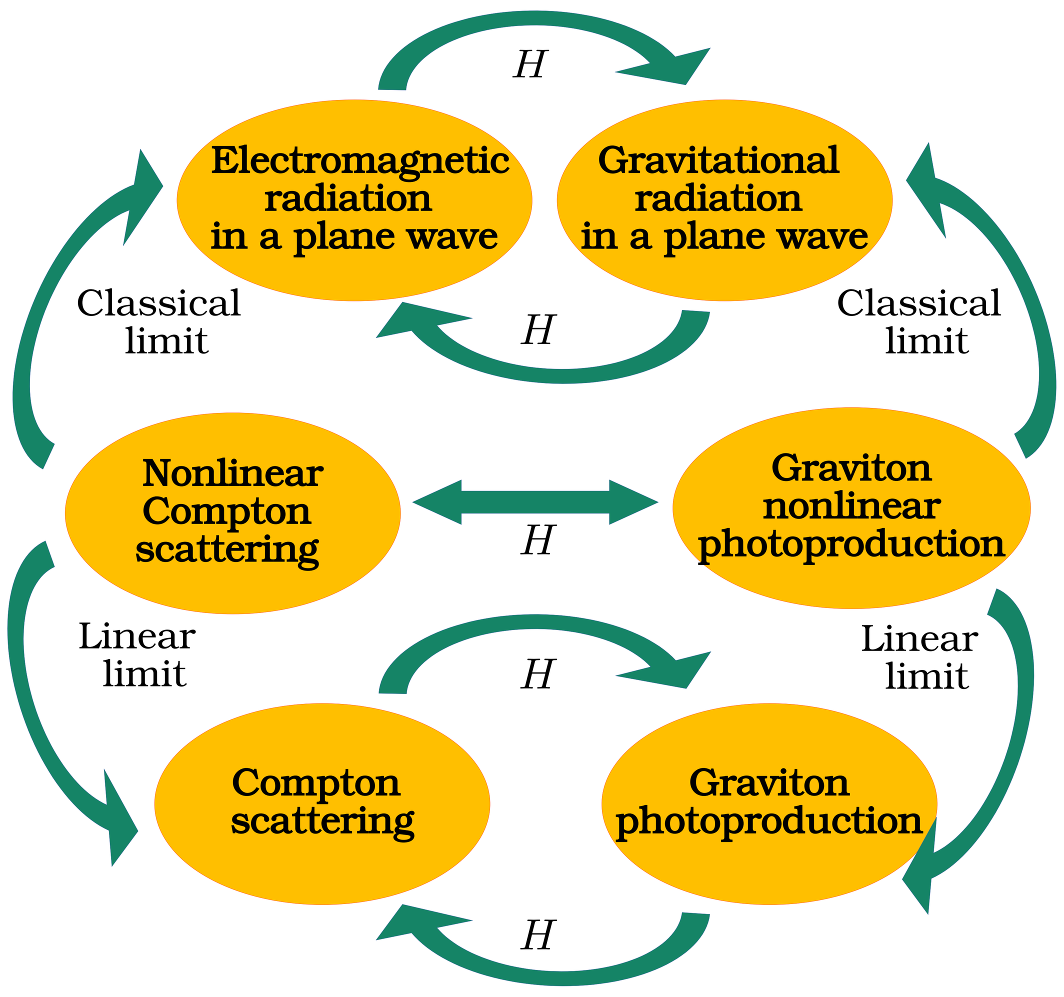
- With these solutions the nonlinear regime of QED can be explored, one interesting example is the emission of a photon by a dressed electron:

Nonlinear Compton scattering



- The first order S-matrix element is given by the three particle vertex, the laser supplies the energy-momentum needed

$$S^\gamma_{if} = -ie\int d^4x\, e^{ik\cdot x}\bar\psi_{p'}(x)\gamma^\mu\psi_p(x)\varepsilon^*_\mu$$
$$= -ieJ_V(k)\cdot\varepsilon^*$$



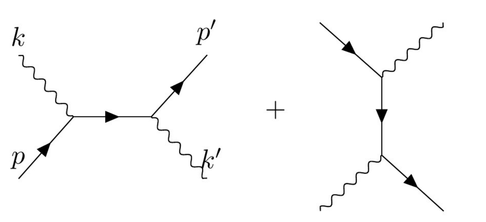
Tree-level quantum gravity and QED

- The linear graviton Lagrangian is what we need, in Dedonder gauge

$$\mathcal{L}_g = -\frac{1}{2}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + \frac{1}{4}\partial^\mu h\partial_\mu h + \frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}$$

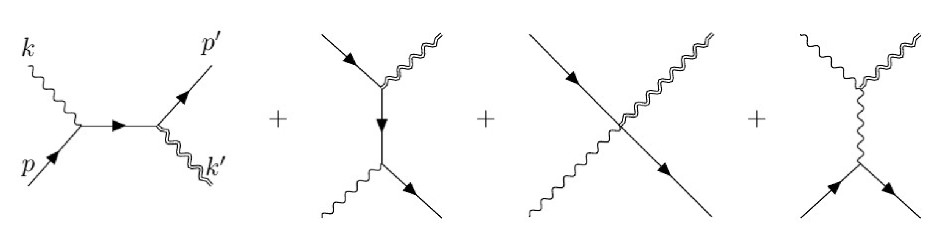
- At tree-level it is easy to show that exists a proportionality between the ampltudes of

Compton scattering



and

Graviton photoproduction



$$\varepsilon_{i,\alpha}\varepsilon^*_{f,\mu}\varepsilon^*_{f,\nu}M^{\alpha\mu\nu}_{e\gamma\to eg} = H\varepsilon_{i,\alpha}\varepsilon^*_{f,\mu}M^{\alpha\mu}_{e\gamma\to e\gamma}$$

- The proportionality constant H is the **same** found in the classical tratment
- This result can be interpreted in this context as a specific case of general theorems concerning on-shell amplitudes [3]

[1] W. H. Furry, Phys. Rev. **81**, 115 (1951)

[2] D. M. Volkov, Z. Phys. **94**, 250 (1935)

[3] H. Elvang and Y.-t. Huang, *Scattering Amplitudes in Gauge Theory and Gravity* (Cambridge University Press, Cambridge, 2015)

[4] A. I. Nikishov and V. I. Ritus, Usp. Fiz. Nauk **180**, 1135 (2010)

[5] G. Audagnotto, C. H. Keitel, and A. Di Piazza, *Proportionality of gravitational and electromagnetic radiation by an electron in an intense plane wave* (2022) arXiv:2208.02215 [hep-ph].

Radiation from moving particles

- A charge driven by a plane wave field produces both electromagnetic and gravitational radiation, in fact every form of energy-momentum is a source of gravity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\kappa^2}{4}T_{\mu\nu}$$

- Here $\kappa^2 = 32\pi G$ is the coupling
- If the gravitational waves are weak enough (very often the case) one can exploit the *weak field* approximation $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ which brings to the linearized Einstein equations for the perturbation

$$\Box h_{\mu\nu} = \frac{\kappa^2}{2}T_{\mu\nu}$$

- There are three possible sources in the system: the particle, the radiated electromagnetic field and the background field. Moreover the two fields can interact

$$T^{\mu\nu} = T^{\mu\nu}_P + T^{\mu\nu}_R + T^{\mu\nu}_B + T^{\mu\nu}_{RB}$$

Particle — *Radiated field* — *Background field* — *Radiated – Background interaction*

- The background alone does not contribute, an electromagnetic plane wave cannot produce a linear gravitational wave — ~~$T^{\mu\nu}_B$~~
- If the radiation-reaction effects are negligible the radiation field self-interaction has to be neglected [4] — ~~$T^{\mu\nu}_R$~~

$$T^{\mu\nu} = T^{\mu\nu}_P + T^{\mu\nu}_{RB}$$

- Introducing the *Classical amplitudes*

$$S^\gamma_c(k) = -ieJ^\mu_P(k)\varepsilon^*_\mu \qquad S^g_c(k) = i\frac{\kappa}{2}T^{\mu\nu}(k)\varepsilon^*_\mu\varepsilon^*_\nu$$

Electromagnetic radiation in a plane wave — *Gravitational radiation in a plane wave*

- A ***Proportionality*** is found between the amplitudes [5]

$$S^g_c(k) = HS^\gamma_c(k)$$

- The proportionality constant reads

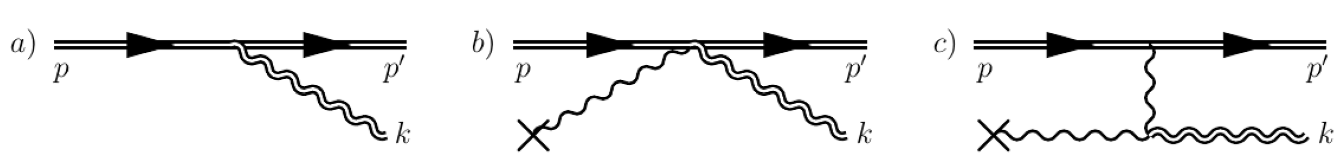
$$H = -\frac{\kappa}{2e}\left(\frac{p_i\cdot\varepsilon^*_f k_f\cdot p_f - p_f\cdot\varepsilon^*_f k_f\cdot p_i}{k_i\cdot k_f}\right)$$

Graviton photoproduction in a plane wave

- Now we want to put everything together and calculate graviton photoproduction in strong-field QED. Will we find the proportionality again in this nonlinear process?
- The first-order contributions to the process are represented by [5]

Graviton nonlinear photoproduction

$$S^g_{fi} = i\frac{\kappa}{2}\mathcal{T}\langle p'|\left\{\int d^4x\, e^{ik\cdot x}\varepsilon^*_\mu\varepsilon^*_\nu\left[T^{\mu\nu}_D(x) - ieT^{\mu\nu}_{QB}(x)\int d^4y\, J_{D,\alpha}(y)A^\alpha_Q(y)\right]\right\}|p\rangle$$



Particle contribution

$$T^{\mu\nu}_D = \bar\psi\left[\frac{i}{4}\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}} - \frac{e}{2}\gamma^{\{\mu}A^{\nu\}}_B\right]\psi$$

Radiated-background field interaction

$$T^{\mu\nu}_{QB} = F^{\mu\alpha}_Q F_{B,\alpha}{}^{\nu} + F^{\mu\alpha}_B F_{Q,\alpha}{}^{\nu} + \frac{1}{2}\eta^{\mu\nu}F^{\alpha\beta}_Q F_{B,\alpha\beta}$$

- The amplitude can be simplified using only the energy-momentum conservation law and the semiclassical nature of Volkov states
- The result is interesting, the proportionality is still found to be there, and no integral had to be done in order to show it

$$S^g_{if}(k) = HS^\gamma_{if}(k)$$

- The proportionality constant is still the **same**

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