Radiation and Reaction at One Loop

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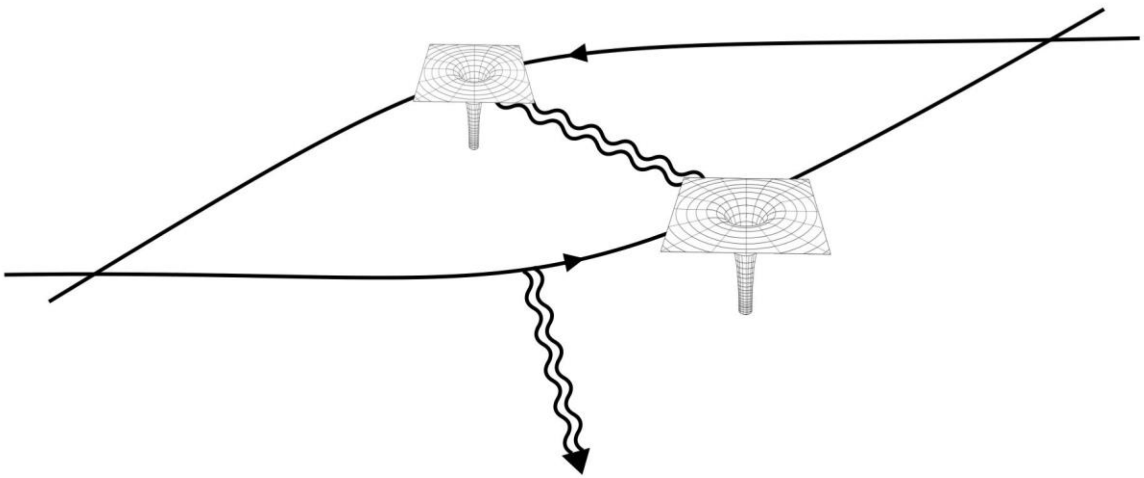
Based on “Radiation and Reaction at One Loop”
With Donal O’Connell & Ingrid A. Vazquez-Holm.

arXiv: soon.

QCD meets Gravity 2022, 16/12/2022

Background and motivation



$$\Rightarrow \langle R_{\mu\nu\rho\sigma}(x) \rangle = \int_k \mathcal{A}(k)\, e^{-ik\cdot x}$$

Kosower, Maybee & O'Connell, 2018
Kosower, O'Connell, Gonzo & Cristofoli, 2019
Sergola, Peinador-Veiga, Monteiro & O'Connell, 2021
Sergola, White, Moynihan, Ross, Cristofoli, Gonzo & O'Connell, 2021

See also Graham's and Radu's talks!

Classical radiation at one loop

$$|\psi\rangle \sim \int |p_1, p_2\rangle$$

$$\langle F_{\mu\nu}(x)\rangle = \sum_\eta \int \mathrm{d}\Phi(k) \left(k_{[\mu}\varepsilon^\eta_{\nu]} \langle\psi|S^\dagger a_\eta(k) S|\psi\rangle e^{-ik\cdot x} + \text{h.c.}\right)$$

On-shell integral support

$$\alpha_\eta(k)$$

$$S = 1 + iT$$

Shen, 2018
Kosower, Maybee & O'Connell, 2018
Kosower, O'Connell, Gonzo & Cristofoli, 2019
Vazquez-Holm & Carrasco, 2020
Vazquez-Holm & Carrasco, 2021
Herrmann, Parra-Martinez, Ruf & Zeng, 2021

Classical radiation at one loop

$$|\psi\rangle \sim \int |p_1, p_2\rangle$$

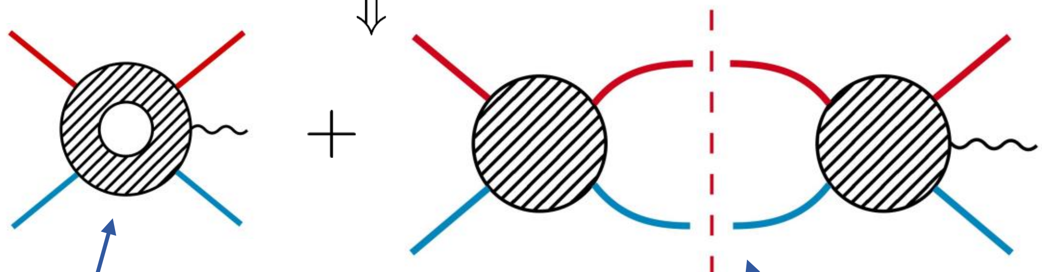
$$\langle F_{\mu\nu}(x)\rangle = \sum_\eta \int \mathrm{d}\Phi(k) \left(k_{[\mu}\varepsilon^\eta_{\nu]} \underbrace{\langle\psi|S^\dagger a_\eta(k) S|\psi\rangle}_{\alpha_\eta(k)} e^{-ik\cdot x} + \text{h.c.}\right)$$

On-shell integral support

$$S = 1 + iT$$

$$\Downarrow$$

$$\alpha_\eta(k) = i$$



$\mathcal{A}_{1L}$ ← Taken in the limit $\hbar \to 0$ → $\mathcal{A}_{tree} \times \mathcal{A}^*_{tree}$

Shen, 2018
Kosower, Maybee & O'Connell, 2018
Kosower, O'Connell, Gonzo & Cristofoli, 2019
Vazquez-Holm & Carrasco, 2020
Vazquez-Holm & Carrasco, 2021
Herrmann, Parra-Martinez, Ruf & Zeng, 2021

Classical radiation at one loop

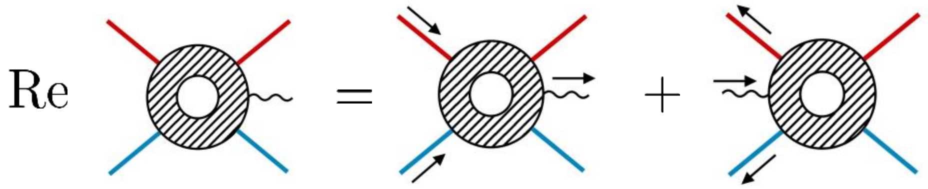
Real part

$$\alpha_\eta(k) = \frac{1}{2}\langle\psi| i a_\eta(k)\left(T + T^\dagger\right) + T^\dagger\left[a_\eta(k), T\right] - \left[a_\eta(k), T^\dagger\right] T|\psi\rangle$$

Imaginary part

Herrmann, Parra-Martinez, Ruf & Zeng, 2021
Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon
& Zeng 2021

Classical radiation at one loop



$$\sim g^5 \int \mathrm{d}^4\ell \, N i(P - P^*)$$

Real part

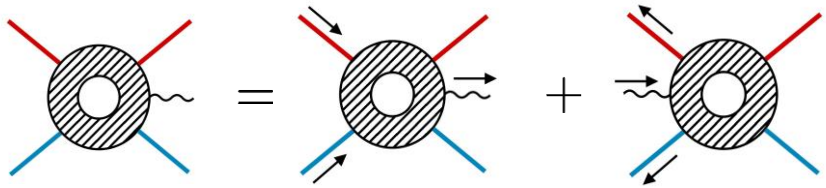
$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathrm{P.V.}\frac{1}{p^2 - m^2} + i\delta(p^2 - m^2)$$

$$\alpha_\eta(k) = \tfrac{1}{2}\langle\psi| i a_\eta(k)\,(T + T^\dagger) + T^\dagger\,[a_\eta(k), T] - [a_\eta(k), T^\dagger]\,T|\psi\rangle$$

Imaginary part

Herrmann, Parra-Martinez, Ruf & Zeng, 2021
Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon & Zeng 2021

Classical radiation at one loop



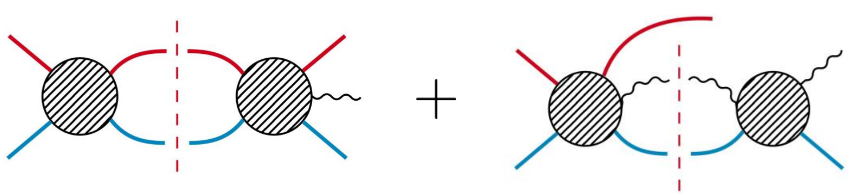
$$\mathrm{Re}\;[\ldots] = [\ldots] + [\ldots] \sim g^5 \int \mathrm{d}^4\ell\, N i(P - P^*)$$

Real part

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathrm{P.V.}\frac{1}{p^2 - m^2} + i\delta(p^2 - m^2)$$

$$\alpha_\eta(k) = \tfrac{1}{2}\langle\psi| i a_\eta(k)\,(T + T^\dagger) + T^\dagger\,[a_\eta(k), T] - [a_\eta(k), T^\dagger]\,T|\psi\rangle$$

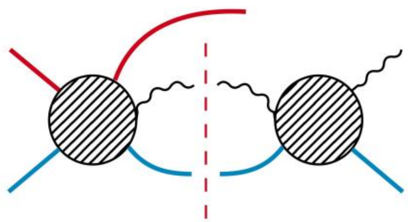
Imaginary part



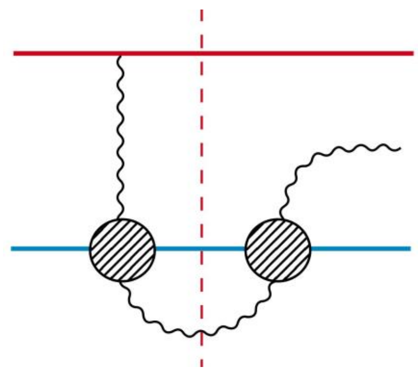
Herrmann, Parra-Martinez, Ruf & Zeng, 2021
Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon & Zeng 2021

Radiation reaction and imaginary part

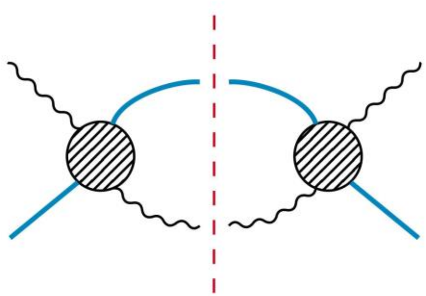
$$\mathrm{Im}\,\alpha_\eta(k) =$$



$$=$$

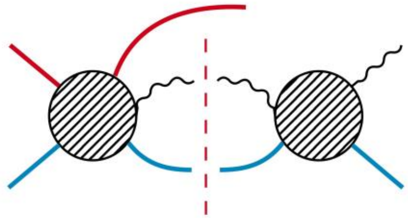


$$\rightarrow$$

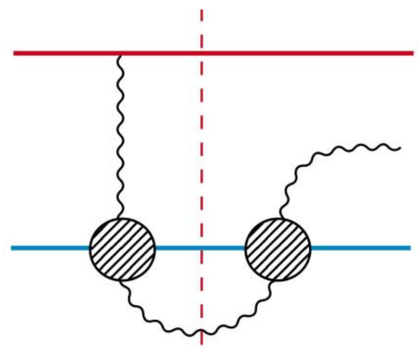


Radiation reaction and imaginary part

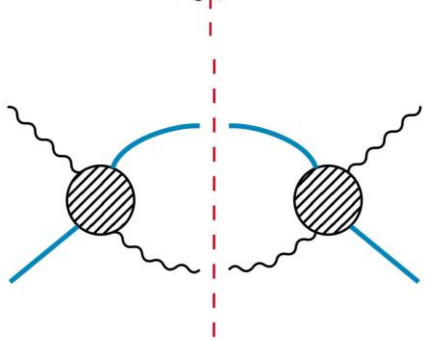
$$\mathrm{Im}\,\alpha_\eta(k) =$$



$$=$$

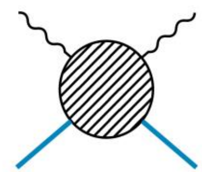


$$\to$$



Geometrical factor

$$\sim \quad \frac{1}{6\pi} \times$$



QED example: ALD force

Radiation reaction force

Lorentz force

$$m_2 \frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} = QF^{\mu\nu}p_\nu + \frac{Q^2}{6\pi}\left(\frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2} + \frac{p_2^\mu}{m_2^2}\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\cdot\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\right)$$

"Schott term"

$$\Rightarrow \quad x^\mu(\tau) = b^\mu + u^\mu\tau + \cdots + Q^4\delta x^\mu_{\mathrm{ALD}}(\tau)$$

ALD, 1892-1905. Schott, 1912.
Kosower, Maybee & O'Connell, 2018

QED example: ALD force

Lorentz force

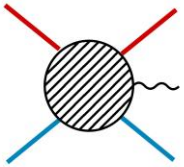
Radiation reaction force

$$m_2 \frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} = QF^{\mu\nu}p_\nu + \frac{Q^2}{6\pi}\left(\frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2} + \frac{p_2^\mu}{m_2^2}\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\cdot\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\right)$$

“Schott term”

$$\Rightarrow \quad x^\mu(\tau) = b^\mu + u^\mu \tau + \cdots + Q^4 \delta x^\mu_{\mathrm{ALD}}(\tau)$$

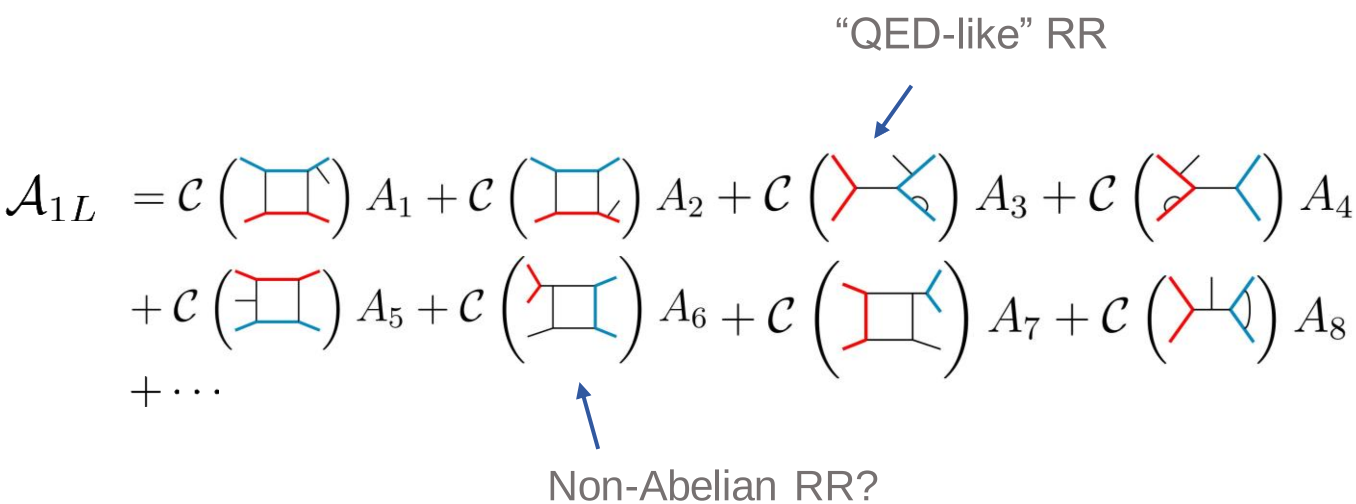
$$\Rightarrow \quad -k^2 \tilde{A}^\mu(k) = \cdots + \frac{Q^5}{6\pi} \times$$



ALD, 1892-1905. Schott, 1912.
Kosower, Maybee & O’Connell, 2018

Radiation in QCD

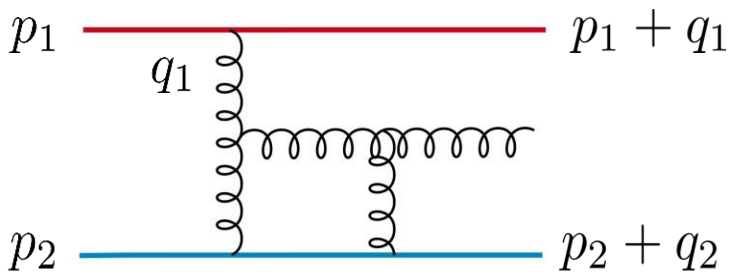
$$\langle F^A_{\mu\nu}(x)\rangle = \sum_\eta \int \mathrm{d}\Phi(k) \left(-ik_{[\mu}\varepsilon^\eta_{\nu]}\langle\psi|S^\dagger a^A_\eta(k) S|\psi\rangle e^{-ik\cdot x} + \text{h.c.}\right)$$



Ross, Maybee, de la Cruz & O'Connell, 2020

Maximally non-Abelian sector: A_6

$A_6 =$



$+ \cdots$

$$= \frac{1}{q_1^2} \int \mathrm{d}^4\ell\, \delta(p_2 \cdot \ell)\, \varepsilon \cdot \sum_i J_i$$

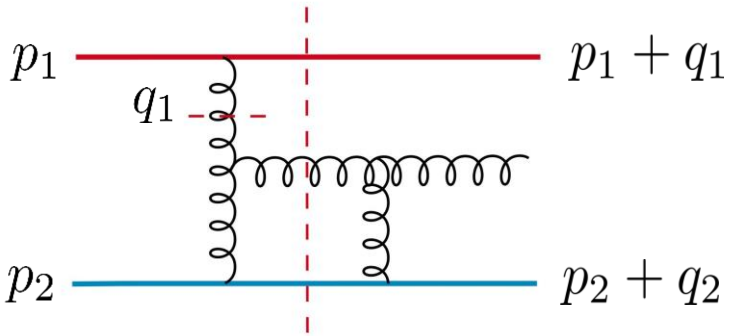
$$J_1^\mu = -\frac{\ell^\mu}{(\ell+q_2)^{\,2}\,(\ell-q_1)^{\,2}} \left(\frac{p_1 \cdot p_2}{4\, p_2 \cdot q_1} + \frac{\ell \cdot p_1}{\ell^2} \right) p_2^2,$$

$$J_2^\mu = \frac{q_1^\mu}{\ell^2\,(\ell+q_2)^{\,2}\,(\ell-q_1)^{\,2}} (p_2^2\, \ell \cdot p_1 + p_1 \cdot p_2\, p_2 \cdot q_1),$$

$$J_3^\mu = \frac{p_1^\mu}{\ell^2} \left(\frac{p_2^2}{4(\ell+q_2)^2} - \frac{(p_2 \cdot q_1)^{\,2}}{(\ell+q_2)^{\,2}(\ell-q_1)^2} \right),$$

$$J_4^\mu = \frac{p_2^\mu}{\ell^2} \left(-\frac{p_2^2\, \ell \cdot p_1}{4\,(\ell-q_1)^{\,2}\, p_2 \cdot q_1} - \frac{p_2^2\, p_1 \cdot p_2}{4\,(p_2 \cdot q_1)^2} + \frac{(p_1 \cdot p_2\, q_1 \cdot q_2 - q_1 \cdot p_2\, q_2 \cdot p_1)}{(\ell+q_2)^{\,2}\,(\ell-q_1)^{\,2}} \right)$$

Maximally non-Abelian sector: A_6

$\mathrm{Im}\, A_6 =$  $+ \cdots$

$$= \frac{1}{q_1^2} \int \mathrm{d}^4\ell\, \delta(p_2 \cdot \ell)\delta((q_1 - \ell)^2)\, \varepsilon \cdot \sum_i J_i$$

$$J_1^\mu = -\frac{\ell^\mu}{(\ell + q_2)^2 (\ell - q_1)^2} \left(\frac{p_1 \cdot p_2}{4\, p_2 \cdot q_1} + \frac{\ell \cdot p_1}{\ell^2} \right) p_2^2,$$

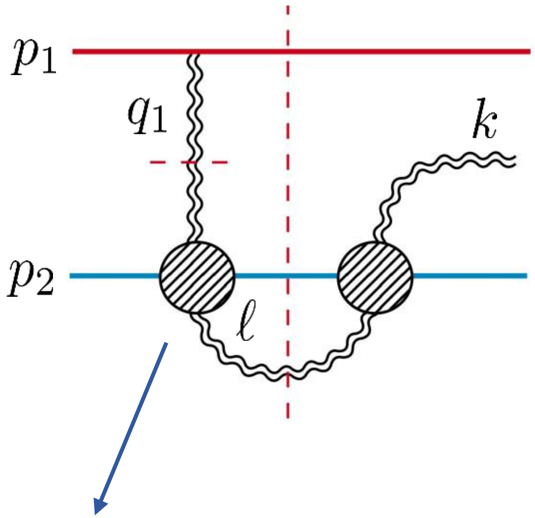
$$J_2^\mu = \frac{q_1^\mu}{\ell^2 (\ell + q_2)^2 (\ell - q_1)^2} (p_2^2\, \ell \cdot p_1 + p_1 \cdot p_2\, p_2 \cdot q_1),$$

$$J_3^\mu = \frac{p_1^\mu}{\ell^2} \left(\frac{p_2^2}{4(\ell + q_2)^2} - \frac{(p_2 \cdot q_1)^2}{(\ell + q_2)^2 (\ell - q_1)^2} \right),$$

$$J_4^\mu = \frac{p_2^\mu}{\ell^2} \left(-\frac{p_2^2\, \ell \cdot p_1}{4\, (\ell - q_1)^2\, p_2 \cdot q_1} - \frac{p_2^2\, p_1 \cdot p_2}{4\, (p_2 \cdot q_1)^2} + \frac{(p_1 \cdot p_2\, q_1 \cdot q_2 - q_1 \cdot p_2\, q_2 \cdot p_1)}{(\ell + q_2)^2 (\ell - q_1)^2} \right)$$

Gravity imaginary part

$$\mathrm{Im}\,\mathcal{A}_{\mathrm{GR}} =$$

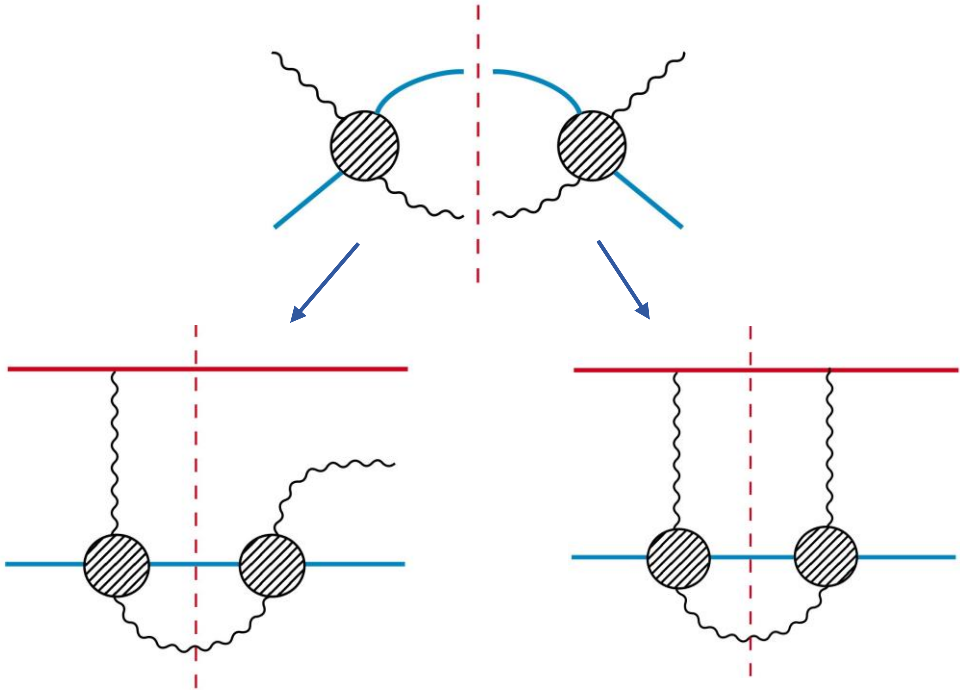


$$= \frac{1}{q_1^2}\int \mathrm{d}\Phi(l)\delta(p_2\cdot(q_1-\ell))\frac{(p_1\cdot k)^4}{q_1\cdot\ell\,k\cdot\ell}$$

$$\times \sum_{\text{helicities}} (\varepsilon\cdot\varepsilon)^2(\varepsilon\cdot\varepsilon)^2$$

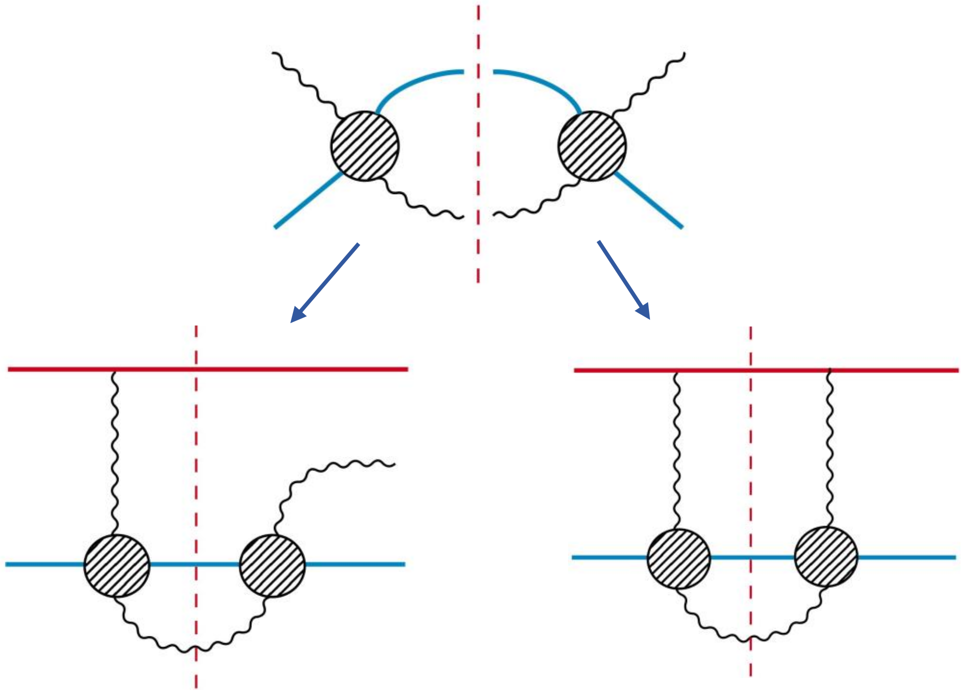
$$\mathcal{A}_{\mathrm{GR}}^{\mathrm{Compt.}} \propto \left(\mathcal{A}_{\mathrm{EM}}^{\mathrm{Compt.}}\right)^2$$

Cut universality



Mino, Sasaki & Tanaka, 1997
Quinn & Wald, 1997
Kosower, Maybee & O'Connell, 2018
Dlapa, Kalin, Liu, Neef & Porto, 2022

Cut universality



$$\frac{Q^2}{6\pi}\frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2}$$

$$\frac{Q^2}{6\pi}\frac{p_2^\mu}{m_2^2}\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\cdot\frac{\mathrm{d}p_2}{\mathrm{d}\tau}$$

Mino, Sasaki & Tanaka, 1997
Quinn & Wald, 1997
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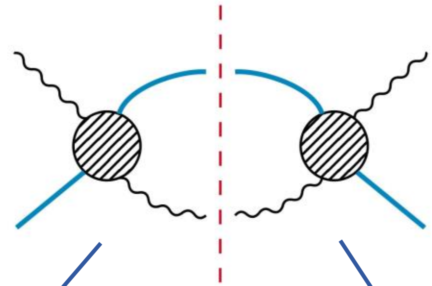
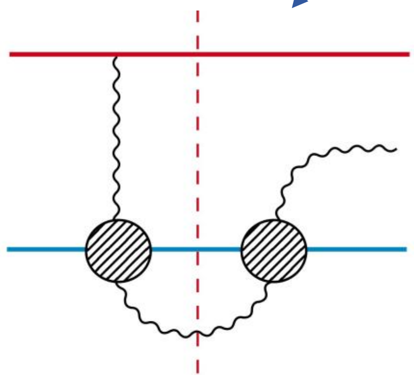
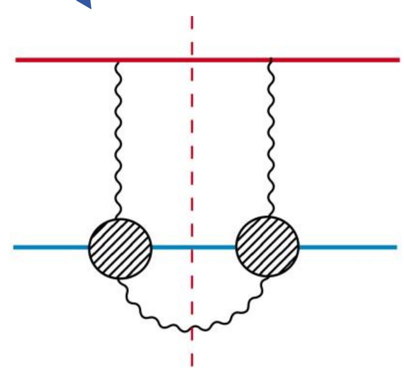
Cut universality



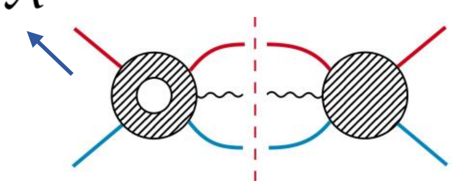


$$\frac{Q^2}{6\pi}\frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2}$$



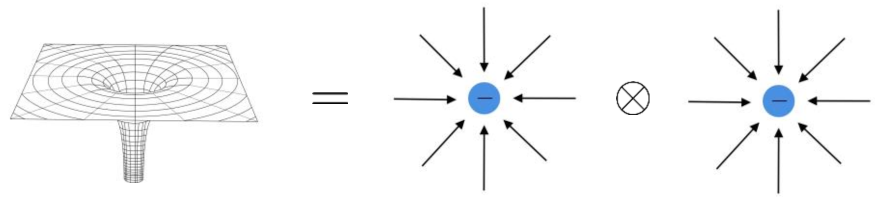
$$\frac{Q^2}{6\pi}\frac{p_2^\mu}{m_2^2}\frac{\mathrm{d}p_2}{\mathrm{d}\tau}\cdot\frac{\mathrm{d}p_2}{\mathrm{d}\tau}$$

$$\Delta p_{3L}^\mu = \int q^\mu \mathcal{A} + \int \ell^\mu \mathcal{A}^* \mathcal{A}$$



Mino, Sasaki & Tanaka, 1997
Quinn & Wald, 1997
Kosower, Maybee & O'Connell, 2018
Dlapa, Kalin, Liu, Neef & Porto, 2022

Gravity



$$R_{\mu\nu\rho\sigma}(x) = -\sum_{\eta} \int \mathrm{d}\Phi(k) \left(k_{[\mu}\varepsilon^{\eta}_{\nu]} k_{[\rho}\varepsilon^{\eta}_{\sigma]} \alpha_{\eta}(k) e^{-ik\cdot x} + \mathrm{h.c.} \right)$$

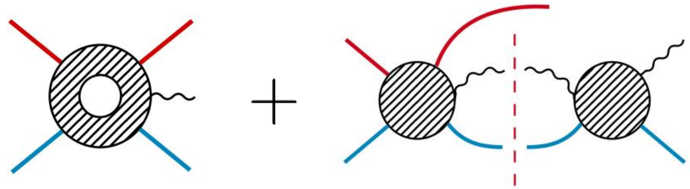
Compute using the DC of
Vazquez-Holm & Carrasco, 2021

Bern, Carrasco & Johansson, 2008
Goldberger & Ridgway, 2016
Luna, Nicholson O'Connell & White, 2017
Vazquez-Holm & Carrasco, 2020
Vazquez-Holm & Carrasco, 2021

Conclusions and future directions

- Amplitudes for classical radiation:

$$\alpha(k) = \mathrm{Re}$$



- We care about gravitational waveforms, it is natural use BCJ numerators.
- The physics is surprisingly rich.
- Things seem to be universal from an amplitude perspective.
- *Next:* Compute gravity waveform.

Thank you!