



Radiation and Reaction at One Loop

Matteo Sergola

matteo.sergola@ed.ac.uk

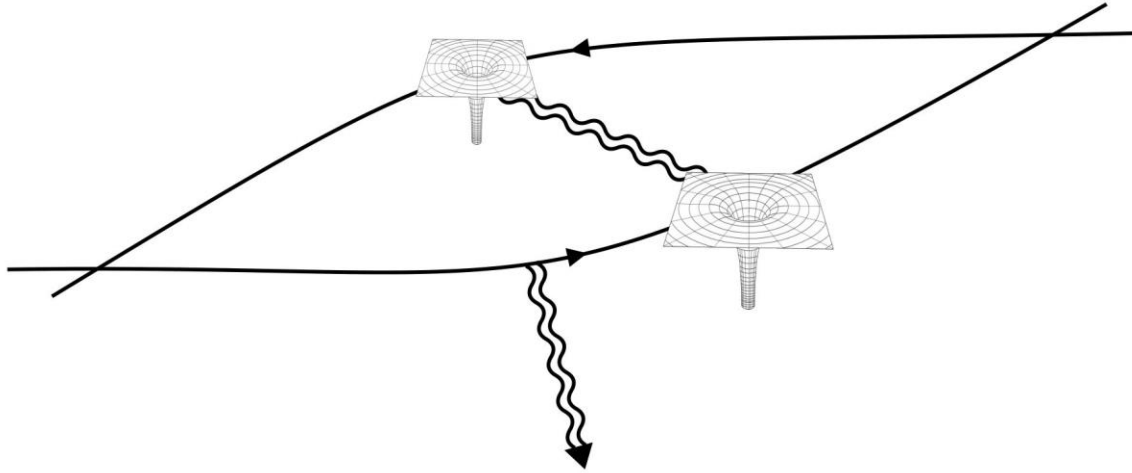
Higgs Centre for Theoretical Physics, The University of Edinburgh.

Based on “Radiation and Reaction at One Loop”
With Donal O’Connell & Ingrid A. Vazquez-Holm.

arXiv: soon.

QCD meets Gravity 2022, 16/12/2022

Background and motivation



$$\Rightarrow \langle R_{\mu\nu\rho\sigma}(x) \rangle = \int_k \mathcal{A}(k) e^{-ik \cdot x}$$

Kosower, Maybee & O'Connell, 2018

Kosower, O'Connell, Gonzo & Cristofoli, 2019

Sergola, Peinador-Veiga, Monteiro & O'Connell, 2021

Sergola, White, Moynihan, Ross, Cristofoli, Gonzo & O'Connell, 2021

See also
Graham's and
Radu's talks!

Classical radiation at one loop

$$\langle F_{\mu\nu}(x) \rangle = \sum_{\eta} \int d\Phi(k) \left(k_{[\mu} \varepsilon_{\nu]}^{\eta} \underbrace{\langle \psi | S^{\dagger} a_{\eta}(k) S | \psi \rangle}_{\alpha_{\eta}(k)} e^{-ik \cdot x} + \text{h.c.} \right)$$

$|\psi\rangle \sim \int |p_1, p_2\rangle$
↙ ↘
On-shell integral support $S = 1 + iT$

Shen, 2018

Kosower, Maybee & O'Connell, 2018

Kosower, O'Connell, Gonzo & Cristofoli, 2019

Vazquez-Holm & Carrasco, 2020

Vazquez-Holm & Carrasco, 2021

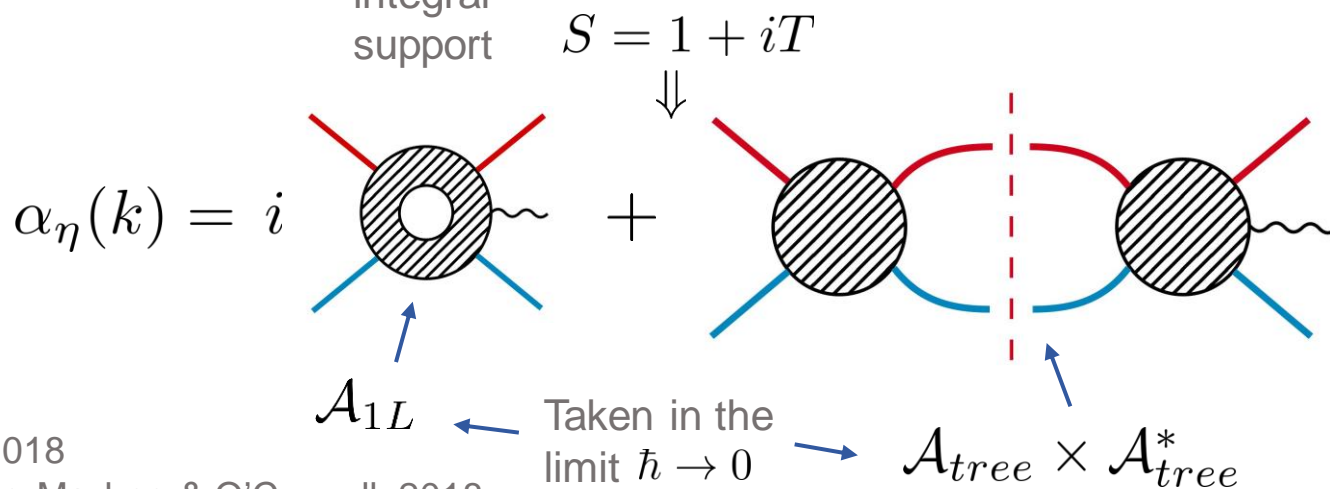
Herrmann, Parra-Martinez, Ruf & Zeng, 2021

Classical radiation at one loop

$$\langle F_{\mu\nu}(x) \rangle = \sum_{\eta} \int d\Phi(k) \left(k_{[\mu} \varepsilon_{\nu]}^{\eta} \underbrace{\langle \psi | S^{\dagger} a_{\eta}(k) S | \psi \rangle}_{\alpha_{\eta}(k)} e^{-ik \cdot x} + \text{h.c.} \right)$$

$|\psi\rangle \sim \int |p_1, p_2\rangle$
 (Arrows point from the bra and ket in the above equation to the integration in this one)

On-shell integral support (Arrow points to the $d\Phi(k)$ integral)



Shen, 2018

Kosower, Maybee & O'Connell, 2018

Kosower, O'Connell, Gonzo & Cristofoli, 2019

Vazquez-Holm & Carrasco, 2020

Vazquez-Holm & Carrasco, 2021

Herrmann, Parra-Martinez, Ruf & Zeng, 2021

Classical radiation at one loop

$$\alpha_\eta(k) = \frac{1}{2} \langle \psi | i a_\eta(k) \overbrace{(T + T^\dagger)}^{\text{Real part}} + \underbrace{T^\dagger [a_\eta(k), T] - [a_\eta(k), T^\dagger] T}_{\text{Imaginary part}} | \psi \rangle$$

Classical radiation at one loop

$$\text{Re} \left[\text{Diagram} \right] = \text{Diagram}_1 + \text{Diagram}_2 \sim g^5 \int d^4 \ell \text{Ni}(P - P^*)$$

The diagram shows a shaded circle with a central white circle. Two red lines enter from the top-left and top-right, and two blue lines enter from the bottom-left and bottom-right. A wavy line exits from the right side. The first diagram on the right has arrows on the red lines pointing towards the circle and on the blue lines pointing away from it. The second diagram on the right has arrows on the red lines pointing away from the circle and on the blue lines pointing towards it.

Real part

$$\frac{1}{p^2 - m^2 + i\epsilon} = \text{P.V.} \frac{1}{p^2 - m^2} + i\delta(p^2 - m^2)$$

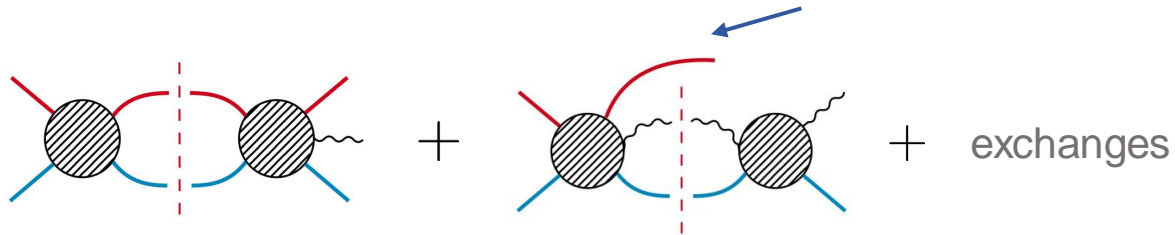
$$\alpha_\eta(k) = \frac{1}{2} \langle \psi | i a_\eta(k) (T + T^\dagger) + \underbrace{T^\dagger [a_\eta(k), T] - [a_\eta(k), T^\dagger] T}_{\text{Imaginary part}} | \psi \rangle$$

Classical radiation at one loop

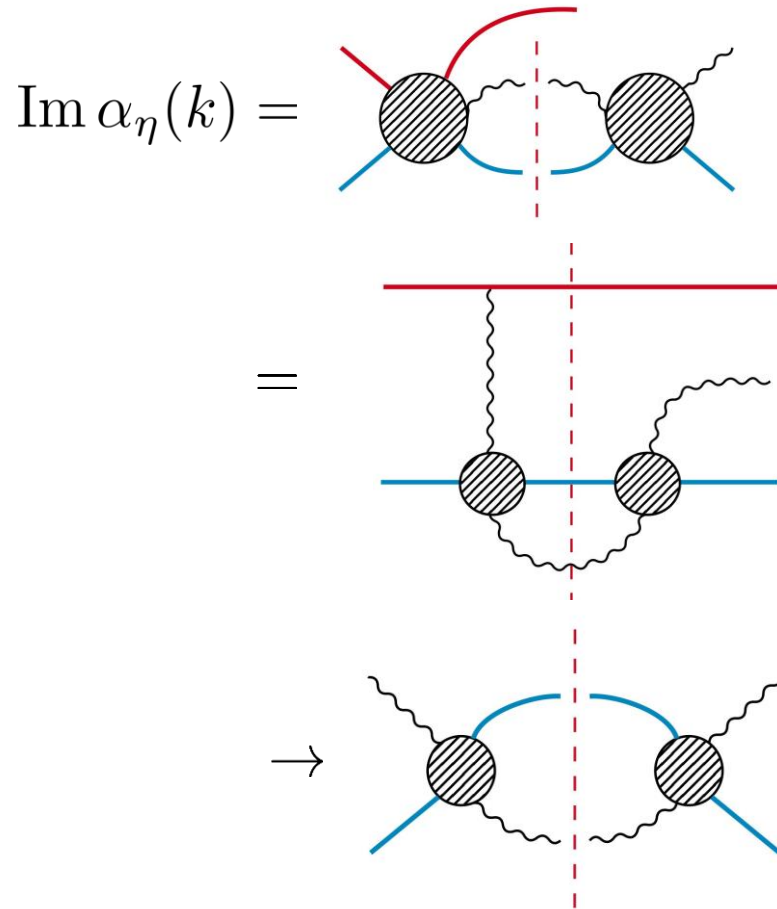
$$\text{Re} \left[\text{Diagram} \right] = \text{Diagram}_1 + \text{Diagram}_2 \sim g^5 \int d^4 \ell \text{Ni}(P - P^*)$$

$$\text{Real part} \quad \frac{1}{p^2 - m^2 + i\epsilon} = \text{P.V.} \frac{1}{p^2 - m^2} + i\delta(p^2 - m^2)$$

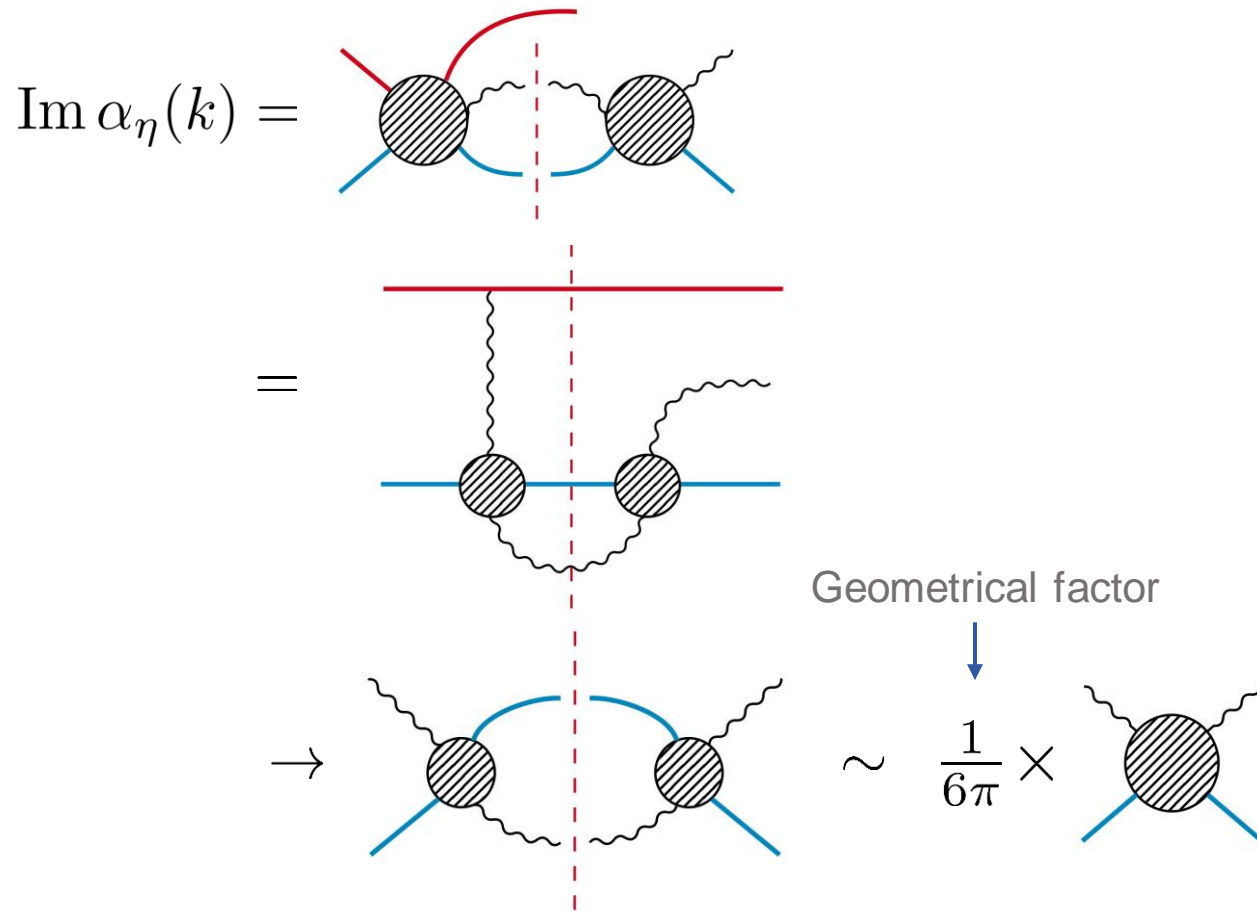
$$\alpha_\eta(k) = \frac{1}{2} \langle \psi | i a_\eta(k) (T + T^\dagger) + \underbrace{T^\dagger [a_\eta(k), T] - [a_\eta(k), T^\dagger] T}_{\text{Imaginary part}} | \psi \rangle$$



Radiation reaction and imaginary part



Radiation reaction and imaginary part



QED example: ALD force



$$m_2 \frac{dp_2^\mu}{d\tau} = Q F^{\mu\nu} p_\nu + \frac{Q^2}{6\pi} \left(\frac{d^2 p_2^\mu}{d\tau^2} + \frac{p_2^\mu}{m_2^2} \frac{dp_2}{d\tau} \cdot \frac{dp_2}{d\tau} \right)$$

Radiation reaction force

↑

“Schott term”

$$\Rightarrow x^\mu(\tau) = b^\mu + u^\mu \tau + \dots + Q^4 \delta x_{\text{ALD}}^\mu(\tau)$$

QED example: ALD force



$$m_2 \frac{dp_2^\mu}{d\tau} = \overset{\text{Lorentz force}}{\downarrow} Q F^{\mu\nu} p_\nu + \frac{Q^2}{6\pi} \left(\overset{\text{Radiation reaction force}}{\downarrow} \frac{d^2 p_2^\mu}{d\tau^2} + \frac{p_2^\mu}{m_2^2} \frac{dp_2}{d\tau} \cdot \frac{dp_2}{d\tau} \right)$$

“Schott term”

$$\Rightarrow x^\mu(\tau) = b^\mu + u^\mu \tau + \dots + Q^4 \delta x_{\text{ALD}}^\mu(\tau)$$

$$\Rightarrow -k^2 \tilde{A}^\mu(k) = \dots + \frac{Q^5}{6\pi} \times \text{diagram}$$

Radiation in QCD



$$\langle F_{\mu\nu}^A(x) \rangle = \sum_{\eta} \int d\Phi(k) \left(-ik_{[\mu} \varepsilon_{\nu]}^{\eta} \langle \psi | S^{\dagger} a_{\eta}^A(k) S | \psi \rangle e^{-ik \cdot x} + \text{h.c.} \right)$$

“QED-like” RR

$$\begin{aligned} \mathcal{A}_{1L} = & \mathcal{C} \left(\text{diagram 1} \right) A_1 + \mathcal{C} \left(\text{diagram 2} \right) A_2 + \mathcal{C} \left(\text{diagram 3} \right) A_3 + \mathcal{C} \left(\text{diagram 4} \right) A_4 \\ & + \mathcal{C} \left(\text{diagram 5} \right) A_5 + \mathcal{C} \left(\text{diagram 6} \right) A_6 + \mathcal{C} \left(\text{diagram 7} \right) A_7 + \mathcal{C} \left(\text{diagram 8} \right) A_8 \\ & + \dots \end{aligned}$$

Non-Abelian RR?

Maximally non-Abelian sector: A_6



$$A_6 = \begin{array}{c} p_1 \text{ --- } p_1 + q_1 \\ | \\ q_1 \text{ (wavy line)} \\ | \\ p_2 \text{ --- } p_2 + q_2 \end{array} + \dots$$

$$= \frac{1}{q_1^2} \int d^4 \ell \delta(p_2 \cdot \ell) \varepsilon \cdot \sum_i J_i$$

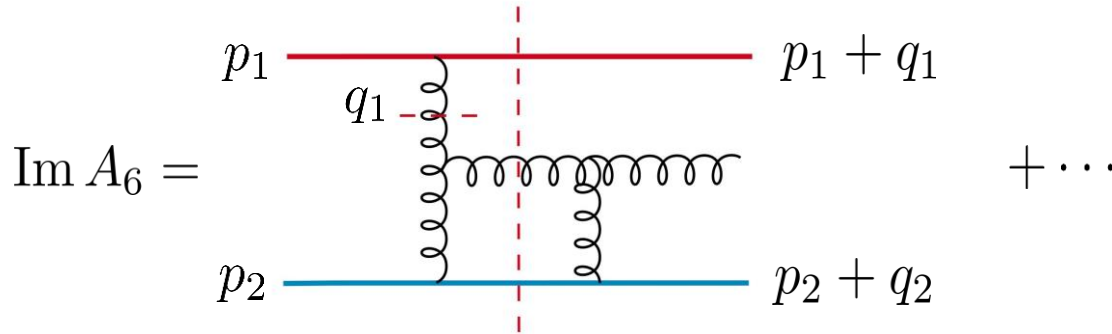
$$J_1^\mu = -\frac{\ell^\mu}{(\ell + q_2)^2 (\ell - q_1)^2} \left(\frac{p_1 \cdot p_2}{4 p_2 \cdot q_1} + \frac{\ell \cdot p_1}{\ell^2} \right) p_2^2,$$

$$J_2^\mu = \frac{q_1^\mu}{\ell^2 (\ell + q_2)^2 (\ell - q_1)^2} (p_2^2 \ell \cdot p_1 + p_1 \cdot p_2 p_2 \cdot q_1),$$

$$J_3^\mu = \frac{p_1^\mu}{\ell^2} \left(\frac{p_2^2}{4(\ell + q_2)^2} - \frac{(p_2 \cdot q_1)^2}{(\ell + q_2)^2 (\ell - q_1)^2} \right),$$

$$J_4^\mu = \frac{p_2^\mu}{\ell^2} \left(-\frac{p_2^2 \ell \cdot p_1}{4 (\ell - q_1)^2 p_2 \cdot q_1} - \frac{p_2^2 p_1 \cdot p_2}{4 (p_2 \cdot q_1)^2} + \frac{(p_1 \cdot p_2 q_1 \cdot q_2 - q_1 \cdot p_2 q_2 \cdot p_1)}{(\ell + q_2)^2 (\ell - q_1)^2} \right)$$

Maximally non-Abelian sector: A_6



$$= \frac{1}{q_1^2} \int d^4 \ell \delta(p_2 \cdot \ell) \delta((q_1 - \ell)^2) \varepsilon \cdot \sum_i J_i$$

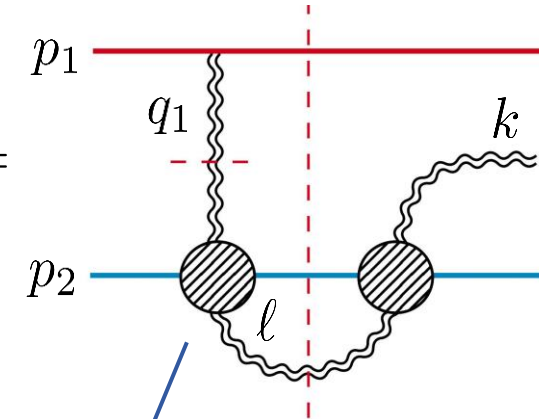
$$J_1^\mu = -\frac{\ell^\mu}{(\ell + q_2)^2 (\ell - q_1)^2} \left(\frac{p_1 \cdot p_2}{4 p_2 \cdot q_1} + \frac{\ell \cdot p_1}{\ell^2} \right) p_2^2,$$

$$J_2^\mu = \frac{q_1^\mu}{\ell^2 (\ell + q_2)^2 (\ell - q_1)^2} (p_2^2 \ell \cdot p_1 + p_1 \cdot p_2 p_2 \cdot q_1),$$

$$J_3^\mu = \frac{p_1^\mu}{\ell^2} \left(\frac{p_2^2}{4(\ell + q_2)^2} - \frac{(p_2 \cdot q_1)^2}{(\ell + q_2)^2 (\ell - q_1)^2} \right),$$

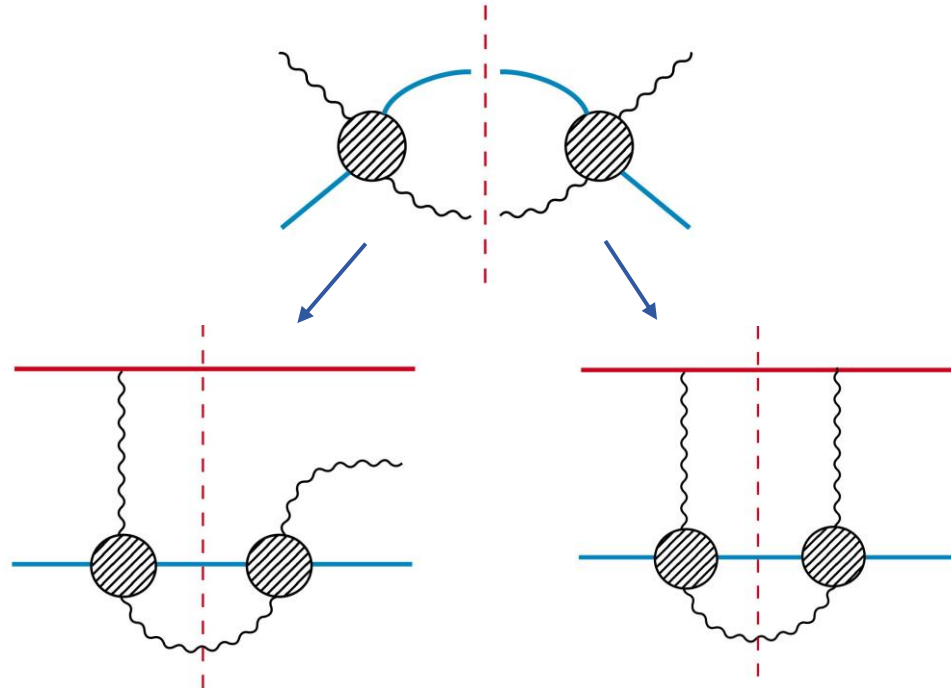
$$J_4^\mu = \frac{p_2^\mu}{\ell^2} \left(-\frac{p_2^2 \ell \cdot p_1}{4(\ell - q_1)^2 p_2 \cdot q_1} - \frac{p_2^2 p_1 \cdot p_2}{4(p_2 \cdot q_1)^2} + \frac{(p_1 \cdot p_2 q_1 \cdot q_2 - q_1 \cdot p_2 q_2 \cdot p_1)}{(\ell + q_2)^2 (\ell - q_1)^2} \right)$$

Gravity imaginary part

$$\text{Im } \mathcal{A}_{\text{GR}} = \int d\Phi(l) \delta(p_2 \cdot (q_1 - l)) \frac{(p_1 \cdot k)^4}{q_1 \cdot l k \cdot l} \times \sum_{\text{helicities}} (\varepsilon \cdot \varepsilon)^2 (\varepsilon \cdot \varepsilon)^2$$


$\mathcal{A}_{\text{GR}}^{\text{Compt.}} \propto \left(\mathcal{A}_{\text{EM}}^{\text{Compt.}} \right)^2$

Cut universality



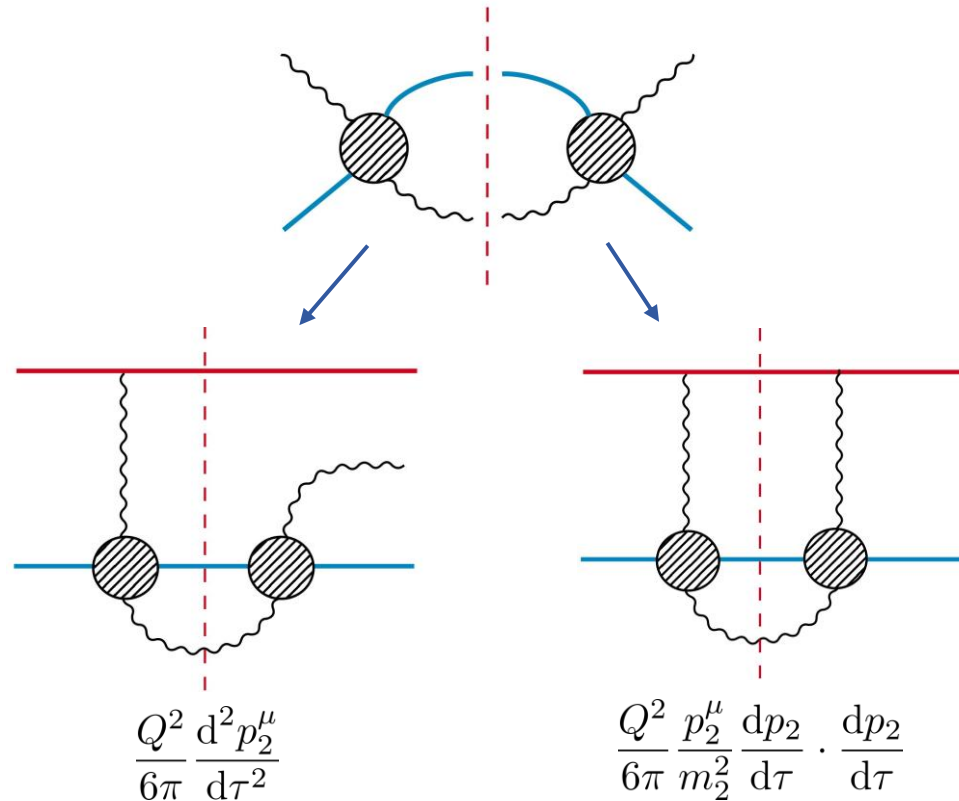
Mino, Sasaki & Tanaka, 1997

Quinn & Wald, 1997

Kosower, Maybee & O'Connell, 2018

Dlapa, Kalin, Liu, Neef & Porto, 2022

Cut universality



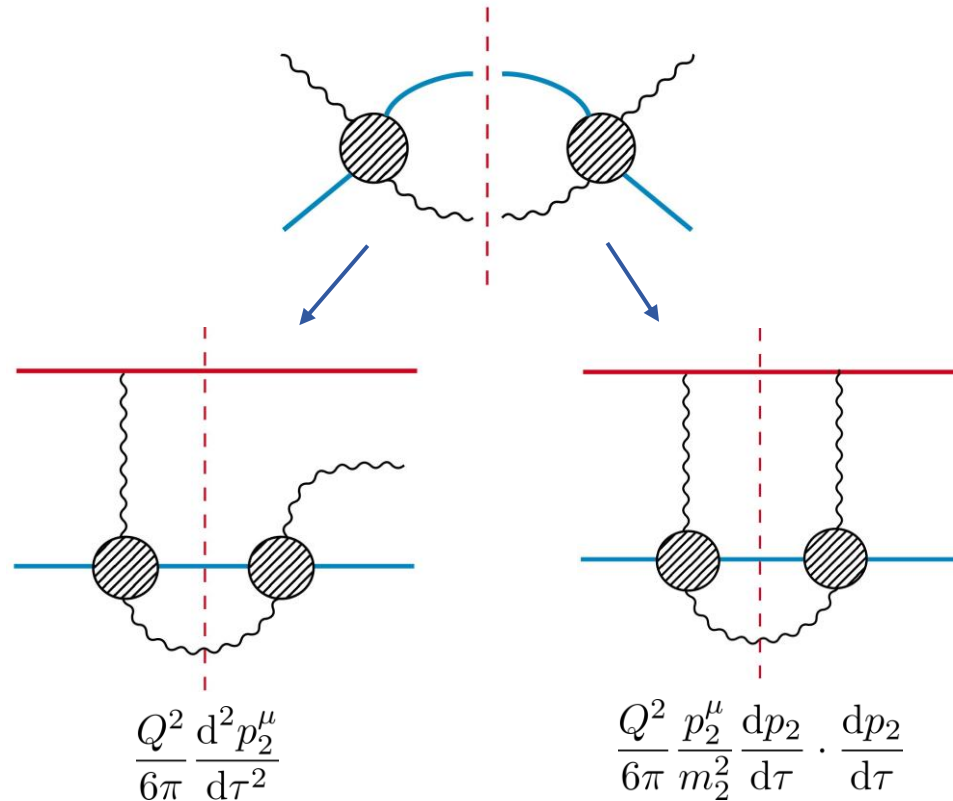
Mino, Sasaki & Tanaka, 1997

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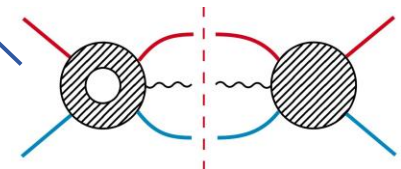
Dlapa, Kalin, Liu, Neef & Porto, 2022

Cut universality

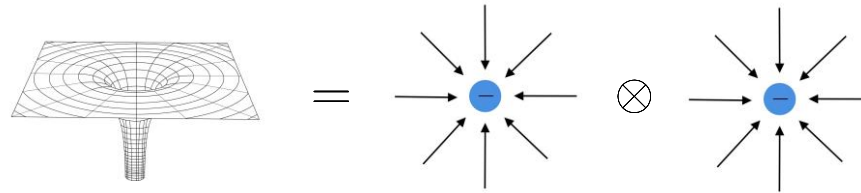


Mino, Sasaki & Tanaka, 1997
 Quinn & Wald, 1997
 Kosower, Maybee & O'Connell, 2018
 Dlapa, Kalin, Liu, Neef & Porto, 2022

$$\Delta p_{3L}^\mu = \int q^\mu \mathcal{A} + \int \ell^\mu \mathcal{A}^* \mathcal{A}$$



Gravity



$$R_{\mu\nu\rho\sigma}(x) = -\sum_{\eta} \int d\Phi(k) \left(k_{[\mu} \varepsilon_{\nu]}^{\eta} k_{[\rho} \varepsilon_{\sigma]}^{\eta} \alpha_{\eta}(k) e^{-ik \cdot x} + \text{h.c.} \right)$$

Compute using the DC of
Vazquez-Holm & Carrasco, 2021

- Bern, Carrasco & Johansson, 2008
- Goldberger & Ridgway, 2016
- Luna, Nicholson O'Connell & White, 2017
- Vazquez-Holm & Carrasco, 2020
- Vazquez-Holm & Carrasco, 2021

Conclusions and future directions



- Amplitudes for classical radiation:

$$\alpha(k) = \text{Re} \left[\text{Diagram 1} + \text{Diagram 2} \right]$$

The equation shows the real part of the sum of two Feynman diagrams. The first diagram is a circle with a central white dot, crossed by two red lines and two blue lines, with a wavy line extending from the right. The second diagram consists of two such circles connected by a wavy line, with a red arc above the left circle and a vertical dashed red line between them.

- We care about gravitational waveforms, it is natural use BCJ numerators.
- The physics is surprisingly rich.
- Things seem to be universal from an amplitude perspective.
- *Next:* Compute gravity waveform.



Thank you!