Evanescent Integrals from local subtractions

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Motivation

- Unitarity methods,beyond one loop, are always d-dimensional and require the computation of “evanescent” (μ) Integrals

- 4d constructions, for example the amplituhedron in $\mathcal{N}=4$, miss these contributions

- μ-Integrals are still rather complicated to compute using traditional methods. All-plus amplitudes are fully characterized by these contributions.

Plan of the talk

- What are μ-Integrals?
- Local subtractions, why μ-Integrals are nice.
- All-plus Amplitudes from divergences:
  Planar five-point example
- Conclusions & Outlook.

μ-Integrals

- At the integrand level decompose the loop momenta
$$\ell^\mu = \ell_4^\mu + \ell_{(D-4)}^\mu$$
- Amplitude integrand can then be written as
$$A^{(\ell)} = A^{(\ell)}_{4D} + \boxed{\mu[A^{(\ell)}]} \longrightarrow \mu_{ij} = \ell_i^{(D-4)} \cdot \ell_j^{(D-4)}$$
- $\mu[A^{(\ell)}]$ naively should be $O(\varepsilon)$ but actually worse due to divergences in integration region!
- At 1-loop they can be understood as dim-shifts.
$$I^d(f(\ell)\mu_{11}^2) = \frac{(d-4)(d-2)}{4} I^{d+4}[f(\ell)]$$

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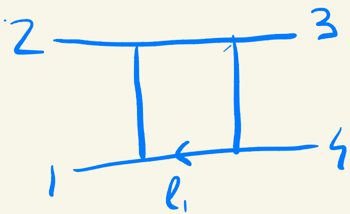
Type of Divergences

- Singular integration regions usually associated to
  - large ℓ $\rightarrow$ UV
  - small ℓ $\rightarrow$ IR (soft)
  - ℓ collinear to ext momenta $\rightarrow$ IR (collinear)

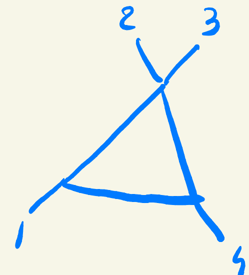
- Our statement is that these regions fully determine u-Integrals up to $O(\varepsilon)$ terms!

IR Soft

- Can be extracted by expanding around vanishing propagators.

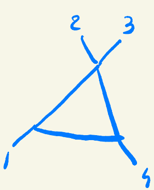


$N(\ell_1, \mu_{11}) \xRightarrow{\text{expand around } \ell_1 = 0}$



$\frac{N(0,\mu_{11})}{(p_1+p_2)^2}$

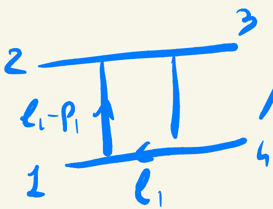
- Starting integral is IR convergent if $N(0,\mu_{11}) \neq N(0,0)$, otherwise we obtain:



$$\frac{N(0,0)}{(p_1+p_2)^2} \simeq \underbrace{\frac{1}{st\epsilon^2}}_{\text{soft pole}} + \underbrace{\frac{1}{st\epsilon}\log(t)}_{?} + \dots$$

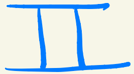
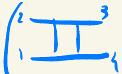
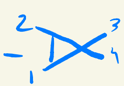
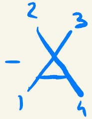
IR Collinear

- Region associated with $l_1 = x p_J$, local subtraction found by [Anastasiou, Sterman 18], requires an integration (transverse loop momenta).



$\mathcal{N}(l_1, \mu_{11})$ Collinear Region exists only for $\mathcal{N}(l, 0)$ $\Longrightarrow$ $\displaystyle \frac{1}{\varepsilon}\int_0^1 dx \, \frac{\mathcal{N}(x p_1)}{s x t (1-x)}$

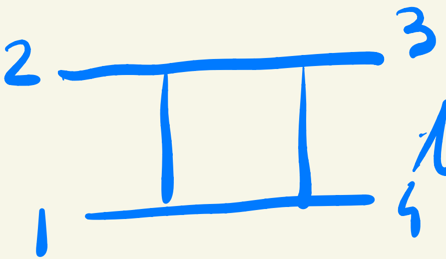
- Integral is divergent, when extracting collinear region need to act on soft subtracted Integral:

$C_1 (1 - S_{12} - S_{14})$  $\Rightarrow C_1 \Big($  $-$  $-$  $\Big) \approx$

$$\frac{1}{\varepsilon}\int_0^1 dx \left[\overbrace{\frac{1}{s x t (1-x)} - \frac{1}{s t (1-x)} - \frac{1}{s x t}}^{+\ \text{presprciption}}\right]$$

UV

We have seen how IR divergences get suppressed when we have μ_{11} in the numerator.

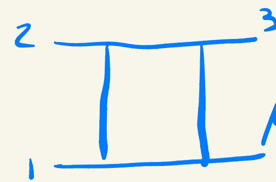


$N(\mu_{11})$

Expand for large ℓ $\Longrightarrow$ $\displaystyle\int d^d\ell_1 \frac{N(\mu_{11})}{(\ell_1^2 - m^2)^4} =$  $N(\mu_{11})$

Power counting to see what happens: $4 - \#\text{exponent of } \mu_{11} - \frac{d}{2}$

μ_{11}^2 first UV divergence!



$\mu_{11}^2 \approx -\dfrac{1}{6\epsilon}$

Why Nice?

- Regions with μ-terms, if they contribute, can only be UV!
- At two-loops, many more combinations but divergences only 1-loop like.

$$\boxed{I^{\text{2-loop}}[\mu_{ij}] = \mathcal{T}_{IR}\, I^{\text{2-loop}}[\mu_{ij}] + (1-\mathcal{T}_{IR})\,\mathcal{T}_{UV}\, I^{\text{2-loop}}[\mu_{ij}] + O(\epsilon)}$$

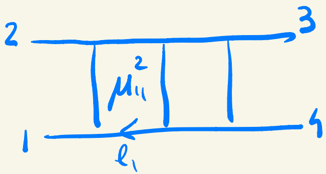
$\mathcal{T}_{IR}$: Soft + collinear

$$\sum_{ij} S_{ij} + \underbrace{\sum C_i\,(1-S_{ij})}_{\sum_k \bar{C}_k}$$

$\mathcal{T}_{UV}$: UV region, large loop momenta expansion.

Examples

• Let us apply our procedure to some 4-point examples



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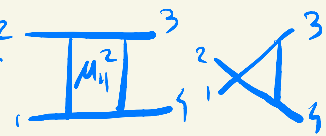
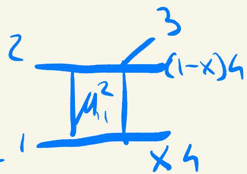
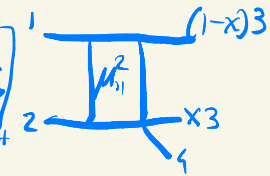


$\frac{1}{s} + \frac{1}{s\varepsilon}\int_0^1 \left[\frac{dx}{1-x}\right]_+$

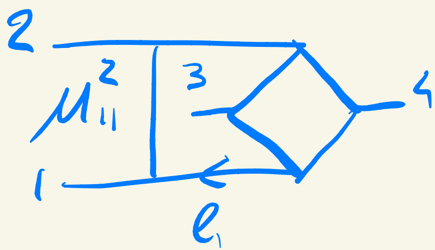


$+ \frac{1}{s\varepsilon}\int_0^1 \left[\frac{dx}{x}\right]_+$

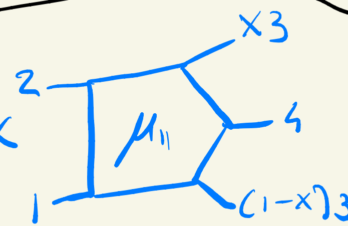


Soft

Collinear



$= \frac{1}{\varepsilon}\int_0^1 dx$



$+ \quad 4 \longleftrightarrow 3$

• In the second case the μ-Integral localizes onto a 1-loop integration

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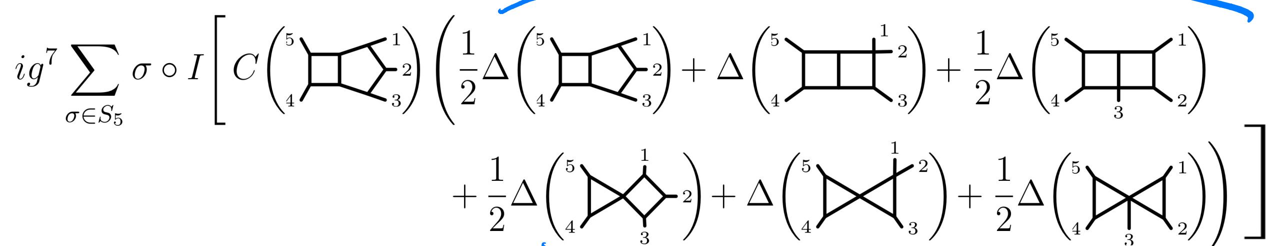
All plus Amplitudes: 5-point Example

- Amplitude is Known, we will focus on planar contributions.
- Weight drop in the amplitude manifest only after combining all pieces together. μ-Integrals can still be weight 4

Non-factorizable

$\mathcal{A}^{(2)}(1^+, 2^+, 3^+, 4^+, 5^+) =$



factorizable

[Badger, Mogull, Ochirov, O'Connell 15]

- Form of the Amplitude Known [Gehrmann, Henn, Lopresti 15]

$$A_+^{(2)} = I^{(1)} A^{(1)} + F^{(2)}_{\text{polylog}} + F^{(2)}_{\text{rational}}$$

- Integrands depend on μ_{ij} in a particular combination

$$F_1 = (D_s - 2)\left(\mu_{11}\mu_{22} + (\mu_{11} + \mu_{22})^2 + 2(\mu_{11} + \mu_{22})\mu_{12}\right) + 16(\mu_{12}^2 - \mu_{11}\mu_{22}), \quad \leftarrow \text{Non factorizable}$$

$$\left.\begin{aligned} F_2 &= 4(D_s - 2)(\mu_{11} + \mu_{22})\mu_{12}, \\ F_3 &= (D_s - 2)^2 \mu_{11}\mu_{22}. \end{aligned}\right\} \text{factorizable}$$

- F_1 only μ_{11}^2, μ_{22}^2 will contribute, both IR and UV
- F_2, F_3 only contribute to UV!

- We can uplift our approach for μ-Integrals to the full amplitude

$$\mathcal{T}_{IR} = \sum_{ij} S_{ij} + \sum_{k} \bar{C}_{k}$$

$$A_{+}^{(2)} = \mathcal{T}_{IR} A_{+}^{(2)} + (1-\mathcal{T}_{IR})\, T_{UV}\left[A_{\text{non fact}}^{(2)} + A_{\text{fact}}^{(2)}\right]$$

- Relating them to the known results:

$$-\sum_{ij} S_{ij} A_{+}^{(2)} = I^{(1)} A^{(2)} \qquad - \quad \bar{C}_{k} A_{+}^{(2)} = 0$$

$$-(1-\mathcal{T}_{IR})\, T_{UV} A_{\text{non fact}}^{(2)} = F_{\text{polylog}}^{(2)}$$

$$-(1-\mathcal{T}_{IR})\, T_{UV} A_{\text{fact}}^{(2)} = F_{\text{rational}}^{(2)}$$

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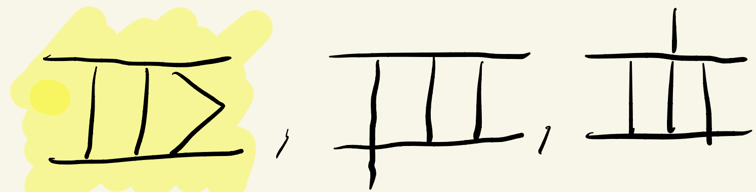
- Relating them to the known results:

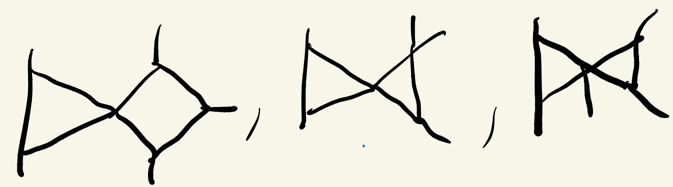
- $\sum_{ij} S_{ij} A_+^{(2)} = I^{(1)} A^{(2)}$

- $\bar{C}_K A_+^{(2)} = 0$

- $(1-\mathcal{T}_{IR})\,\mathcal{T}_{UV} A^{(2)}_{\text{non fact}} = F^{(2)}_{\text{polylog}}$



- $(1-\mathcal{T}_{IR})\,\mathcal{T}_{UV} A^{(2)}_{\text{fact}} = F^{(2)}_{\text{rational}}$



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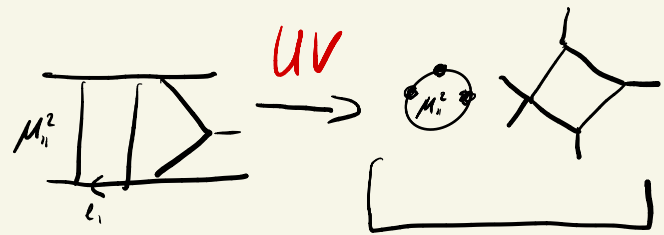

$$-(1-\mathcal{T}_{IR})\,\mathcal{T}_{UV} A^{(2)}_{\text{fact}} = F^{(2)}_{\text{rational}}$$

Once IR subtracted finite 1-mass box!

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Conclusions & Outlook

- Implemented [Anastasiou, Sterman 18] for μ-Integrals.

- Explicitly shown cancellation of collinear contributions and understood structure of finite reminder.

- Apply this method to higher point Amplitudes and beyond All-plus.

- Can we construct μ-parts of the amplitude with the same nice factorization property?

Thank you!