Intersection theory: Amplitudes and Double copies

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Pouria Mazloumi

QCD meets Gravity 2022
Zurich
Dec 15th

This talk is based on

Einstein Yang-Mills Amplitudes from Intersections of Twisted Forms

Pouria Mazloumi and Stephan Stieberger

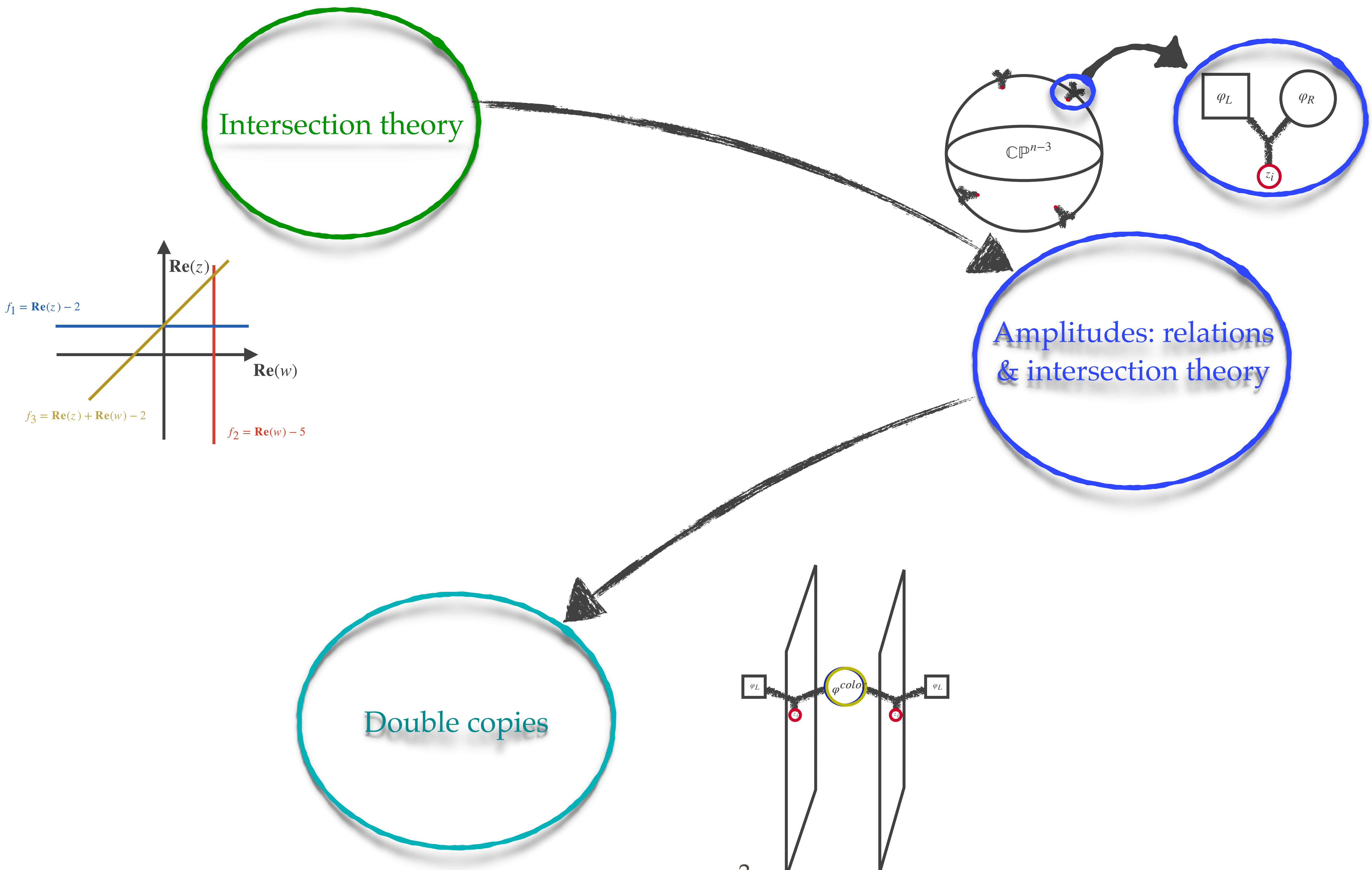
arXiv:2201.0083

Intersections of Twisted Forms: New theories and Double copies

Pouria Mazloumi and Stephan Stieberger

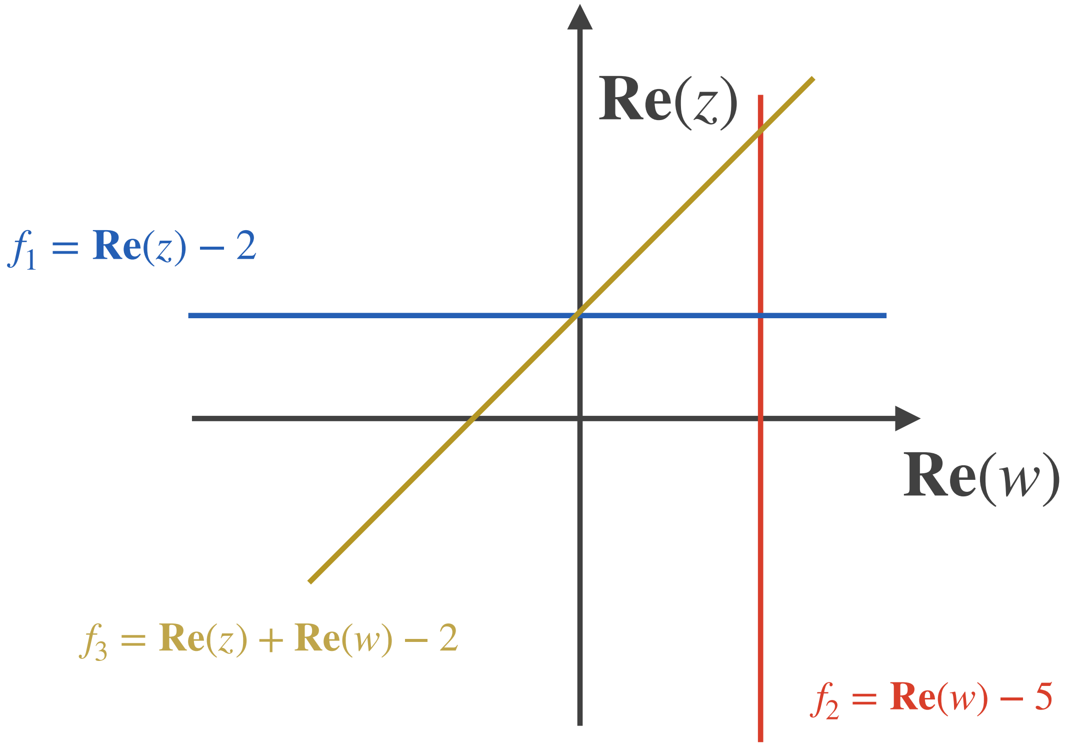
arXiv:2212.xxxx

Overview



Intersection theory
Amplitudes: relations & intersection theory
Double copies
Re(z)
Re(w)
f1 = Re(z) − 2
f2 = Re(w) − 5
f3 = Re(z) + Re(w) − 2
CP^{n−3}
φL
φR
zi
φ^{colo}

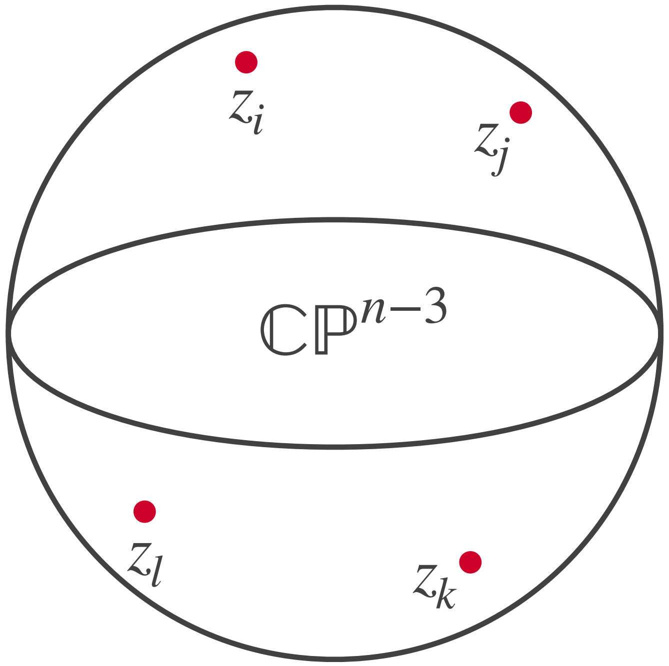
Intersection theory



A tale of a Riemann Surface

- We start by defining the "configuration space" of genus zero punctured Riemann sphere with n-punctures z_i :

$$\mathcal{M}_{0,n} = \{(z_1, \ldots, z_i, \ldots, z_n) \in (\mathbf{CP})^{n-3}\ ,\ i \neq j, k, l \mid \underset{m \neq n}{\forall}\ z_m \neq z_n\}\ .$$

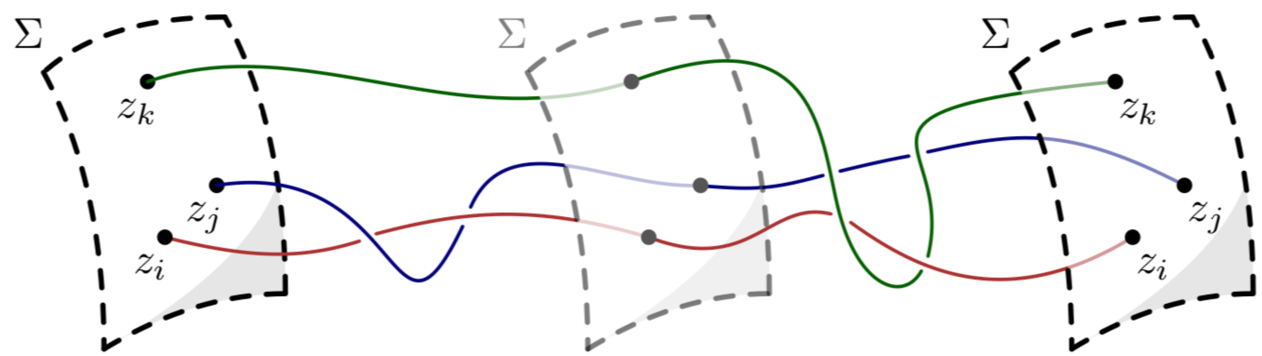


Taking the punctured Riemann Sphere one can define the cohomology of $(n-3)$-forms φ with the equivalent classes with the Gauss-Manin connection $\nabla_{\pm\omega}$:

$\mathscr{L}_\omega : \pi_1(\mathscr{M}_{0,n}) \to C^\times$

$$\nabla_{\pm\omega} := d \pm \omega \wedge$$

$$d(\eta \otimes \exp \int_\gamma \omega)$$
$$= (d\eta + \omega \wedge \eta) \otimes \exp \int_\gamma \omega$$



So the twisted cohomology group is then given by:

$$H^{n-3}_{\pm\omega}(\mathscr{M}_{0,n}, \nabla_{\pm\omega}) = \frac{\{\varphi \in \Omega^{n-3}(\mathscr{M}_{0,n}) \mid \nabla_{\pm\omega}\varphi = 0\}}{\nabla_{\pm\omega}\Omega^{n-4}(\mathscr{M}_{0,n})} .$$

Mizera '17

$$[\varphi] \simeq \varphi + \nabla_{\pm\omega}\xi ,$$

$$\xi \in \Omega^{n-4}(\mathscr{M}_{0,n})$$

Example

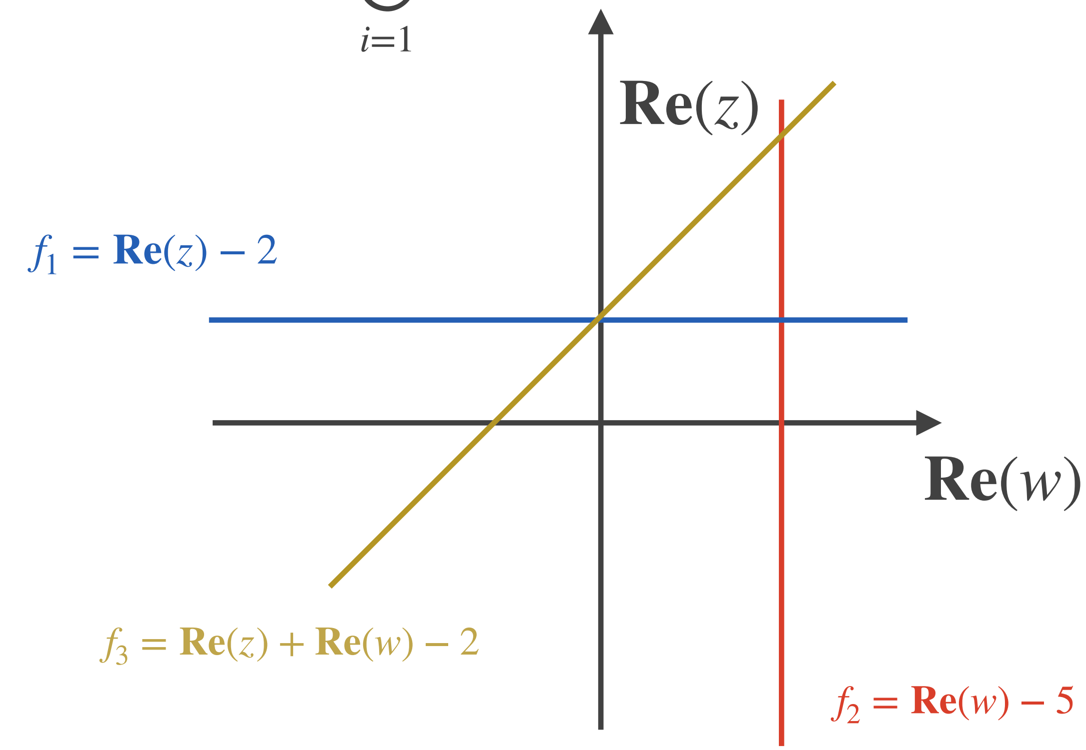
Logarithmic singularities

(z, w)

$X = \mathbf{CP}^1 \backslash \bigcup_{i=1}^{3} \{f_i = 0\}$

$$\omega = \sum_i^3 \alpha_i d \log f_i = \alpha_1 d \log f_1 + \alpha_2 d \log f_2 + \alpha_3 d \log f_3$$

$$\varphi_K = d \log \frac{f_i}{f_j}$$



The intersection number on the twisted cohomology group is the invariant paring between two forms:

$$\langle \varphi_L, \varphi_R \rangle_\omega := \Lambda \int_X \iota_\omega(\varphi_L) \wedge \varphi_R$$

All intersection vertices of the hyperplane associated to φ_k

$\varphi_L(i = 1, j = 2)$

$\varphi_R(i = 3, j = 2)$

$$\langle \varphi_L, \varphi_R \rangle_\omega = \frac{1}{\alpha_2}$$

Matsumoto '98

For the amplitudes one can define:

$$\omega = \alpha' \sum_{1 \le i,j \le n} 2p_i p_j \; d\ln(z_i - z_j) \ ,$$

↓

The α_i that we had before $= \dfrac{1}{\alpha_2} \to \dfrac{1}{s_2}$

Scattering equation

QFT

$$\omega = \sum_{\substack{j=1 \\ j \neq i}} \frac{2p_i \cdot p_j}{z_i - z_j}$$

Further the Koba-Nielsen factor in string amplitudes is given by:

String theory

$$KN \equiv \prod_{1 \le i,j \le n} |z_i - z_j|^{2\alpha' p_i \cdot p_j} = e^{\int_\gamma \omega}$$

Twisted Cycles ← Homology Cohomology → Twisted Co-Cycles

$$H_{n-3}(\mathcal{M}_{0,n}, \mathcal{L}_\omega) \cong H^{n-3}(\mathcal{M}_{0,n}, \mathcal{L}_\omega)$$

Punctured circles with ordering "a"

$$C_a \otimes KN := C_a \otimes e^{i\pi\phi(\alpha)} \prod_{1\le i<j\le n} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

Relation to string amplitudes by Poincaré duality:

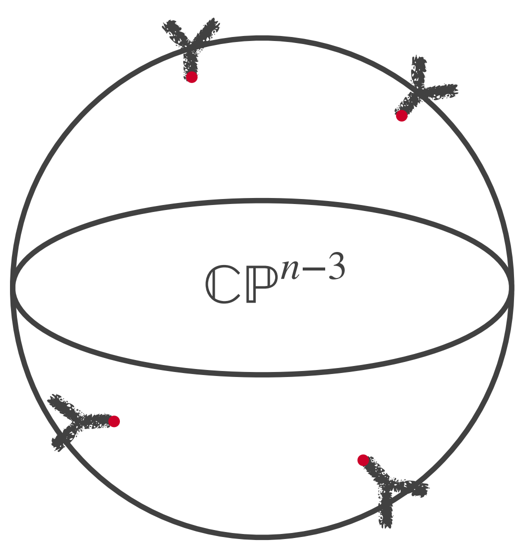
$$\langle C_a \otimes KN \,|\, \mathcal{O}(z)\rangle = \int_{C_a} KN\ \mathcal{O}(z) \ \sim \mathcal{A}_n^{string}(a)$$

The $\alpha' \to 0$ limit of the relation we contact QFT and intersection theory:

$$\lim_{\alpha'\to 0} \int_{C_a} KN\ \mathcal{O}(z)^{open} = \langle PT(a), \mathcal{O}(z)^{open}\rangle_\omega \ = \mathcal{A}^{YM}(a)\,.$$

$\alpha' \to 0$

Intersection of Park-Taylor and gauge forms



The setup of the CHY formalism is produced through the saddle point approximation

$$\langle \varphi_L, \varphi_R \rangle_\omega := \left(-\frac{\alpha'}{2\pi i} \right)^{n-3} \int_X \iota_\omega(\varphi_L) \wedge \varphi_R$$

$$\lim_{\alpha' \to \infty} \langle \varphi_+, \varphi_- \rangle_\omega = \alpha'^{\frac{n-3}{2}} \int_{\mathcal{M}_{0,n}} d\mu_n \prod_{k=2}^{n-2} \delta(f_k) \lim_{\alpha' \to \infty} \hat{\varphi}_+ \hat{\varphi}_- \ ,$$

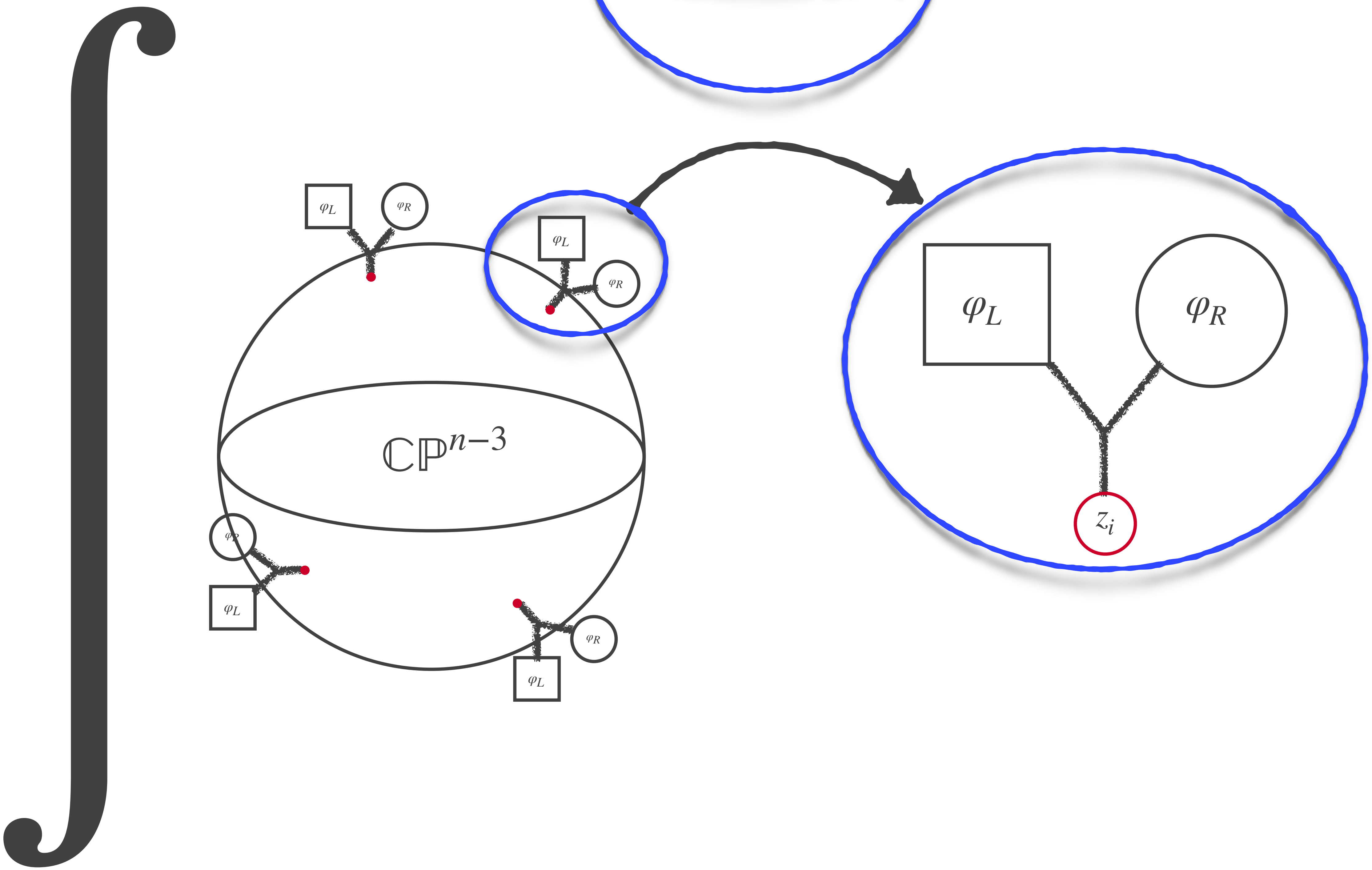
$$d\mu_n = z_{jk} z_{jl} z_{kl} \prod_{\substack{i=1 \\ i \notin \{j,k,l\}}}^{n} dz_i \ ,$$

Localisation on Scattering equations

$$f_k := \sum_{j \neq k} \frac{p_k p_j}{z_j - z_k} = 0 \ , \quad 1 \leq k \leq n \ .$$

Functional part of the forms $\varphi = \hat{\varphi}(z) dz$

Amplitudes: relations & intersection theory



Further, the well known amplitude relations have trivial definition in the cohomology:

Equivalence classes

$$[\varphi] \sim [\varphi + \nabla \xi] \longrightarrow \nabla \xi = \nabla \varphi^{color} = (d + \omega \wedge)\varphi^{color}_{n-1} = \omega \wedge \varphi^{color}_{n-1} \sim 0$$

$$\mathscr{A}_{YM}(1,2,\ldots,n)\ Tr(T^{c_1}T^{c_2}\ldots T^{c_n}) = \langle \varphi^{color}_n, \varphi^{gauge}_{+,n} \rangle_\omega$$

$$\langle \varphi^I, \nabla \xi \rangle \sim 0$$

BCJ-KK relations:

$$\sum_i A(1,2,3,\ldots,i,p,i+1,\ldots,N) = 0$$

$$\sum_i x_{ip}\ A(1,2,3,\ldots,i,p,i+1,\ldots,N) = 0$$

$$x_{ip} = p \cdot \sum_{j=1}^{i} p_j$$

Back to the punctured sphere, two important examples are the twisted form corresponding to Park Taylor factor:

$$\varphi_n^{color}(\sigma) = d\mu_n \frac{Tr(T^{\sigma(1)}T^{\sigma(2)}T^{\sigma(3)}\dots T^{\sigma(n)})}{(z_{\sigma(1)}-z_{\sigma(2)})\dots(z_{\sigma(n-1)}-z_{\sigma(n)})}, \quad \sigma \in S_n \ .$$

$$d\mu_n = \bigwedge_{i=1}^{n} dz_i$$

$$\langle \varphi_4^{color}, \varphi_4^{color}\rangle_\omega = Tr(T^{c_1}T^{c_2}T^{c_3}T^{c_4})Tr(T^{c_1}T^{c_2}T^{c_3}T^{c_4}) \left\{ \frac{1}{(p_1+p_2)^2} + \frac{1}{(p_1+p_3)^2} \right\} \ .$$

and the so called gauge form which relates to Yang-Mills respectively

$$\varphi_{\pm,n}^{gauge} = d\mu_n \int \prod_{i=1}^{n} d\theta_i d\bar{\theta}_i \frac{\theta_k\theta_l}{z_k - z_l} \exp\left\{ -\sum_{i\neq j} \frac{\theta_i\theta_j p_i\cdot p_j + \bar{\theta}_i\bar{\theta}_j\varepsilon_i\cdot\varepsilon_j + 2(\theta_i-\theta_j)\bar{\theta}_i\varepsilon_i\cdot p_j}{z_i - z_j \mp \alpha'^{-1}\theta_i\theta_j} \right\} ,$$

$$\langle \varphi_n^{color}, \varphi_{+,n}^{gauge}\rangle_\omega = \mathscr{A}_{YM}(1,2,\dots,n)Tr(T^{c_1}T^{c_2}\dots T^{c_n}) \ .$$

Color structure

Spin-1 kinematics

Mizera '17

In general one can take different twisted forms and construct different field theory amplitudes:

Theory	φ_-	φ_+
Bi-adjoint scalar	φ_-^{color}	φ_+^{color}
Yang-Mills	φ_-^{color}	φ_+^{gauge}
Einstein gravity	φ_-^{gauge}	φ_+^{gauge}
YM+(DF)^2	φ_-^{color}	$\varphi_+^{bosonic}$
Weyl-Einstein	φ_-^{gauge}	$\varphi_+^{bosonic}$
NLSM	φ_-^{color}	φ_+^{scalar}

$$\varphi^{color}_{\pm,n} = d\mu_n \frac{Tr(T^{c_1}T^{c_2}\ldots T^{c_n})}{(z_1-z_2)(z_2-z_3)\ldots(z_n-z_1)} \equiv Tr(T^{c_1}T^{c_2}\ldots T^{c_n})\; PT(1,2,\ldots,n)$$

$$\varphi^{gauge}_{\pm,n} = d\mu_n \int \prod_{i=1}^{n} d\theta_i d\bar{\theta}_i \frac{\theta_k\theta_l}{z_k - z_l} \exp\left\{ -\alpha'^2 \sum_{i\neq j} \frac{\theta_i\bar{\theta}_j p_i\cdot p_j + \bar{\theta}_i\bar{\theta}_j \varepsilon_i\cdot\varepsilon_j + 2(\theta_i-\theta_j)\bar{\theta}_i\varepsilon_i\cdot p_j}{z_i - z_j \mp \alpha'^{-1}\theta_i\theta_j} \right\},$$

$$\varphi^{scalar}_{\pm,n} = d\mu_n (\mathrm{Pf}'A)_n = -d\mu_n \frac{\det A_{[kl]}}{(z_k-z_l)^2}$$

$$\varphi^{bosonic}_{\pm,n} = d\mu_n \int \prod_{i=1}^{n} d\theta_i d\bar{\theta}_i \exp\left\{ -\alpha' \sum_{i\neq j}^{n} \left(\pm\alpha' \frac{2\theta_i\bar{\theta}_j p_i\cdot\varepsilon_j}{z_i-z_j} + \frac{\theta_i\bar{\theta}_i\theta_j\bar{\theta}_j \varepsilon_i\cdot\varepsilon_j}{(z_i-z_j)^2} \right) \right\}$$

Mizera '19

Using String amplitude as a source of twisted forms for a generic amplitude we have:

$$\int_{\mathscr{M}} \prod_{k=1}^{r} dz_k d\bar{z}_k \left\langle \prod_{j=1}^{r} V_o(\varepsilon_j, k_j, z_j, \bar{z}_j) \right\rangle_{S^2} \approx \int_{C_a} KN\ \mathcal{O}(z) = \langle C_a \otimes KN | \mathcal{O}(z) \rangle$$

This relation can be used construct $\mathcal{O}(z)$

$\varphi^{bosonic}_{\pm,n}$ $\varphi^{gague}_{\pm,n}$

What about the mixed open-closed amplitude?

$$EYM := \mathscr{A}(1,2,\ldots,n;\ 1,,2,\ldots,.r)$$

$$\left\langle \prod_{i=1}^{n} V_o(\varepsilon_i, k_i, z_i) \prod_{j=1}^{r} V_o(\varepsilon_j, k_j, z_j, \bar{z}_j) \right\rangle_{D^2} \longrightarrow \left\langle \prod_{i=1}^{n} V_o(\varepsilon_i, k_i, z_i, \bar{z}_i) \prod_{j=1}^{r} V_o(\varepsilon_j, k_j, z_j, \bar{z}_j) \right\rangle_{S^2}.$$

We can embed this integral onto the sphere

$$V_o(\varepsilon_i, k_i, x_i) \longmapsto V_o(\varepsilon_i, k_i, z_i)\ e^{ik_i \widetilde{X}(\bar{z}_i)}\ J^{c_i}(\bar{z}_i), \qquad i = 1, \ldots, n\ ,$$

$$V_c(\varepsilon_s, q_s, z_{n+s}, \bar{z}_{n+s}) \longmapsto V_c = V_o(\varepsilon_s, q_s, z_{n+s})\ V_o(\widetilde{\varepsilon}_s, \widetilde{q}_s, \bar{z}_{n+s})\ , \qquad s = 1, \ldots, r\ .$$

Putting these two inside the intersection formula we get:

$$\mathscr{A}(1,2,\ldots,n;\,1,,2,\ldots,.r) = \lim_{\alpha'\to\infty} \langle \widetilde{\varphi}^{EYM}_{+,n;r}, \varphi^{gauge}_{-,n+r}\rangle_\omega = \int_{\mathscr{M}_{0,n+1}} d\mu_{n+r} \prod_{a=1}^{n+1}{}' \delta(f_a) \lim_{\alpha'\to\infty} \hat{\varphi}^{gauge}_{n+r} \hat{\widetilde{\varphi}}^{EYM}_{n;r}$$

$$= \int_{\mathscr{M}_{0,n+r}} d\mu_{n+r} \prod_{a=1}^{n+r}{}' \delta(f_a) \frac{\mathrm{Pf}\Psi_{S_r}\Big|_{\bar{z}_l\to z_l} \mathrm{Pf}'\Psi_{n+r}}{(z_1-z_2)(z_2-z_3)\ldots(z_n-z_1)} \,.$$

Then we have the following twisted forms:

$$\tilde{\varphi}^{EYM}_{\pm,n;r} = d\mu_{n+r} \int \prod_{i=1}^{n+r} \frac{\theta_1\theta_2}{z_1-z_2} d\theta_i d\bar{\theta}_i \exp\left\{ \frac{1}{2}\alpha'^2 \sum_{i,j=1}^{n+r} (\theta_j\ \bar{\theta}_j)\Psi_{S_r}\begin{pmatrix}\theta_i\\ \bar{\theta}_i\end{pmatrix}\right\} \times \exp\left\{ \pm\frac{1}{2}\alpha' \sum_{\substack{i,j=1\\ i\neq j}}^{n+1} \frac{\theta_i\theta_j\bar{\theta}_i\bar{\theta}_j(\xi_i\cdot\xi_j)}{(z_i-z_j)^2}\right\} .$$

$$\tilde{\varphi}^{EM}_{+}(r;n) = \, = d\mu_{n+r} \int \prod_{i=1}^{n+r} \frac{\theta_1\theta_2}{z_1-z_2} d\theta_i d\bar{\theta}_i \exp\left\{ \frac{1}{2}\alpha'^2 \sum_{i,j=1}^{n+r} (\theta_j\ \bar{\theta}_j)\Psi_{S:n}\begin{pmatrix}\theta_i\\ \bar{\theta}_i\end{pmatrix}\right\}$$

PM, Stieberger '22

PM, Stieberger to apear

One can use this embedding further on the bosonic string amplitude :

$$\varphi^{bosonic}_{\pm,n} = d\mu_n \int \prod_{i=1}^{n} d\theta_i d\bar{\theta}_i \exp\left\{ -\alpha' \sum_{i\neq j}^{n} \left(\pm\, \alpha' \frac{2\theta_i\bar{\theta}_j p_i \cdot \varepsilon_j}{z_i - z_j} + \frac{\theta_i\bar{\theta}_i\theta_j\bar{\theta}_j \varepsilon_i \cdot \varepsilon_j}{(z_i - z_j)^2} \right) \right\}$$

$$V_o(\varepsilon_i, k_i, x_i) \longmapsto V_o(\varepsilon_i, k_i, z_i)\; e^{ik_i \widetilde{X}(\bar{z}_i)}\; J^{c_i}(\bar{z}_i), \qquad i = 1,\ldots,n\ ,$$

$$V_c(\varepsilon_s, q_s, z_{n+s}, \bar{z}_{n+s}) \longmapsto V_c = V_o(\varepsilon_s, q_s, z_{n+s})\; V_o(\widetilde{\varepsilon}_s, \widetilde{q}_s, \bar{z}_{n+s})\ , \qquad s = 1,\ldots,r\ .$$

$$\widetilde{\varphi}^{Bosonic}_{\pm,n;r} = d\mu_{n+r}\, \mathscr{C}_n\{z_i\} \int \prod_{i=1}^{r} d\theta_i d\bar{\theta}_i \exp\left\{ \alpha'^2 \left(\sum_{j=1}^{n} \frac{\varepsilon_i \cdot p_j}{z_{ij}} + \sum_{\substack{j=1 \\ j\neq i}}^{r} \frac{\varepsilon_i \cdot p_j}{z_{ij}} + \frac{1}{\alpha'} \sum_{\substack{i\neq j \\ j,i\in r}} \frac{\theta_i\bar{\theta}_i\theta_j\bar{\theta}_j \varepsilon_i \cdot \varepsilon_j}{(z_i - z_j)^2} \right) \right\}.$$

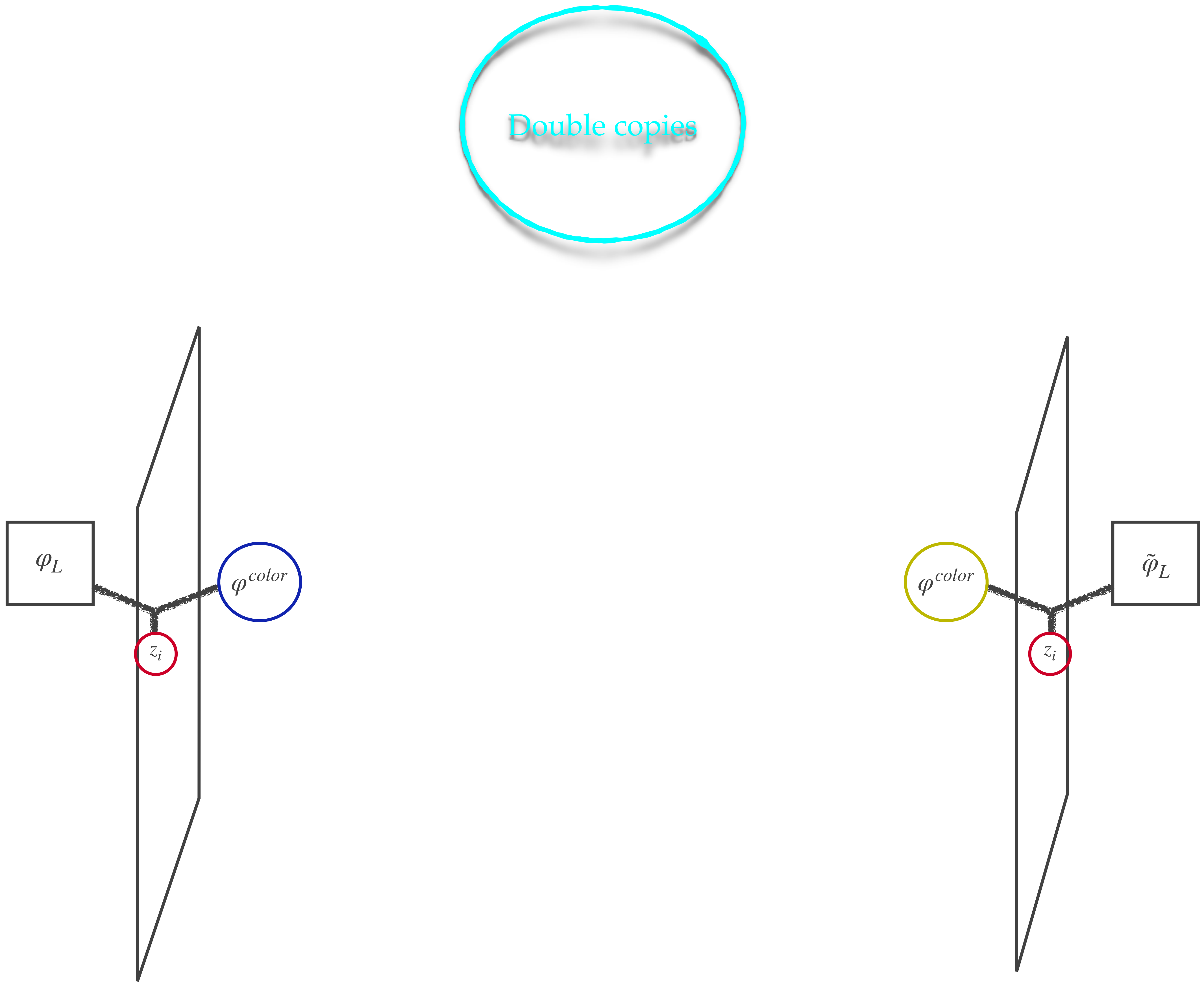
In general one can take different twisted forms and construct different field theory amplitudes:

$\tilde{\varphi}_+^{Bosonic}$ $\tilde{\varphi}_+^{EM}$ $\tilde{\varphi}_+^{EYM}$

Theory	φ_-	φ_+
Bi-adjoint scalar	φ_-^{color}	φ_+^{color}
Yang-Mills	φ_-^{color}	φ_+^{gauge}
Einstein gravity	φ_-^{gauge}	φ_+^{gauge}
YM+(DF)²	φ_-^{color}	$\varphi_+^{bosonic}$
Weyl-Einstein	φ_-^{gauge}	$\varphi_+^{bosonic}$
NLSM	φ_-^{color}	φ_+^{scalar}

Theory	φ_-	φ_+
Einstein Yang-Mills	φ_-^{gauge}	
gen.YMS	φ_-^{color}	
ext.DBI	φ_-^{scalar}	
Einstein Maxwell	φ_-^{gauge}	
DBI	φ_-^{scalar}	
YM-scalar	φ_-^{color}	
Weyl Yang-Mills	φ_-^{gauge}	

PM, Stieberger to apear



Double copies
φ_L
φ^{color}
z_i
φ^{color}
$\tilde{\varphi}_L$
z_i

The amplitude BCJ-KK relations can be seen as equivalence classes on the twisted Cohomology:

$$\varphi^{color} \sim BCJ - KK$$

So what can one learn on the Theory level?

Lets look at two theories satisfying these relation through color form

$$T_1 = \langle \varphi^1_+, \varphi^{color} \rangle \qquad T_2 = \langle \varphi^2_+, \varphi^{color} \rangle$$

$$T_1 \otimes T_2 = \langle \varphi^1_+, \varphi^{color}_a \rangle \; \mathbf{1} \; \langle \varphi^{color}_b, \varphi^2_- \rangle \ .$$

$$T_1 \otimes T_2 = \langle \varphi^1_{a,+}, \varphi^2_{b,-} \rangle \ .$$

PM, Stieberger to apear

There is a close connection between BCJ-kk relation and Double copy (see Double copy section talks)

$$\mathbf{1} = \sum | \varphi^{color^\vee}_{+,a} \rangle \langle \varphi^{color^\vee}_{-,b} |_\omega$$

$$\langle \varphi^{color}_{+,a}, \varphi^{color^\vee}_{-,b} \rangle_\omega = \delta_{ab} \, , \varphi_{+,a} = PT(a) \ ,$$

On the amplitude level we can see we can write:

$$T_1 \otimes T_2 = \langle \varphi^1_{a,+}, \varphi^2_{b,-} \rangle$$

$$\varphi^1_{a,+} = \sum_{a=1}^{(m-3)!} \langle \varphi^{color}(a), \varphi^1_{+} \rangle_\omega \; \varphi^{color^\vee}(a)$$

$$\varphi^2_{b,-} = \sum_{b=1}^{(m-3)!} \langle \varphi^2_{-}, \varphi^{color}(b) \rangle_\omega \; \varphi^{color^\vee}(b) \;,$$

So the "Double copies" amplitude will be:

$$\langle \varphi^1_+, \varphi^2_- \rangle = \sum_a \langle \varphi^{color}, \varphi^1_+ \rangle_\omega \langle \varphi^{color^\vee}(a), \varphi^{color^\vee}(b) \rangle \langle \varphi^2_-, \varphi^{color}(a) \rangle_\omega$$

$$= \sum_{a,b} \langle \varphi^2_-, \varphi^{color}(a) \rangle_\omega \; S[a|b] \; \langle \varphi^{color}(b), \varphi^1_+ \rangle_\omega$$

$$= \sum_{a,b} \mathscr{A}(a) \, S[a|b] \, \mathscr{A}(b)$$

This is the KLT matrix coming from the bi-adjoint scalar

Theory	φ_-	φ_+
Yang-Mills	φ_-^{color}	φ_+^{gauge}
Bi-adjoint scalar	φ_-^{color}	φ_+^{color}
YM+(DF)^2	φ_-^{color}	$\varphi_+^{bosonic}$
NLSM	φ_-^{color}	φ_+^{scalar}
gen.YMS	φ_-^{color}	$\tilde{\varphi}_+^{EYM}$
YMS	φ_-^{color}	$\tilde{\varphi}_+^{EM}$

Lets start with the simplest example:

$$T_1 \otimes T_2 = YM \otimes \text{bi-adjoint} = \langle \varphi_+^{gague}, \varphi_a^{color} \rangle \; \mathbf{1} \; \langle \varphi_b^{color}, \varphi_-^{color} \rangle = \langle \varphi_{a,+}^{gauge}, \varphi_{b,-}^{gcolor} \rangle = YM\,.$$

$$T_1 \otimes T_2 = YM \otimes YM = \langle \varphi_+^{gague}, \varphi_a^{color} \rangle \; \mathbf{1} \; \langle \varphi_b^{color}, \varphi_-^{gague} \rangle = \langle \varphi_{a,+}^{gauge}, \varphi_{b,-}^{gauge} \rangle = GR\,.$$

In fact one more advantage of the Intersection formalism is the naturally of the double copy through φ^{color} :

Theory \ Theory	**YM** φ_+^{gauge}	DF^2 $\varphi_+^{bosonic}$	**NLSM** φ_+^{scalar}
YM φ_+^{gauge}	**GR**	**Conformal Gravity**	**Born-Infeld**
DF^2 $\varphi_+^{bosonic}$	**Conformal Gravity**	$Weyl^3$	DF^2-photon
NLSM φ_+^{scalar}	**Born-Infeld**	DF^2-photon	**Special Galiliean**

Mizera '19
Johansson, Schlotterer, et all '18

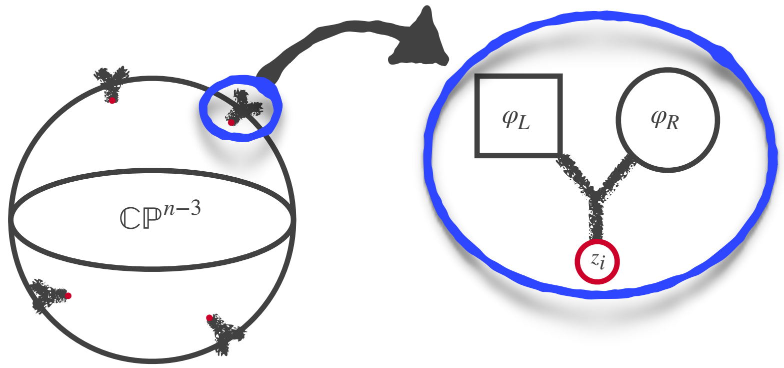
In fact one more advantage of the Intersection formalism is the naturally of the double copy through φ^{color} :

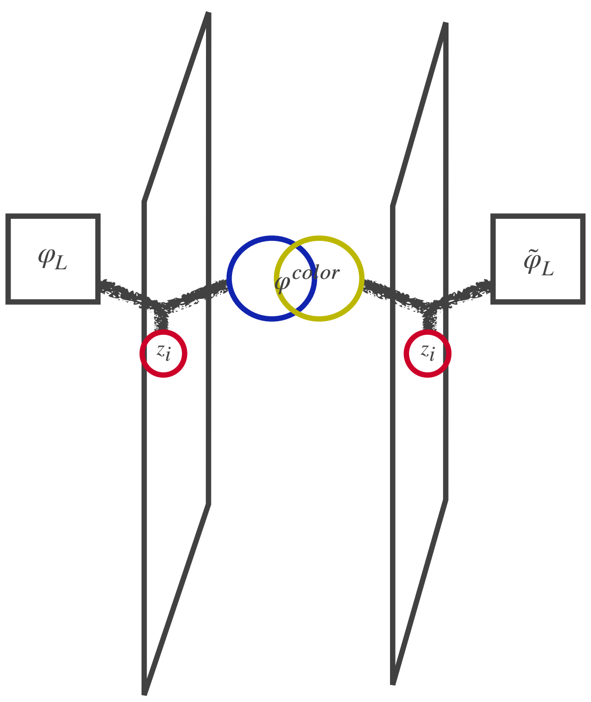
Theory \ Theory	$\tilde{\varphi}_+^{EM}$ YMS	$\tilde{\varphi}_+^{EYM}$ gen.YM	φ_+^{scalar} NLSM	$\varphi_+^{bosonic}$ YM+(DF)²
$\tilde{\varphi}_+^{EM}$ YMS	(EM)²	EM⊗EYM	DBI	??
$\tilde{\varphi}_+^{EYM}$ gen.YM	EM⊗EYM	(EYM)²	??	??
φ_+^{scalar} NLSM	DBI	??	**Special Galilean**	Born-Infeld
YM+(DF)² $\varphi_+^{bosonic}$	??	??	BI-(DF)² photon	??

PM, Stieberger to apear

Summary

- We looked at the intersection theory of twisted forms and how to construct new ones through embeddings on Riemann surfaces





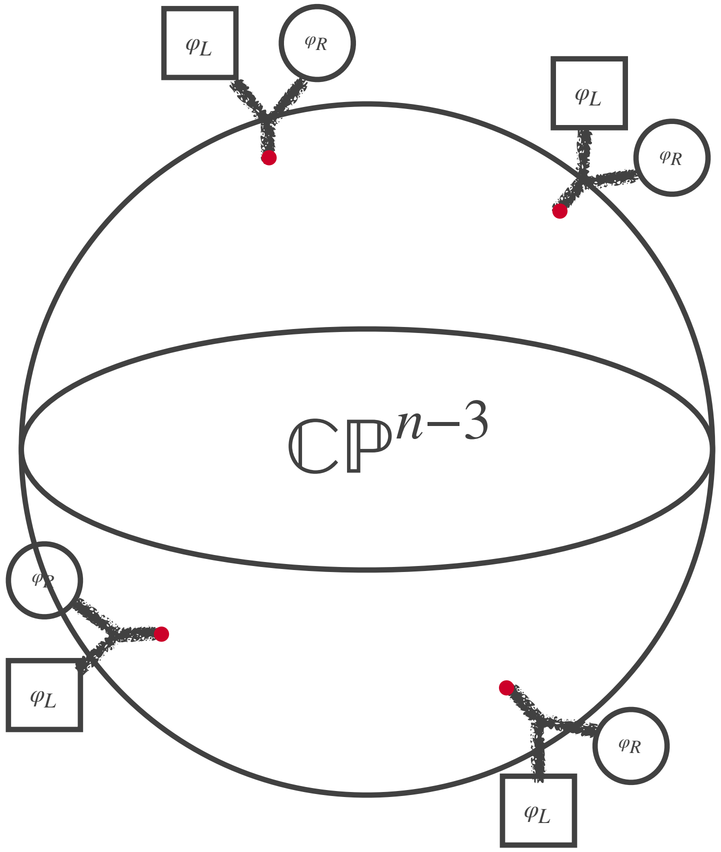
- Using the relation between color twisted form and BCJ relations we proposed new method to construct double copies

- Using the new twisted forms we introduced more double copies

Outlook

- Can one produce more theories through string relations ?
- Loop level calculations on the string amplitude level ?
- Can one understand massive amplitudes and produce massive twisted forms ?
- One can use the relation between twisted forms on Riemann surfaces and Ribbon Graphs to make contact with Matrix models

Thank you for your attention



Supplement

For Pure Yang Mills, Gravity and EYM one has the following integrands

$$\mathscr{A}_{CHY}(m) = \int_{\mathscr{M}_{0,m}} d\mu_m \prod_{a=1}^{m}{}' \delta(f_a)\ \mathscr{I}_m(p,\varepsilon,\sigma)\ .$$

Theory	$\mathscr{I}_m(p,\varepsilon,\sigma)$
YM	$\mathscr{C}(1,2,\ldots,n)\ \mathrm{Pf}'\Psi_m(k_a,q,\varepsilon,\sigma)$,
GR	$\mathrm{Pf}'\Psi_m(k_a,q,\varepsilon,\sigma)\ \mathrm{Pf}'\Psi_m(k_a,q,\varepsilon,\sigma)$,
EYM	$\mathscr{C}(1,2,3,\ldots,n)\ \mathrm{Pf}\Psi_{S_r}\ \mathrm{Pf}'\Psi_S(k_a,q_a,\varepsilon,\sigma)$,

$$\Psi_m = \begin{pmatrix} A & -C^T \\ C & B \end{pmatrix}, \quad A = \begin{cases} 0\ , & i=j \\ \dfrac{p_i p_j}{\sigma_i-\sigma_j}\ , & i\neq j \end{cases}, \quad C = \begin{cases} -\displaystyle\sum_{k\neq i}^{m} \dfrac{\varepsilon_i p_k}{\sigma_i-\sigma_k}\ , & i=j \\ \dfrac{\varepsilon_i p_j}{\sigma_i-\sigma_j}\ , & i\neq j \end{cases}, \quad B = \begin{cases} 0\ , & i=j\ , \\ \dfrac{\varepsilon_i\varepsilon_j}{\sigma_i-\sigma_j}\ , & i\neq j\ . \end{cases}$$

$$\mathscr{C}(1,2,\ldots,n) = \frac{1}{(\sigma_1-\sigma_2)(\sigma_2-\sigma_3)\ldots(\sigma_n-\sigma_1)}$$

Cachazo, He, Ye Yuan '14

Embedding onto Sphere

Motivation: we require factorisation of the amplitude into two Pfaffians

Pfaffians ↔ amplitudes

$$x_i \longmapsto z_i \quad , \quad i = 1, \ldots, n \ .$$

For the vertex operators taking into account equations of motion

We can extend the fields too:

$$X^\mu(x) \longmapsto X^\mu(z) + \widetilde{X}^\mu(\bar{z}) \ \Rightarrow \ \begin{cases} \partial X & \mapsto \partial X \\ k \cdot X & \mapsto k \cdot (X + \widetilde{X}) \end{cases} ,$$

$$\psi^\mu(x) \longmapsto \psi^\mu(z) \ .$$

$$V_o(\varepsilon_i, k_i, x_i) \longmapsto V_o(\varepsilon_i, k_i, z_i) \ e^{ik_i \widetilde{X}(\bar{z}_i)} \ J^{c_i}(\bar{z}_i), \qquad i = 1, \ldots, n \ ,$$

$$V_c(\varepsilon_s, q_s, z_{n+s}, \bar{z}_{n+s}) \longmapsto V_c = V_o(\varepsilon_s, q_s, z_{n+s}) \ V_o(\widetilde{\varepsilon}_s, \widetilde{q}_s, \bar{z}_{n+s}) \ , \qquad s = 1, \ldots, r \ .$$

We can compute a string like amplitude using this vertex operators on the sphere

Understanding each part helps us better understand the amplitude

$$V_o(\varepsilon_i, k_i, z_i) \longmapsto V_o(\varepsilon_i, k_i, z_i) \quad e^{ik_i \widetilde{X}(\bar{z}_i)} \quad J^{c_i}(\bar{z}_i), \qquad i = 1,\ldots,n \ ,$$

The open string vertex on the sphere

$$\left[i\partial X^\beta + \frac{\alpha'}{2}(k_i\psi)\psi^\beta(z_i)\right] e^{ik_i X(z_i)}$$

Result of the continuation to the sphere

We add this current to produce the Park-Taylor factor $\mathscr{C}(1,2,\ldots,n)$

$$V_c(\varepsilon_s, q_s, z_{n+s}, \bar{z}_{n+s}) \longmapsto V_c = V_o(\varepsilon_s, q_s, z_{n+s}) \quad V_o(\widetilde{\varepsilon}_s, \widetilde{q}_s, \bar{z}_{n+s}) \ , \qquad s = 1,\ldots,r \ .$$

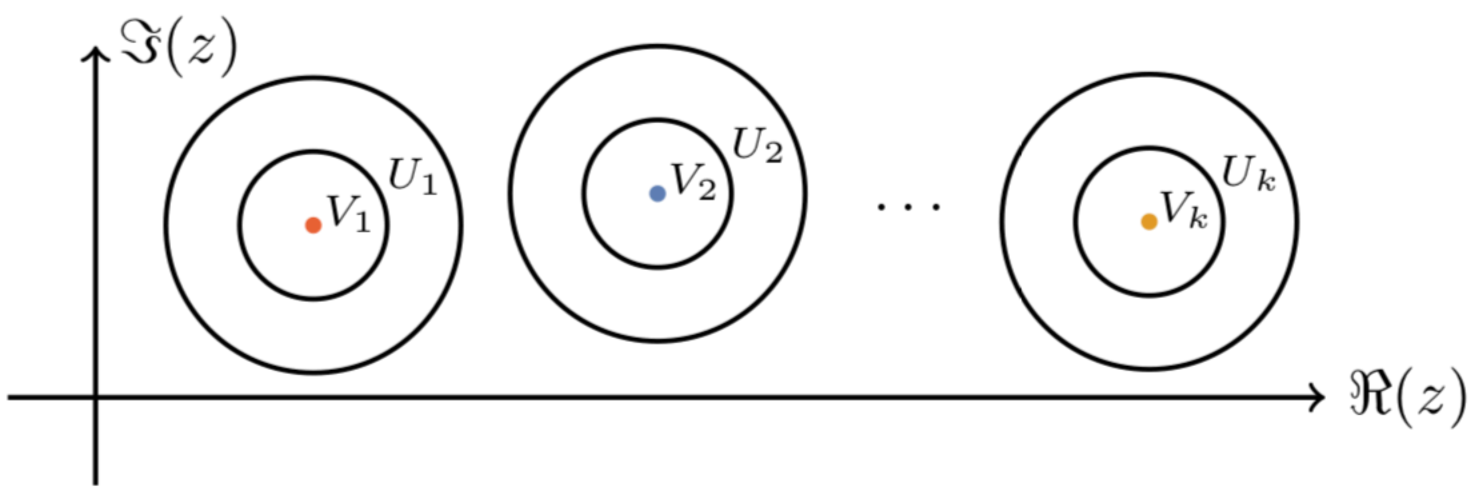
The open string vertex on the sphere → The double copy Structure ← The open string vertex on the sphere

Rabbit Hole

The intersection number is the product of two holomorphic top form it vanishes!!

$$\iota_\omega : H^{n-3}(\mathcal{M}_{0,n}, \nabla_\omega) \rightarrow H_c^{n-3}(\mathcal{M}_{0,n}, \nabla_\omega)$$

We compactify the Cohomology therefore the forms would be on compact support!
Therefore, the form $\iota_\omega(\varphi)$ will be non-holomorphic.

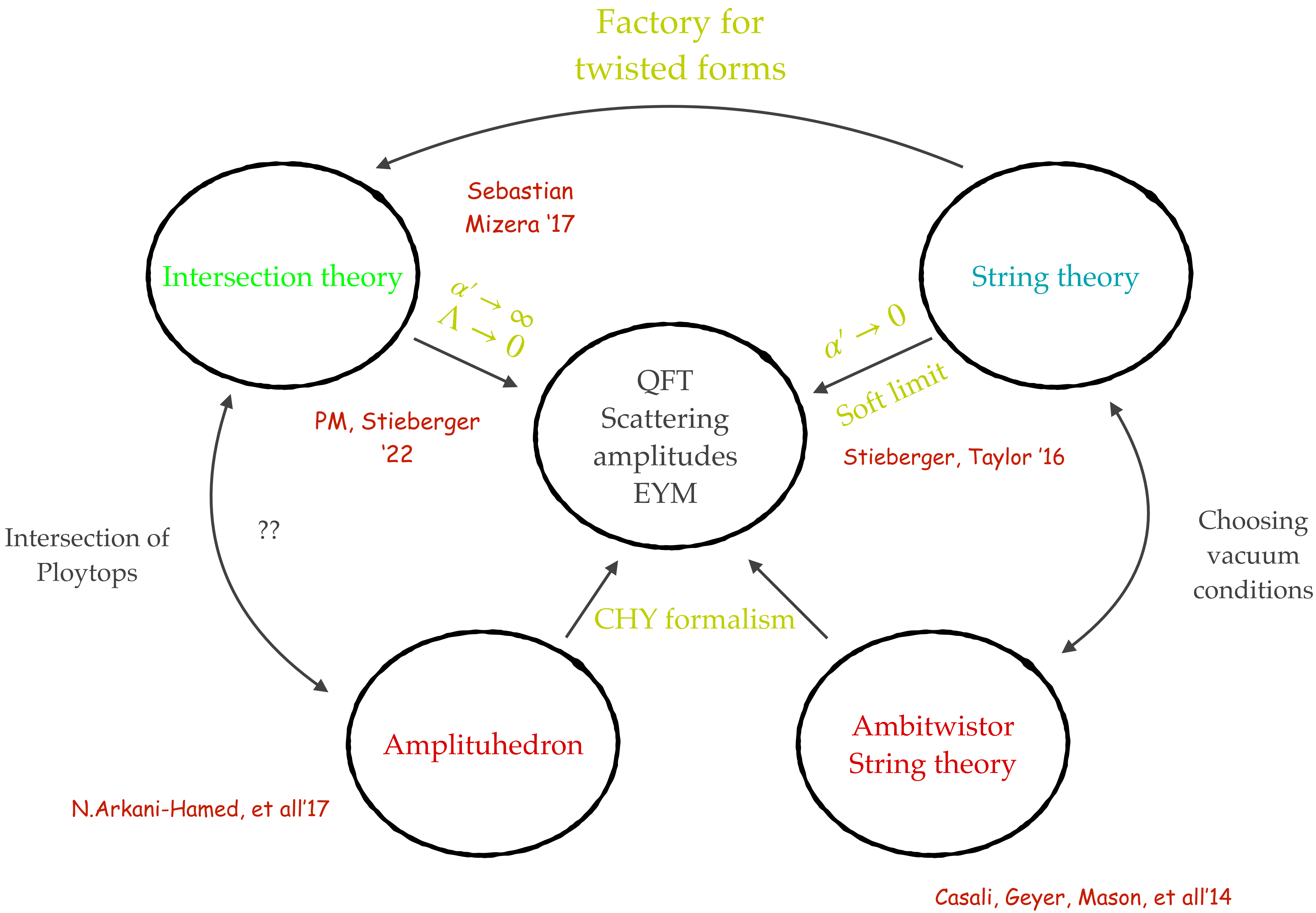


Non holomorphic Surgical removal

$$h_p(z,\bar{z}) = \begin{cases} 1 & (z,\bar{z}) \in V_p \\ [0,1] & (z,\bar{z}) \in U_p \\ 0 & otherwise \end{cases} \longrightarrow \quad \iota_\omega(\varphi) = \varphi - \sum_{i=1}^{k} \nabla_\omega(h_i\psi_i)$$

$$\nabla_\omega \psi_i = \varphi \rightarrow \psi_i = \nabla_\omega^{-1}\varphi$$

Solve by Series Expansion


Factory for
twisted forms
Sebastian
Mizera '17
Intersection theory
String theory
$\alpha' \to \infty$
$\Lambda \to 0$
$\alpha' \to 0$
Soft limit
QFT
Scattering
amplitudes
EYM
PM, Stieberger
'22
Stieberger, Taylor '16
Intersection of
Ploytops
??
Choosing
vacuum
conditions
CHY formalism
Amplituhedron
Ambitwistor
String theory
N.Arkani-Hamed, et all'17
Casali, Geyer, Mason, et all'14