Universität Zürich[UZH]

Queen Mary University of London

Celestial Twistor Amplitudes

Joshua Gowdy

Queen Mary University of London

QCD Meets Gravity '22

University of Zurich

Based on 2212.01327 with Graham Brown and Bill Spence

Goal and Methodology

- To study light transformed celestial correlators
  - Free of delta function singularities from bulk momentum conservation
  - Possibly candidate for THE basis celestial holographic states
  - Extract CCFT data

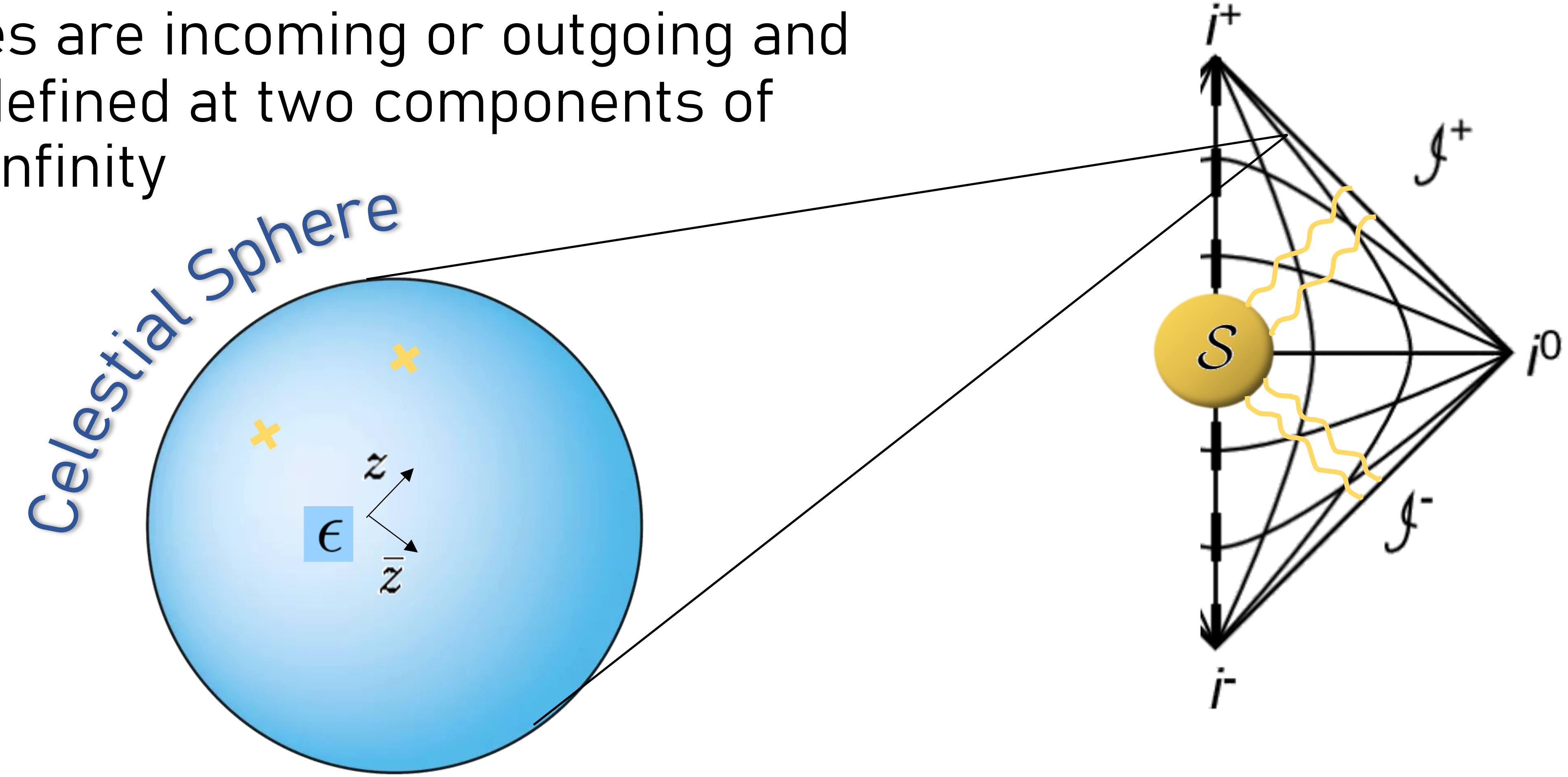
Sharma 2107.06250
Jorge-Diaz, Pasterski, Sharma 2212.00962

De, Hu, Srikant, Volovich 2206.08875

- By studying celestial twistor amplitudes
  - Precise twistor/light correspondence
  - Twistor amplitudes have already been computed
  - Mellin transforms are easy-ish!

Lorentzian Scattering (Warm Up)

- States are incoming or outgoing and are defined at two components of null infinity



Describing an Amplitude (Warm Up)

- Scattering amplitudes of massless particles are functions of spinors $\lambda_\alpha \ \tilde{\lambda}_{\dot{\alpha}}$
- Spinors act as homogeneous coordinates on the null cone.
- Little group action

$$\lambda_\alpha \mapsto e^{i\theta}\,\lambda_\alpha$$

- States/amplitudes are homogeneous functions labelled by representations of $\mathrm{U}(1)$, namely helicity J.

Describing a Celestial Amplitude

- Celestial amplitudes of massless particles are *also* functions of spinors.
- Spinors act as homogeneous coordinates on the celestial sphere.
- Extended Little group action

$$\lambda \to y\,\lambda$$

Banerjee, 1801.10171

- Celestial states/amplitudes are homogeneous functions labelled by representations of $\mathbb{C}_* = \mathrm{U}(1) \times \mathbb{R}_+$ namely conformal dimension Δ and helicity J

4d Lorentz = 2d Conformal

Only ever need to check these!

- Homogeneity property

$$|y\,\lambda, \bar{y}\,\tilde{\lambda}; h, \bar{h}\rangle = y^{-2h}\bar{y}^{-2\bar{h}}|\,\lambda, \tilde{\lambda}; h, \bar{h}\rangle\,,$$

- Plus Lorentz action

$$|\lambda_\alpha, \tilde{\lambda}_{\dot{\alpha}}; h, \bar{h}\rangle \to |M_\alpha^{\ \beta}\lambda_\beta, \bar{M}_{\dot{\alpha}}^{\dot{\beta}}\tilde{\lambda}_{\dot{\beta}}; h, \bar{h}\rangle,$$

- Implies 2d conformal covariance in affine coordinates

$$\left|\begin{pmatrix} z \\ 1 \end{pmatrix}, \epsilon\begin{pmatrix} \bar{z} \\ 1 \end{pmatrix}; h, \bar{h}\right\rangle \to (cz+d)^{-2h}(\bar{c}\bar{z}+\bar{d})^{-2\bar{h}}\left|\begin{pmatrix} z' \\ 1 \end{pmatrix}, \epsilon\begin{pmatrix} \bar{z}' \\ 1 \end{pmatrix}; h, \bar{h}\right\rangle$$

Chiral Mellin Transform

- In general, to build such states/amplitudes in homogeneous coordinates we use a chiral Mellin transform given by

$$|\,\lambda, \tilde{\lambda}; h, \bar{h}, l\rangle := \frac{1}{2\pi i}\int_{\mathbb{C}_*} \frac{d\bar{u}}{\bar{u}} \wedge \frac{du}{u}\, u^{2h} \bar{u}^{2\bar{h}} |u\,\lambda, \bar{u}\,\tilde{\lambda}; l\rangle,$$

- The spin condition appears naturally

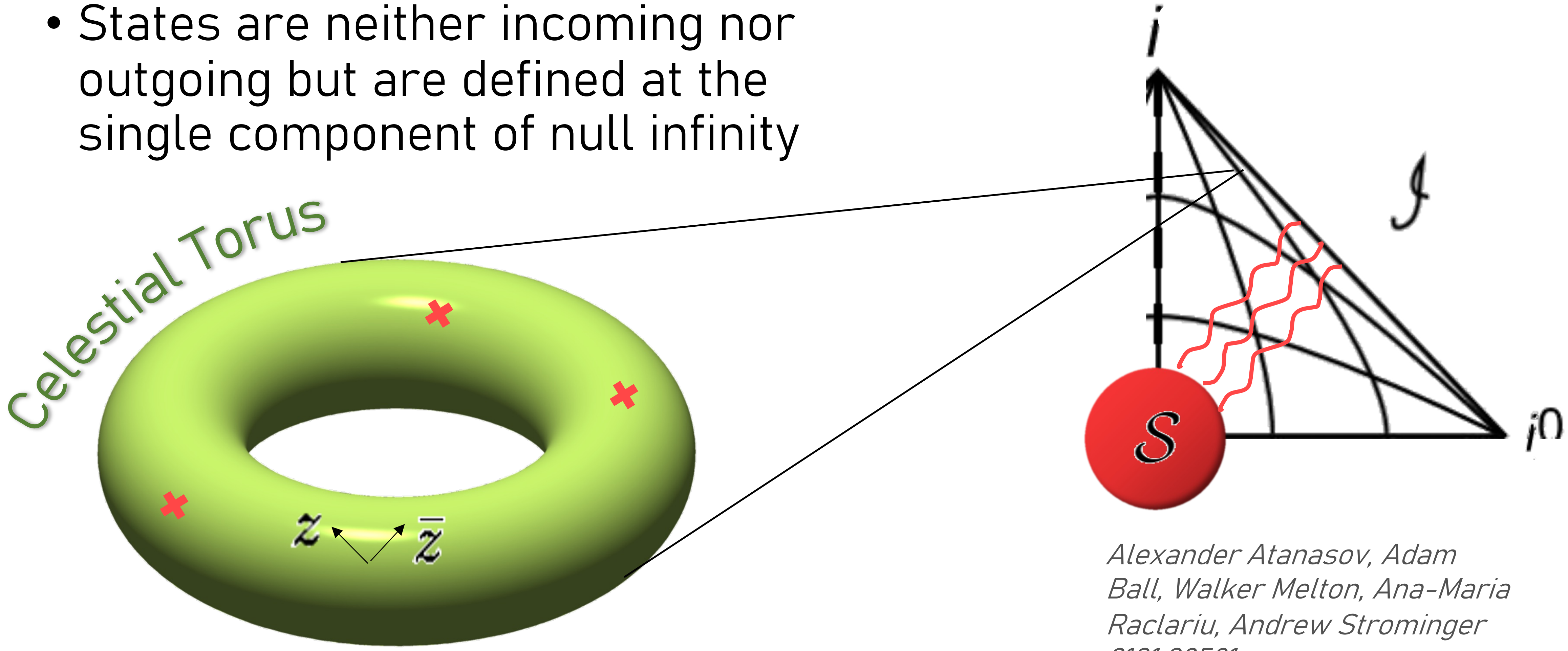
$$h - \bar{h} = l$$

Brandhuber, JG, Brown, Spence, Travaglini 2105.10263

Gelfand et al. Generalized Functions, Volume 5

Split Signature Scattering

- States are neither incoming nor outgoing but are defined at the single component of null infinity



Alexander Atanasov, Adam Ball, Walker Melton, Ana-Maria Raclariu, Andrew Strominger 2101.09591

(2,2) Little Group

- Spinors λ_α and $\tilde{\lambda}_{\dot{\alpha}}$ are real and independent.
- Little group in (2,2) is $\mathbb{R}_* = \mathbb{R}_+ \times \mathbb{Z}_2$

$$\lambda_\alpha \mapsto c\,\lambda_\alpha, \quad \tilde{\lambda}_{\dot{\alpha}} \mapsto c^{-1}\tilde{\lambda}_{\dot{\alpha}},$$

- States are labelled by a continuous helicity J labelling reps of $\mathbb{R}_+$ and a discrete parameter s=0,1 labelling even and odd states under the action of $\mathbb{Z}_2$

(2,2) Extended Little Group

- Boosts may flip the sign of the momenta so Extended Little Group action is $\mathbb{R}_* \times \mathbb{R}_*$

$$\lambda \to y\,\lambda, \quad \tilde{\lambda} \to \tilde{y}\,\tilde{\lambda}\,,$$

- Each spinor acts as a homogeneous coordinate on one of the null circles of the Lorentzian celestial torus.

- States/amplitudes are labelled by reps of the extended little group and are homogeneous functions

$$|y\,\lambda, \tilde{y}\,\tilde{\lambda}; h, s_h, \bar{h}, s_{\bar{h}}\rangle = |y|^{-2h}\mathrm{sgn}(y)^{-s_h}|\tilde{y}|^{-2\bar{h}}\mathrm{sgn}(\tilde{y})^{-s_{\bar{h}}}|\,\lambda, \tilde{\lambda}; h, s_h, \bar{h}, s_{\bar{h}}\rangle\,,$$

(2,2) Chiral Mellin Transform

- To build conformal primaries/correlators in homogeneous coordinates we use a (2,2) chiral Mellin transform

$$|\lambda_\alpha, \tilde{\lambda}_{\dot{\alpha}}; h, s_h, \bar{h}, s_{\bar{h}}, l\rangle := \frac{1}{4}\int_{\mathbb{R}_*\times\mathbb{R}_*} \frac{d\tilde{u}}{|\tilde{u}|} \wedge \frac{du}{|u|}\, |u|^{2h}\, |\tilde{u}|^{2\bar{h}}\, \mathrm{sgn}(u)^{s_h} \mathrm{sgn}(\tilde{u})^{s_{\bar{h}}} |u\lambda_\alpha, \tilde{u}\tilde{\lambda}_{\dot{\alpha}}, l\rangle.$$

Brown, JG, Spence
2212.01327

- Mod functions and sgn functions!
- Simply a product of independent Mellin transforms.

Light Transforms

- Map $h \to 1-h \iff \Delta \to 1-J,\ J \to 1-\Delta,$
- Light transform

$$\left|\mu, \tilde{\lambda}; 1-h, \bar{h}, s_h, s_{\bar{h}}, l\right\rangle = i^{-s_h} \frac{\Gamma(2-2h)}{\Gamma(\frac{3}{2}-h+\frac{s_h}{2})\Gamma(h-\frac{s_h}{2}-\frac{1}{2})} \times \int_{\mathbb{RP}^1} \langle\lambda\, d\,\lambda\rangle\, |\langle\lambda\,\mu\rangle|^{2h-2} \mathrm{sgn}(\langle\lambda\,\mu\rangle)^{s_h} \left|\lambda, \tilde{\lambda}; h, \bar{h}, s_h, s_{\bar{h}}, l\right\rangle .$$

Brown, JG, Spence
2212.01327

Twistor ~ Light

- Half-Fourier Transform $\left|\mu, \tilde{\lambda}, l\right\rangle = \frac{1}{2\pi} \int_{\mathbb{R}^2} d^2\lambda \, e^{i\langle\lambda\,\mu\rangle} \left|\lambda, \tilde{\lambda}, l\right\rangle$

Witten
hep-th/0312171

$$\begin{array}{ccc} \text{Momentum Basis} & \xrightarrow{\quad T \quad} & \text{Twistor Basis} \\ \Big\downarrow C_{h,\bar{h}} & & \Big\downarrow C_{1-h,\bar{h}} \\ \text{Mellin Basis} & \xrightarrow{\quad L \quad} & \text{Light Basis} \end{array}$$

Sharma
2107.06250

Brown, JG, Spence
2212.01327

Ambidextrous Twistor Amplitudes

Arkani-Hamed, Cachazo, Cheung, Kaplan 0903.2110

- Incredibly simple!
- Three Points Yang-Mills

$$\tilde{A}_3^{--+} := \text{sgn}(\langle\lambda_1\lambda_2\rangle\langle\lambda_1\mu_3\rangle\langle\lambda_2\mu_3\rangle)\,\text{sgn}(1+\theta_{31}^{-1})\,\text{sgn}(1+\theta_{32}^{-1}).$$

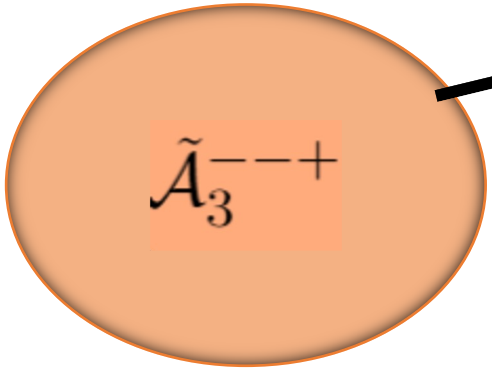
- Three Points Gravity

$$\tilde{M}_3^{--+} := |\langle\lambda_1\lambda_2\rangle\langle\lambda_1\mu_3\rangle\langle\lambda_2\mu_3\rangle|\left|1+\theta_{31}^{-1}\right|\left|1+\theta_{32}^{-1}\right|.$$

- Nice double copy-esque structure!

$$\theta_{ij} = \frac{\langle\lambda_j\mu_i\rangle}{[\tilde{\lambda}_i\tilde{\mu}_j]}$$

3 point MHV Ambidextrous Celestial Twistor Amplitudes



Example: Plus Helicity Leg 3

1. Apply Dual Twistor transform $\lambda_3 \to \mu_3$
2. Apply half Mellin transform over μ_3

Two steps:

- (anti)symmetrise over μ_3
- And integrate over positive scale of μ_3

$\mathbb{Z}_2$ Homogeneity

$\mathbb{R}_+$ Homogeneity

Key Integral at 3 points:
Mellin transform of an (anti)symmetrised sgn function

$$G_0(x) := \frac{1}{2}\Big(\operatorname{sgn}(1+x) + \operatorname{sgn}(1-x)\Big),$$

$$G_1(x) := \frac{1}{2}\Big(\operatorname{sgn}(1+x) - \operatorname{sgn}(1-x)\Big),$$

$$\int_0^\infty \frac{dx}{x}\, x^{2k+(-1)^s\epsilon}\, G_s(x\theta_{ij}^{-1}) = \operatorname{sgn}(-\theta_{ij})^s \frac{|\theta_{ij}|^{2k}}{2k+(-1)^s\epsilon}.$$

$$k = 1 - h$$

Three Point MHV

- Yang Mills

$$\tilde{\mathcal{A}}_3^{--+}\{s_{\bar{k}_1}, s_{\bar{k}_2}, s_{k_3}\} = \frac{\pi}{4}\mathrm{sgn}(\langle\lambda_1\lambda_2\rangle\langle\lambda_1\mu_3\rangle\langle\lambda_2\mu_3\rangle)\,\delta\Big(\sum_i 2k_i\Big)\tilde{\delta}\Big(\sum_i s_{k_i}\Big)$$
$$\times\,\mathrm{sgn}(-\theta_{31})^{s_{\bar{k}_1}}\frac{|\theta_{31}|^{2\bar{k}_1}}{2\bar{k}_1+(-1)^{s_{\bar{k}_1}}\epsilon}\mathrm{sgn}(-\theta_{32})^{s_{\bar{k}_2}}\frac{|\theta_{32}|^{2\bar{k}_2}}{2\bar{k}_2+(-1)^{s_{\bar{k}_2}}\epsilon}.$$

- Gravity

$$\tilde{\mathcal{M}}_3^{--+}\{s_{\bar{k}_1}, s_{\bar{k}_2}, s_{k_3}\} = \frac{\pi}{4}\Big|\langle\lambda_1\lambda_2\rangle\langle\lambda_1\mu_3\rangle\langle\lambda_2\mu_3\rangle\Big|\,\delta\Big(\sum_i 2k_i+2\Big)\delta\Big(\sum_i s_{k_i}\Big)$$
$$\times\,\mathrm{sgn}(-\theta_{31})^{s_{\bar{k}_1}}\frac{|\theta_{31}|^{2\bar{k}_1}}{2\bar{k}_1(2\bar{k}_1+1)}\mathrm{sgn}(-\theta_{32})^{s_{\bar{k}_2}}\frac{|\theta_{32}|^{2\bar{k}_2}}{2\bar{k}_2(2\bar{k}_2+1)},$$

Four Point YM Twistor Amplitude

- Twistor amplitude remains simple even at four points.
- For Yang Mills,

$$\tilde{A}_4^{+-+-} = \mathrm{sgn}(\langle \lambda_2 \mu_1 \rangle + [\tilde{\lambda}_1 \tilde{\mu}_2])\,\mathrm{sgn}(\langle \lambda_4 \mu_1 \rangle + [\tilde{\lambda}_1 \tilde{\mu}_4])$$
$$\times\, \mathrm{sgn}(\langle \lambda_2 \mu_3 \rangle + [\tilde{\lambda}_3 \tilde{\mu}_2])\,\mathrm{sgn}(\langle \lambda_4 \mu_3 \rangle + [\tilde{\lambda}_3 \tilde{\mu}_4])$$

Arkani-Hamed, Cachazo, Cheung, Kaplan 0903.2110

Four-Point YM Celestial Twistor Amp

$$\tilde{\mathcal{A}}_4^{+-+-}\{0,0,0,0\} = 4\pi^2 \,\mathrm{sgn}(\langle\mu_1\,\lambda_2\rangle\langle\mu_3\,\lambda_2\rangle\langle\mu_3\,\lambda_4\rangle\langle\mu_1\,\lambda_4\rangle)\delta\Big(\sum_a 2k_a\Big)\,\tilde{\delta}\Big(\sum_a s_{k_a}\Big)$$

$$\times\,|\theta_{12}|^{2\bar{k}_2}|\theta_{14}|^{2k_3+2\bar{k}_4}|\theta_{34}|^{-2k_3}[I(0,0,0,0)+\mathrm{sgn}(\theta_{12}\theta_{32}\theta_{34}\theta_{14})\,I(1,1,1,1)]$$

$$\begin{aligned}
&I(0,0,0,0)\\
&=\Big(\Theta(|\theta|-1)\Big[-\frac{|\theta|^{2k_3}}{(2k_3-\epsilon)(2\bar{k}_4+\epsilon)(2\bar{k}_2+2k_3)}-\frac{|\theta|^{-2\bar{k}_2}}{(2\bar{k}_2+\epsilon)(2\bar{k}_2+2k_3)(2\bar{k}_2+2k_3+2\bar{k}_4+\epsilon)}\Big]\\
&+\Theta(1-|\theta|)\Big[-\frac{1}{(2\bar{k}_2+\epsilon)(2k_3-\epsilon)(2k_3+2\bar{k}_4)}-\frac{|\theta|^{2k_3+2\bar{k}_4}}{(2\bar{k}_4+\epsilon)(2k_3+2\bar{k}_4)(2\bar{k}_2+2k_3+2\bar{k}_4+\epsilon)}\Big]\Big).
\end{aligned}$$

$$\theta := \frac{\theta_{12}\theta_{34}}{\theta_{14}\theta_{32}} = \frac{\frac{\langle\lambda_2\,\mu_1\rangle\langle\lambda_4\,\mu_3\rangle}{\langle\lambda_4\,\mu_1\rangle\langle\lambda_2\,\mu_3\rangle}}{\frac{[\tilde{\lambda}_1\,\tilde{\mu}_4][\tilde{\lambda}_3\,\tilde{\mu}_2]}{[\tilde{\lambda}_1\,\tilde{\mu}_2][\tilde{\lambda}_3\,\tilde{\mu}_4]}} =: \frac{r}{\tilde{r}}.$$

Celestial Twistor BCFW

Arkani-Hamed, Cachazo, Cheung, Kaplan 0903.2110

- Twistor BCFW recursion relations imply recursion relations for light transformed correlators on celestial torus, this gives a path to higher multiplicity correlators.
- Checked four point expression and nicely explains the conformally invariant functions

$$I(0,0,0,0) := \int \frac{d\bar{K}}{2\pi i} \frac{|\theta|^{-2\bar{K}}}{2k_3 + 2\bar{k}_4 + 2\bar{K} + \epsilon} \frac{1}{2\bar{K} + \epsilon} \frac{1}{2k_3 + 2\bar{K} - \epsilon} \frac{1}{-2\bar{k}_2 + 2\bar{K} - \epsilon}$$

- Massless BCFW = multi-collinear factorisation and so contains OPE data!

Outlook

- Road to higher multiplicity is clear but not easy
- Ready to extract Lorentzian CCFT data via conformal block decomposition/partial waves
- How exactly is this related to BCFW?
- Supersymmetric extensions are well known in twistor space and give corresponding extensions of the formulae presented here, including celestial supertwistor BCFW.
- Are light/twistor celestial correlators THE basis for CCFT?