Celestial chiral algebras and colour-kinematics duality

Ricardo Monteiro

Queen Mary University of London

QCD meets Gravity

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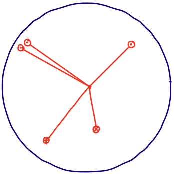
Based on 2208.11179 & to appear.

See also 2208.13750 Bu, Heuveline, Skinner; 2209.00696 Guevara.

Motivation

Celestial holography



4D amplitudes structures $\longleftrightarrow$ 2D celestial structures

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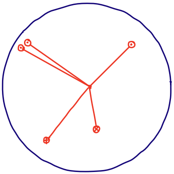

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Gravity self-dual/chiral sector:

- kinematic algebra [RM, O'Connell 11] $\longleftrightarrow$ wedge of $w_{1+\infty}$ [Strominger 21]

  [Guevara, Himwich, Pate, Strominger 21]
- Moyal deformation $\longleftrightarrow$ wedge of $W_{1+\infty}$

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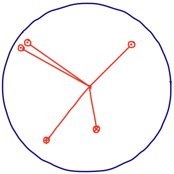

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Bonus: relation to chiral higher-spin theories.

Self-dual sectors of YM and gravity

Light-cone coords $(u, v, w, \bar{w})$, $ds^2 = -du\,dv + dw\,d\bar{w}$.

Yang-Mills: light-cone gauge $A_\mu = (0, A_v, \bar{A}, A)$ [Chalmers, Siegel '98]

$$S_{\text{YM}}(A, \bar{A}) = \int d^4x \; \mathrm{tr}\left(\bar{A}\,\Box A + (\partial_u \bar{A})\left[A, \frac{\partial_w}{\partial_u} A\right] + (\partial_u A)\left[\bar{A}, \frac{\partial_{\bar{w}}}{\partial_u} \bar{A}\right] + [A, \partial_u \bar{A}] \frac{1}{\partial_u^2} [\bar{A}, \partial_u A] \right)$$

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Self-dual Yang-Mills ($\star F = iF$): truncate to $(-++)$ vertex.

Redefine $\bar{A} = \partial_u^{-1} \bar{\Psi}$, $A = \partial_u \Psi$:

$$S_{\text{SDYM}}(\Psi, \bar{\Psi}) = \int d^4x \;\; \mathrm{tr} \;\; \bar{\Psi}\left(\Box \Psi + [\partial_u \Psi, \partial_w \Psi]\right)$$

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Self-dual gravity: similar [Siegel 92] [Plebanski 75]

$$S_{\text{SDGR}}(\phi, \bar{\phi}) = \int d^4x \;\bar{\phi}\left(\Box\phi + \{\partial_u\phi, \partial_w\phi\}\right) \qquad \{f, g\} := \partial_u f\, \partial_w g - \partial_w f\, \partial_u g$$

Colour-kinematics duality & double copy

Simplest double copy setting!

$$ds^2 = -du\,dv + dw\,d\bar{w}$$

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BCJ colour-kinematics duality [Bern, Carrasco, Johansson 08] manifest! [RM, O'Connell 11]

Cubic vertex $(-++)$: $\boxed{V_{\text{SDYM}} = X(k_1, k_2)\, f^{a_1 a_2 a_3}\,, \quad V_{\text{SDGR}} = X(k_1, k_2)^2}$

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Lie algebra of area-preserving diffeomorphisms in null plane (u, w).

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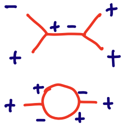
Lie algebra of area-preserving diffeomorphisms in null plane (u, w).

spinor helicity notation: $X(k_1,k_2) = \langle\eta|k_1k_2|\eta\rangle$

Amplitudes in self-dual sector

Amplitudes with only $(++-)$ vertex?

- tree level: $-++\cdots+$ (one-minus YM/GR)
- one loop: $+++\cdots+$ (all-plus YM/GR)
- that's it! $\Rightarrow$ SD theories one-loop exact



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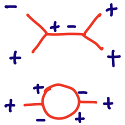

Tree level:

Amplitudes $-++\cdots+$ vanish for $n > 3$. Classical integrability!

$$\text{e.g.} \quad A_{\text{YM}}(1234) \propto \frac{X(k_1,k_2)\,X(k_3,k_4)}{s_{12}} + \frac{X(k_2,k_3)\,X(k_4,k_1)}{s_{23}} = 0$$

'Maximal helicity violation' (MHV) sector is $--++\cdots+$.

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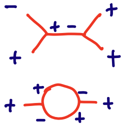

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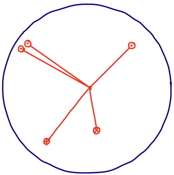
One loop:

Amplitudes are rational functions. **Monday talk by Sam Wikeley**

Celestial holography

See amplitude as a 2D CFT correlator on celestial sphere. [Strominger et al]



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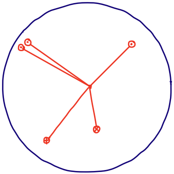

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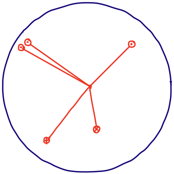

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Celestial CFT OPE $\longleftrightarrow$ collinear limit of amplitude.

Tree amplitude: $$\mathcal{A}_n(k_1, k_2, k_3, \dots) \to A_{3\text{pt}} \frac{1}{s_{12}} \mathcal{A}_{n-1}(k_1 + k_2, k_3, \dots)$$

Tree celestial OPE: $$\mathcal{O}(k_1)\, \mathcal{O}(k_2) \sim A_{3\text{pt}} \frac{1}{s_{12}} \mathcal{O}(k_1 + k_2)$$

Momentum basis better than conformal basis for our purposes.

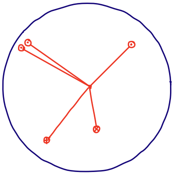
Celestial chiral OPEs

Massless kinematics:

$$k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}\,, \quad \lambda = (1, z)\,, \quad \tilde{\lambda} = (k_u, k_w)\,, \quad z = \frac{k_{\bar{w}}}{k_u} = \frac{k_v}{k_w}$$

$$X(k_1, k_2) := k_{1w}k_{2u} - k_{1u}k_{2w} = \epsilon^{\dot{A}\dot{B}}\,\tilde{\lambda}_{1\,\dot{A}}\,\tilde{\lambda}_{2\,\dot{B}} =: [12]$$

$$s_{12} := (k_1 + k_2)^2 = (z_1 - z_2)\,X(k_1, k_2)$$



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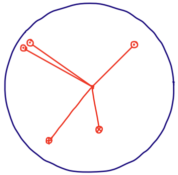

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Collinear limit $z_1 \to z_2$

SDYM & YM:

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SDGR & GR:

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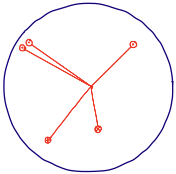

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Colour-kinematics duality $\Rightarrow$ OPE associativity (tree level)

$w_{1+\infty}$ from soft expansion in SDGR

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$\omega_{1+\infty}$ from soft expansion in SDGR

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Soft expansion is $k_u\,,\ k_w \to 0$ at fixed z,

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = \sum_{a,b\geq 0} \frac{(ik_u)^a}{a!}\,\frac{(ik_w)^b}{b!}\,\ell_{a,b} \qquad \text{with} \qquad \ell_{a,b} = \{(u + z\bar{w})^a (w + zv)^b, \cdot\}$$

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self-dual kinematic algebra

wedge subalgebra of Lie algebra $\omega_{1+\infty}$

[Strominger 21] [Guevara, Himwich, Pate, Strominger 21]
[Adamo, Mason, Sharma 21] [older: Penrose, Bakas, Pope et al, . . .]

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self-dual kinematic algebra — wedge subalgebra of Lie algebra $\omega_{1+\infty}$

[Strominger 21] [Guevara, Himwich, Pate, Strominger 21]
[Adamo, Mason, Sharma 21] [older: Penrose, Bakas, Pope et al, ...]

Similarly, expand $\mathcal{O}_+(k)$ in soft generators $\varpi_{a,b}(z)$:

$$\varpi_{a,b}(z_1)\,\varpi_{c,d}(z_2) \sim \frac{ad-bc}{z_1 - z_2}\,\varpi_{a+c-1,b+d-1}(z_2)$$

Moyal-deformed SDGR

Motivation: $\omega_{1+\infty}$ can be deformed into $W_{1+\infty}$ [Pope, Romans, Shen 90] (extends spin-2 Virasoro to spins ≥ 1).

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Idea: $\omega_{1+\infty}$ is associated to Poisson bracket of SDGR.

Poisson bracket can be deformed into Moyal bracket.

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Moyal-deformed self-dual gravity [Strachan 92]

$$S_{\text{Moyal-SDGR}}(\phi, \bar{\phi}) = \int d^4x\ \bar{\phi}(\Box\phi + \{\partial_u\phi, \partial_w\phi\}^M)$$

Moyal-deformed SDGR: double copy and OPE

Action: $S_{\text{Moyal-SDGR}}(\phi,\bar{\phi}) = \int d^4x \;\; \bar{\phi}(\Box\phi + \{\partial_u\phi, \partial_w\phi\}^M)$

Vertex: $V_{\text{Moyal-SDGR}} = X(k_1,k_2)\, X^M(k_1,k_2)$ [Chacón, García-Compeán, Luna, RM, White 20]

$$X^M(k_1,k_2) = \frac{1}{\alpha}\sinh\left(\alpha\, X(k_1,k_2)\right)$$ still structure constant

$$\frac{1}{\alpha^2}\sinh\left(\alpha\, X(k_1,k_2)\right)\sinh\left(\alpha\, X(k_3,k_4)\right) + \text{cyc}(123) = 0$$

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Celestial OPE: tree-level associativity guaranteed.

$$\mathcal{O}_+(k_1)\; \mathcal{O}_+(k_2) \;\sim\; \frac{X(k_1, k_2)\, X(k_1, k_2)^M}{s_{12}}\; \mathcal{O}_+(k_1 + k_2) \;=\; \frac{X(k_1, k_2)^M}{z_1 - z_2}\; \mathcal{O}_+(k_1 + k_2)$$

Soft expansion gives algebra equivalent to wedge subalgebra of $W_{1+\infty}$.

[Pope, Romans, Shen 90; Fairlie, Nuyts 90] [Wu, Heuveline, Skinner 22]

Moyal deformation and chiral higher-spin theory

Moyal deformation not Lorentz invariant,

but related to Lorentz invariant 'chiral higher-spin theory' [Metsaev 91; Skvortsov, Ponomarev 16]:

$$V_{\text{Moyal-SDGR}} = \frac{1}{\alpha^2} \sum_{\sigma \geq 1} \frac{\big(\alpha X(k_1, k_2)\big)^{2\sigma}}{(2\sigma - 1)!} \qquad V_{\text{chs}} = \frac{1}{\alpha^2} \frac{\big(\alpha X(k_1, k_2)\big)^{h_1+h_2+h_3}}{(h_1 + h_2 + h_3 - 1)!}$$

Generating chiral higher spins: $\phi \mapsto \eta^2 \sum_{h=-\infty}^{\infty} \eta^{-h} \phi_h \qquad \alpha \mapsto \alpha\,\eta$ [c.f. Ponomarev 17]

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$$0 = \Box\phi + \{\partial_u \phi, \partial_w \phi\}^M \mapsto \eta^2 \sum_h \eta^{-h} \Bigg(\Box\phi_h + \sum_{\substack{h_1, h_2 \\ \text{even } h_1+h_2-h>0}} \phi_{h_1} \frac{(\alpha \overset{\leftrightarrow}{P})^{h_1+h_2-h}}{\alpha^2 (h_1 + h_2 - h - 1)!} \phi_{h_2} \Bigg)$$

C.h.s. tree amplitudes ($n > 3$) vanish like SDGR, but no parity-invariant parent (?).

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C.h.s. has associative celestial OPE, c.f. [Ren, Spradlin, Srikant, Volovich 22].

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Moyal-deformed U(N) SDYM: $\boxed{\Box\Psi + [\partial_u\Psi, \partial_w\Psi]^M = 0}$

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Double-copy structure and celestial OPE analogous to Moyal-deformed SDGR,

but with new type of colour-kinematic algebra!

$$V_{\text{Moyal-SDYM}} = X(k_1,k_2)\left(\cosh\left(\alpha\,X(k_1,k_2)\right) f^{a_1a_2a_3} + \sinh\left(\alpha\,X(k_1,k_2)\right) d^{a_1a_2a_3}\right)$$

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Related to gluonic chiral higher-spin theory of [Skvortsov, Tran, Tsulaia 18].

Integrability in chiral sector

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Celestial OPE coefficients ${C_{IJ}}^K$:

SDYM: $f^{a_1 a_2 a_3}$, SDGR: $X(k_1, k_2)$, Moyal-SDGR: $X^M(k_1, k_2)$, ...

$$\mathcal{O}_I(k_1)\,\mathcal{O}_J(k_2) \sim \frac{{C_{IJ}}^K}{z_1 - z_2}\,\mathcal{O}_K(k_1 + k_2) = \frac{X(k_1, k_2)\,{C_{IJ}}^K}{s_{12}}\,\mathcal{O}_K(k_1 + k_2)$$

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If chiral OPEs fully determine theory, then double copy ensures integrability (similar to Ward conjecture).

KLT [Kawai, Lewellen, Tye 86] amplitude formula:

$$\mathcal{A} = \sum_{\rho,\rho' \in S_n} A_X(\rho)\, S_{\text{KLT}}(\rho, \rho')\, A_C(\rho')$$

A_X / A_C : ordered amp's with vertices $X(k_1, k_2)$ / ${C_{IJ}}^K$. $A_X = 0 \Rightarrow \mathcal{A} = 0$.

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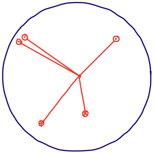
Non-vanishing quantities? Form factors. [Boels, Isermann, RM, O'Connell 11] [Costello, Paquette 22]

Conclusion



- Double-copy structure is closely connected to celestial OPEs.
- Chiral sector:

$$V_{\text{3-pt}} = X\, C_{IJ}{}^{K} \quad \Rightarrow \quad \mathcal{O}_I(z_1)\,\mathcal{O}_J(z_2) \sim \frac{C_{IJ}{}^{K}}{z_1 - z_2}\,\mathcal{O}_K(z_2)$$

  X-algebra: vanishing amplitudes $\leftrightarrow$ classical integrability.

- Celestial appearance of (loop of wedge of) $w_{1+\infty}$, $W_{1+\infty}$, etc.
- Moyal-deformed SDGR/SDYM generate chiral higher-spin theories.