Effective Field Theories with Celestial Duals

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2111.11356 and 2206.08322

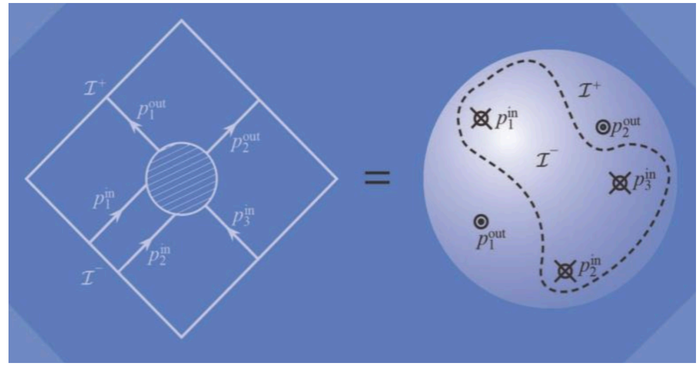
Introduction

Celestial Holography program is interesting both for understanding holographic description of flat space and for exploring properties of scattering amplitudes.

Stieberger's review talk

I will discuss necessary conditions for a 4d theory to have a celestial dual with an associative OPE.

Celestial Holography



Massless amplitude in 4d

$$\mathcal{A}_n(p_i, s_i) \rightarrow \int \prod_i d\omega_i\, \omega_i^{\Delta_i - 1}\, \mathcal{A}_n(p_i, s_i) \rightarrow$$

momentum   helicity

$p = \omega(1+z\bar{z}, z+\bar{z}, i(z-\bar{z}), 1-z\bar{z})$

$s = h - \bar{h},\ \Delta = h + \bar{h}$

conformal weights

$$\rightarrow \langle \mathcal{O}_{h_1,\bar{h}_1}(z_1,\bar{z}_1) \ldots \mathcal{O}_{h_n,\bar{h}_n}(z_n,\bar{z}_n) \rangle$$

Correlation function in 2d

Celestial Conformal Field Theory

Pasterski, Shao, Strominger

Plan

- Review $w_{1+\infty}$ - algebra in CCFT

  Strominger
- Deform $w_{1+\infty}$ due to non-minimal couplings

  MRSV
- Jacobi identity $\Rightarrow$ constraints on couplings

  MRSV
- Implications for amplitudes in theories w/ celestial duals

  RSSV
- Conclusion / open questions

Graviton OPEs

Two positive helicity gravitons

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$G^+_{\Delta_1}(z,\bar{z})\, G^+_{\Delta_2}(0,0) \sim -\frac{1}{2}\frac{\kappa_{2,2,-2}}{z}\sum_{n=0}^{\infty} B(\Delta_1 - 1 + n, \Delta_2 - 1)\frac{\bar{z}^{n+1}}{n!}\partial^n G^+_{\Delta_1+\Delta_2}$$

$$\kappa_{2,2,-2} \to \sqrt{G_N}$$

- from collinear limit of amplitudes
  Fan, Fotopoulos, Stieberger, Taylor, Zhu
- from symmetries
  Pate, Raclariu, Strominger, Yuan

Conformally soft gravitons

- Noting that OPE coefficients B have poles, define conformally soft graviton operators

$$H^{k}(z,\bar{z}) = \lim_{\Delta \to k} (\Delta - k)\, G^{+}_{\Delta}(z,\bar{z}) \qquad k = 2, 1, 0, -1, \ldots$$

Pate, Raclariu, Strominger

- Mode expansion

$$H^{k}(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^{k}_{n}(z)}{\bar{z}^{\,n+\frac{k-2}{2}}}$$

- We can work out OPE of $H^{k}_{n}(z)$

Soft current algebra

- Defining the commutator

$$[A,B](z) = \oint_z \frac{dw}{2\pi i} A(w)B(z)$$

- Guevara, Himwich, Pate, Strominger worked out soft current algebra for gravity → long expressions

- Strominger defined $w_n^p = \frac{1}{\varkappa_{2,2,-2}} (p-n-1)!(p+n-1)!\, H_n^{-2p+4}$ "light-transform"

and showed that commutation relations are

$$[w_m^p, w_n^q] = \left(m(q-1) - n(p-1)\right) w_{m+n}^{p+q-2}$$

$p = 1, \frac{3}{2}, 2, \frac{5}{2} \dots$

$1-p \leq m \leq p-1$

w-algebra

$$[w_m^p, w_n^q] = (m(q-1) - n(p-1))\, w_{m+n}^{p+q-2}$$

$p = 1, \frac{3}{2}, 2, \frac{5}{2} \ldots$

$1-p \leq m \leq p-1$

"$w_{1+\infty}$"

Penrose 1976
Bakas 1989
Pope Romans Shen 1990
Klebanov Polyakov 1991

$p=2 \Rightarrow c=0$ Virasoro ✓

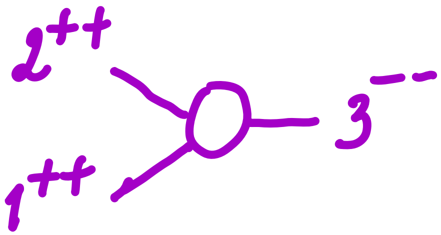
- There is a similar story for gluons

$$[S_n^{q,a}, S_m^{p,b}] = -i f^{ab}{}_c\, S_{n+m}^{q+p-1,c}$$

$$[w_m^p, S_n^{q,a}] = (m(q-1) + n(p-1))\, S_{m+n}^{p+q-2,a}$$

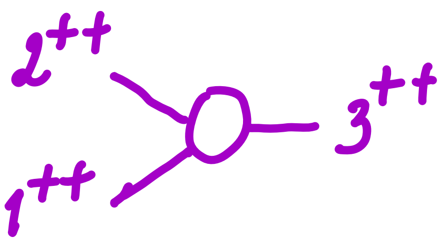
Deformed OPE

- Himwich, Pate, Singh considered corrections to OPE from non-minimal couplings.



$$= \kappa_{2,2,-2}\,\frac{[12]^6}{[23]^2[31]^2}$$

R



$$= \kappa_{2,2,2}\,[12]^2[23]^2[13]^2$$

R^3

$\hookrightarrow \kappa_{h_1,h_2,h_3}$ = 3pt coupling for helicities $h_1 h_2 h_3$

$$G^+_{\Delta_1}(z,\bar z)\,G^+_{\Delta_2}(0,0) \sim \frac{1}{z}\Big(\kappa_{2,2,-2}\,B(\Delta_1-1,\Delta_2-1)\,\bar z\,G^+_{\Delta_1+\Delta_2} + \kappa_{2,2,2}\,B(\Delta_1+3,\Delta_2+3)\,\bar z^5\,G^-_{\Delta_1+\Delta_2+4}+\dots\Big)$$

Deformed w-algebra

- The algebra becomes

$$[W^{q_1,2}_{m_1}, W^{q_2,2}_{m_2}] = -\frac{x_{2,2,-2}}{2}\left(m_1(q_1-1)-m_2(q_2-1)\right)W^{q_1+q_2-2,2}_{m_1+m_2}$$

$$-\frac{x_{2,2,2}}{2}N(q_1,q_2,m_1,m_2,5)W^{q_1+q_2-6,-2}_{m_1+m_2}$$

$$N(q_1,q_2,m_1,m_2,p)=\sum_{n=0}^{p}(-1)^{p-n}\binom{p}{n}[m_1+q_1-1]_{p-n}[-m_1+q_1-1]_n$$
$$[m_2+q_2-1]_n[-m_2+q_2-1]_{p-n}$$

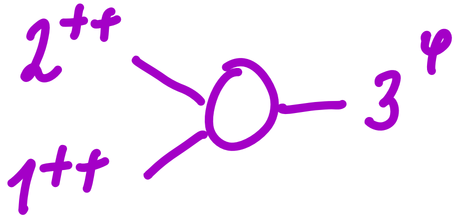
where $[a]_n \equiv a(a-1)\cdots(a-n+1)$

Pope, Romans, Shen 1990

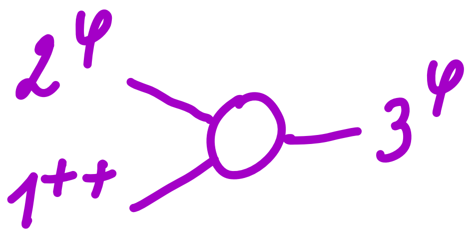
Jacobi identity

$$[[W^{q_1,2}_{m_1}, W^{q_2,2}_{m_2}], W^{q_3,2}_{m_3}] + \text{cyclic} \neq 0 \quad ?$$

Add scalars



$$x_{0,2,2}\,[12]^4 \qquad \varphi R^2$$



$$x_{0,0,2}\,\frac{[12]^2[13]^2}{[23]^2} \qquad R\varphi^2$$

Jacobi identity requires

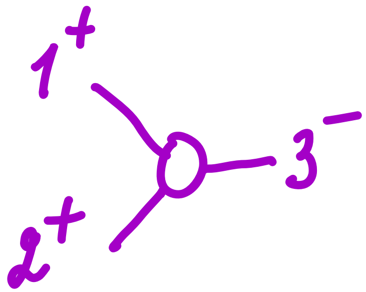
$$(x_{-2,2,2} - x_{0,0,2})\,x_{0,2,2} = 0$$

$$(x_{-2,2,2} - x_{0,0,2})\,x_{0,0,2} = 0$$

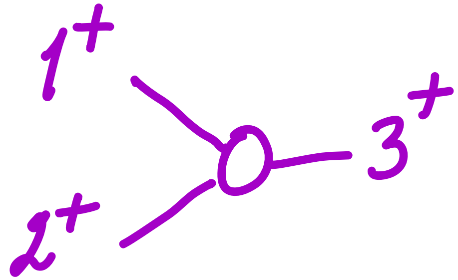
$$3\,x_{0,2,2}^{2} = 10\,x_{-2,2,2}\,x_{2,2,2}$$

MRSV

Include gluons

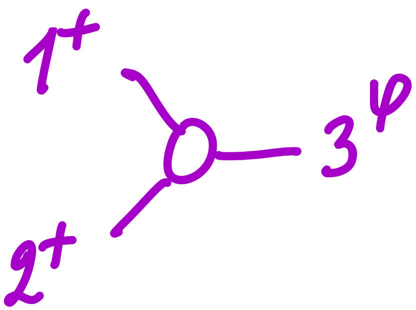


$x_{-1,1,1}$



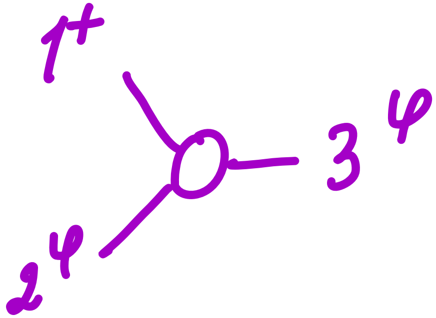
$x_{1,1,1}$

F^3



$x_{0,1,1}$

φF^2



$x_{0,0,1}$

$\varphi^2 F$

Constraints

$$x_{0,1,1}^2 = 2\,\overset{\text{YM}}{x_{-1,1,1}}\,\overset{F^3}{x_{1,1,1}}$$

$$x_{-1,1,2}\,x_{1,1,1} = 3\,x_{1,1,2}\,x_{-1,1,1}$$

MRSV

Relation to W-algebra

Pope, Shen, Romans constructed $W_{1+\infty}$ by deforming $w_{1+\infty}$

$$[V^{q_1}_{m_1}, V^{q_2}_{m_2}] := \sum_{r=0}^{\infty} \lambda^{2r} g^{q_1 q_2}_{2r}(m_1, m_2; \mu) V^{q_1+q_2-2r-2}_{m_1+m_2} +$$

$$+ \lambda^{2(q_1-2)} c_{q_1}(m_1; \mu) \delta^{q_1, q_2} \delta_{m_1+m_2, 0}$$

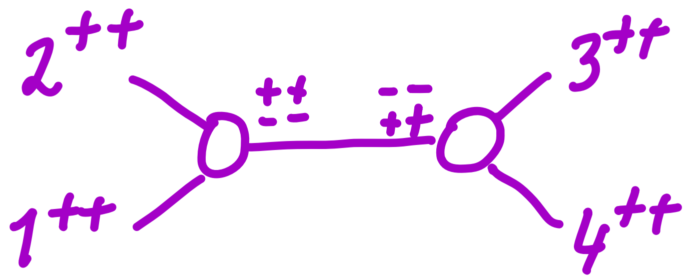
- $\mu = -3/16$, $g^{q_1 q_2}_{2r} = \frac{1}{2(2r+1)} N(q_1, q_2, m_1, m_2, 2r+1)$
- $c_q = 0$
- wedge subalgebra $q = 1, 3/2, 2, \ldots$ $1-q \le m \le q-1$
- $[V^{q_1, s_1}_{m_1}, V^{q_2, s_2}_{m_2}] = \sum_{r=0}^{\infty} \frac{\lambda^{2r}}{2(2r+1)!} N(q_1, q_2, m_1, m_2, 2r+1) \cdot V^{q_1+q_2-2r-2s, s_1+s_2-2r-2}_{m_1+m_2}$

OUR ALGEBRA $s = 0, \pm 2$

Implications for Amplitudes 1

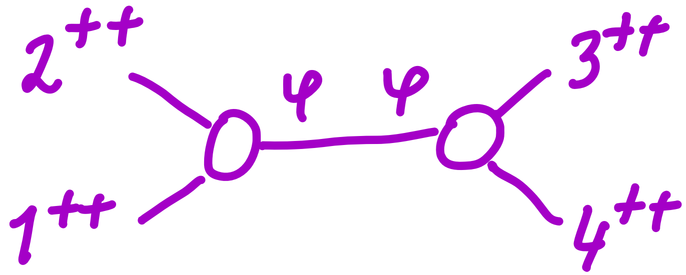
$A(1^{++}, 2^{++}, 3^{++}, 4^{++}) =$

Broedel, Dixon



graviton exchange

+



scalar exchange

+ permutations

$$= \left(10\, x_{-2,2,2}\, x_{2,2,2} - 3\, x_{0,2,2}^{2}\right) \#$$

$= 0$ VANISHES IF WE IMPOSE OUR CONSTRAINT!

RSSV

Implications for Amplitudes 2

- Similarly $A(1^{++},2^{++},3^{++},4^{\varphi})=0$, $A(1^{+},2^{+},3^{+},4^{+})=0$, $A(1^{+},2^{+},3^{+},4^{\varphi})=0$ on support of our constraints

- We showed that all (all-line shift constructible) four-point amplitudes vanish on the support of our constraints

$$\Updownarrow$$

- These amplitudes are constructed from purely holomorphic/antiholomorphic vertices.

Implications for Amplitudes 3

- Higher-point amplitudes

$$A(1^+, 2^+, 3^+, 4^+, 5^+) \neq 0$$

- all-line shift constructible
- has a term involving $A(1^+, 2^+, 3^+) A(1^-, 2^+, 3^+, 4^+)$

includes holomorphic + antiholomorphic vertices

- All-line shift constructible vanish only if they don't receive contributions from any channels other than those involving only pure holomorphic / antiholomorphic vertices as building blocks

OPE associativity

RSSV

- We can derive the coupling constraints directly from the associativity of the OPEs (w/o W-algebra)

$$\left(\operatorname*{Res}_{2\to3}\operatorname*{Res}_{1\to2} - \operatorname*{Res}_{1\to3}\operatorname*{Res}_{2\to3} + \operatorname*{Res}_{2\to3}\operatorname*{Res}_{1\to3}\right)\langle \mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_n}(z_n,\bar{z}_n)\rangle = 0$$

- The correlator is Mellin transform of the amplitude.
- We can check associativity of celestial OPE by evaluating double residues directly on the amplitude.

$$\sum_{s_I} A_3(\tilde{\lambda}_1^{s_1}, \tilde{\lambda}_2^{s_2}, \hat{\tilde{\lambda}}_I^{-s_I})\frac{1}{[12][34]}A_3(\hat{\tilde{\lambda}}_I^{s_I}, \tilde{\lambda}_3^{s_3}, \tilde{\lambda}_4^{s_4}) + \text{permutations} = 0$$

- This immediately gives coupling constraints equations.

The following are equivalent:

1. Soft Jacobi/double residue condition (wedge part)
2. Hard celestial double residue condition
3. Hard momentum space double residue condition
4. $\nexists$ 4-pt amplitudes w/ $\underbrace{\text{angle bracket weight } -1}_{\text{e.g. } \frac{[12]\langle 12\rangle}{\langle 13\rangle^2}}$

Purely an amplitudes statement.
Ready to use straight out of box!

Adam Ball arXiv:2211.09151

Self-dual gravity

- In quantum self-dual gravity $w_{1+\infty}$ algebra persists w/o corrections (one-loop exact all-plus amplitudes Bern et al.)
  Ball, Narayanan, Salzer, Strominger
- Moyal deformation of self-dual gravity leads to $W_{1+\infty}$ algebra Monteiro
  Bu, Heuveline, Skinner
- Celestial $w_{1+\infty}$ symmetries from twistor space
  Adamo, Mason, Sharma
- Celestial dual of self-dual YM form factors don't have associative chiral algebra unless one adds an 'axion'
  Costello, Paquette

Summary

- worked out deformations of $w_{1+\infty}$ due to non-minimal couplings
- Jacobi identity / OPE associativity implies constraints on coupling
- worked out implifications of these constraints on amplitudes

What theories satisfy our constraints ?

- Chiral higher-spin theory

$$\kappa_{s_1,s_2,s_3} \sim \frac{(l_P)^{s_1+s_2+s_3-1}}{\Gamma(s_1+s_2+s_3)} \qquad s_1+s_2+s_3>0$$

Metsaev '91

- String theory compactified to 4d?