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Black Holes and Massive Higher Spins

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Based on: **2107.14779, 2212.06120, 2301.xxxxx**

December 16, 2022

Content

2107.14779 (M Chiodaroli, H Johansson, **PP**)
2212.06120, 2301.xxxxx (L Cangemi, M Chiodaroli, H Johansson, A Ochirov, **PP**, E Skvortsov)

Review: Three-point

Kerr energy-momentum tensor contracted into on-shell graviton [Vines (2017)]

$$\varepsilon_{\mu\nu}(k)\, T^{\mu\nu}_{\mathrm{BH}}(k) = (\varepsilon_k \cdot p)^2 \exp\left(k_\mu S^\mu_{\mathrm{BH}}\right), \quad S^\mu_{\mathrm{BH}} = (0, 0, 0, ma)$$

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$$S^\mu(p) \equiv \langle \frac{1}{2m} \epsilon^{\mu\nu\rho\sigma} p_\nu M_{\rho\sigma} \rangle = \frac{1}{m} \langle \hat{W}^\mu \rangle$$

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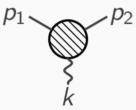
Massive spinning three-point amplitude: [Arkani-Hamed, Huang, Huang (2017)]



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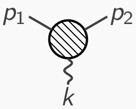

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[Guevara, Ochirov, Vines (2018); Huang,...(2018); Guevara, Bautista(2019); ...]

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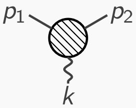

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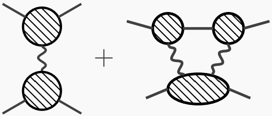
[Guevara, Ochirov, Vines (2018); Huang,...(2018); Guevara, Bautista(2019); ...]

Remarkable: Kerr three-point amplitude for $s \rightarrow \infty$

Classical observables in terms of amplitudes:

[Bjerrum-Bohr,...; Guevara,...; Haddad,...; Huang,...; Kosower,...; Luna,...; Mogull,...; O'Connell,..; ...]

$\Delta\mathcal{O} \sim$  $+ \dots$

Review: Compton

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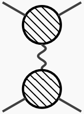
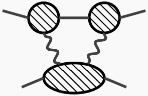
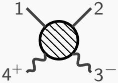
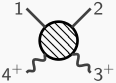
$\Delta\mathcal{O} \sim$  $+$  $+ \ldots$

Compton amplitude needed at NLO. BCFW results: [Arkani-Hamed,...;Johansson,...]



$$= \frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}(s_{13}-m^2)(s_{14}-m^2)}$$



$$= \frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}(s_{13}-m^2)(s_{14}-m^2)}$$

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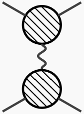
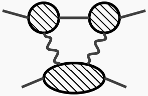
$\Delta\mathcal{O} \sim$  $+$  $+ \dots$

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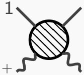

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Remarkable: $s \leq 2$ matches Kerr, **BUT** $s > 2$ has spurious pole

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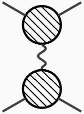
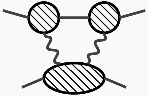
$\Delta\mathcal{O} \sim$  $+$  $+\ldots$

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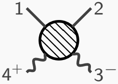

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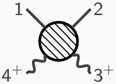

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Meaning: BCFW does not work for higher spin.

Recap

What we know: (potentials)

- $\mathcal{O}(G^2)$: Hamiltonian up to $\mathcal{O}(S^4)$, $\mathcal{O}(S^{>5})$ conjectured

  [Guevara, Ochirov, Vines; Luna, Kosmopoulos; Porto, Liu; Chen, Chung, Huang, Kim; Aoude, Haddad, Helset; Bern, Kosmopoulos, Luna, Roiban, Teng]

- $\mathcal{O}(G^3)$: results at $\mathcal{O}(S^2)$, $\mathcal{O}(S^\infty)$ radiation-reaction

  [Jakobsen, Mogull; Cordero, Kraus, Lin, Ruf, Zeng] [Alessio,Di Vecchia]

- probe limit to $\mathcal{O}(S^\infty)$ [Menezes,Sergola; Damgaard,Hoogeveen,Luna,Vines]

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Open questions:

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? Higher-spin theories that give rise to the Kerr amplitudes?

$$m^{-2s}(\varepsilon_k \cdot p_1)^2 [\mathbf{12}]^{2s}$$

Generating function for root-Kerr amplitudes:

$$\sum_{s=0}^{\infty} m^{-2s}(\varepsilon_k \cdot p_1)[\mathbf{12}]^{2s} = A_{\phi\phi A} + \frac{A_{WWA} - (\varepsilon_1 \cdot \varepsilon_2)^2 A_{\phi\phi A}}{(1+\varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1 \cdot p_2\, \varepsilon_2 \cdot p_1}$$

Low Spin
Gauge Theory

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Extract Lagrangians:

- $s = 0, 1/2$ minimally-coupled

$$\mathcal{L}^{s=0} = \overline{D_\mu \phi} D^\mu \phi - m^2 \bar{\phi}\phi$$

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! **Lesson:** high-energy behaviour important

Low Spin
Gravity

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$$\begin{aligned}\mathcal{L}^{s=2} \;=\; & \nabla_\mu\overline{H}_{\nu\rho}\nabla^\mu H^{\nu\rho} - 2\nabla_\nu\overline{H}^\nu_\mu\nabla^\rho H^\mu_\rho - \overline{H}^\rho_\rho\nabla_\mu\nabla_\nu H^{\mu\nu} \\ & - H^\rho_\rho\nabla_\mu\nabla_\nu\overline{H}^{\mu\nu} - \nabla_\mu\overline{H}^\nu_\nu\nabla^\mu H^\rho_\rho - m^2\overline{H}_{\mu\nu}H^{\mu\nu} \\ & + m^2\overline{H}^\mu_\mu H^\nu_\nu - 2R^{\mu\nu\rho\sigma}\overline{H}_{\mu\rho}H_{\nu\sigma}\end{aligned}$$

High-Energy Unitarity
Example: s = 1 gauge

Massive spin-1 field coupled to photon:

$$\mathcal{L} = 2D_{[\mu}\overline{W}_{\nu]}D^{[\mu}W^{\nu]} - m^2\overline{W}_\mu W^\mu + ie\alpha F_{\mu\nu}\overline{W}^\mu W^\nu$$

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Compton Ingredients

Three-point current (P off-shell, p_1 and k on-shell; $f_k \equiv k\varepsilon_k - \varepsilon_k k$):

$$J(P,p_1,k) = \varepsilon_P\cdot\varepsilon_1\,\varepsilon_k\cdot(p_1-P) - \varepsilon_1\cdot\varepsilon_k\,\varepsilon_P\cdot p_1 + \varepsilon_k\cdot\varepsilon_P\,\varepsilon_1\cdot P - \alpha\, f_k^{\mu\nu}\varepsilon_{1\mu}\varepsilon_{P\nu}$$

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$$\Delta_{\mu\nu}(P) = \frac{1}{P^2 - m^2}\left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{m^2}\right)$$

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root-Kerr three-point and Compton matched!

Massive spin-3/2 field coupled to photon: [Deser,Waldron,Pascalutsa(2000)]

$$\mathcal{L} = \bar{\psi}^{\mu}\gamma_{\mu\nu\rho}\Big(iD^{\nu} - \frac{1}{2}m\gamma^{\nu}\Big)\psi^{\rho} - \frac{ie}{m}\Big(l_1\bar{\psi}_{\mu}F^{\mu\nu}\psi_{\nu} + l_2\bar{\psi}_{\mu}F_{\rho\sigma}\gamma^{\rho}\gamma^{\sigma}\psi^{\mu}$$

$$+ l_3 F^{\mu\nu}(\bar{\psi}_{\mu}\gamma_{\nu}\gamma\cdot\psi + \bar{\psi}\cdot\gamma\gamma_{\mu}\psi_{\nu}) + l_4\bar{\psi}\cdot\gamma F_{\rho\sigma}\gamma^{\rho}\gamma^{\sigma}\gamma\cdot\psi + i\,l_5 F^{\mu\nu}(\bar{\psi}_{\mu}\gamma_{\nu}\gamma\cdot\psi - \bar{\psi}\cdot\gamma\gamma_{\mu}\psi_{\nu})\Big)$$

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Spin-3/2 propagator:

$$\Delta^{\mu\nu}(P) \sim \left(\eta^{\mu\nu} - \frac{P^\mu P^\nu}{m^2}\right)(\not{P} + m) + \frac{1}{3}\left(\frac{P^\mu}{m} + \gamma^\mu\right)(\not{P} - m)\left(\frac{P^\nu}{m} + \gamma^\nu\right)$$

High-Energy Unitarity
s = 3/2 gauge

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Impose current constraint

$$P\cdot J = \mathcal{O}(m) \Rightarrow l_1 = -2,\ l_2 = 1/2,\ l_3 = 1,\ l_5 = 0$$

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root-Kerr three-point matched!

Spin-3/2 Gauge Theory

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_\mu \left(F^{\mu\nu} - \frac{i}{2}\gamma_5 \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \right) \psi_\nu$$

$$\mathcal{A}_4 = \frac{[4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle}{[4|p_1|3\rangle} \left(\frac{([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^2}{(s_{13}-m^2)(s_{14}-m^2)} - \frac{[\mathbf{1}4][\mathbf{2}4]\langle \mathbf{1}3\rangle\langle \mathbf{2}3\rangle}{m^4} \right)$$

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Spin-5/2 Gravity

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_{\mu\rho} \left(R^{\mu\nu\rho\sigma} - \frac{i}{2}\gamma_5 \epsilon^{\rho\sigma\alpha\beta} R^{\mu\nu}{}_{\alpha\beta} \right) \psi_{\nu\sigma}$$

$$\mathcal{M}_4 = \frac{[\mathbf{41}]\langle\mathbf{32}\rangle + [\mathbf{42}]\langle\mathbf{31}\rangle}{[4|p_1|3\rangle} \left(\frac{([\mathbf{41}]\langle\mathbf{32}\rangle + [\mathbf{42}]\langle\mathbf{31}\rangle)^4}{s_{12}(s_{13}-m^2)(s_{14}-m^2)} - \frac{([\mathbf{14}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle)^2}{m^6} \right)$$

! $P \cdot J = \mathcal{O}(m)$ not enough for $s \geq 2$ gauge / $s \geq 3$ gravity

- ! $P \cdot J = \mathcal{O}(m)$ not enough for $s \geq 2$ gauge / $s \geq 3$ gravity
- ? What are we missing?

UV theory:

$$\mathcal{L} = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \frac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i + \frac{\mu^2}{2}\|\vec{\phi}\|^2 - \frac{\lambda}{4!}\|\vec{\phi}\|^4$$

Massive Gauge Symmetry

Spontaneous Symmetry Breaking

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Spontaneous Symmetry Breaking (Goldstone φ):

$$\begin{aligned} \mathcal{L} =& 2\overline{D_{[\mu} W_{\nu]}}\, D^{[\mu} W^{\nu]} + (m\overline{W}_\mu - \overline{D_\mu \varphi})\,(mW^\mu - D^\mu \varphi) \\ &+ ieF_{\mu\nu} \overline{W}^\mu W^\nu + \{\text{Higgs}, W^4, \ldots\} \end{aligned}$$

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Massive (Stückelberg) gauge symmetry

$$W_\mu \to (W_\mu, \varphi) \quad \text{with} \quad \delta W_\mu = D_\mu\lambda,\ \delta\phi = m\lambda + \ldots$$

Symmetry = no unphysical degrees of freedom

Massive Gauge Symmetry
Spontaneous Symmetry Breaking

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? Higher-spins? $\Rightarrow$ no SSB, yes gauge symmetry!

Fields $\{H_{\mu\nu}, B_\mu, \varphi\}$. Gauge symmetry:

$$\delta H_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu + m g_{\mu\nu} \lambda + \dots,$$
$$\delta B_\mu = D_\mu \lambda + m \xi_\mu + \dots, \delta\varphi = m\lambda + \dots.$$

Massive Gauge Symmetry
Spin-2 Example

Fields $\{H_{\mu\nu}, B_\mu, \varphi\}$. Gauge symmetry:

$$\delta H_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu + m g_{\mu\nu} \lambda + \ldots,$$
$$\delta B_\mu = D_\mu \lambda + m \xi_\mu + \ldots, \delta\varphi = m\lambda + \ldots.$$

Non-minimal Lagrangian (lowest derivative):

$$\frac{-ie}{m^2} F_{\mu\nu} \Big(\overline{D_\mu H_{\alpha\beta}} D_\alpha H_{\beta\nu} - \overline{D_\alpha H_{\beta\nu}} D_\mu H_{\alpha\beta} + \overline{D_\alpha H_{\beta\mu}} D_\alpha H_{\beta\nu}$$
$$-2\overline{D_\alpha H_{\beta\mu}} D_\beta H_{\alpha\nu} - \overline{D_\mu H_{\alpha\beta}} D_\nu H_{\alpha\beta} + \overline{D_\mu H_{\nu\alpha}} D \cdot H_\alpha$$
$$- \overline{D \cdot H_\alpha} D_\mu H_{\nu\alpha} + \overline{D \cdot H_\mu} D \cdot H_\nu - \overline{D \cdot H_\mu} D_\nu H$$
$$+ \overline{D_\nu H} D \cdot H_\mu - \overline{D_\mu H_{\nu\alpha}} D_\alpha H + \overline{D_\alpha H} D_\mu H_{\nu\alpha} + \overline{D_\mu H} D_\nu H \Big)$$
$$\frac{-ie}{m^2} F_{\mu\nu} \Big\{ m \Big(\overline{D \cdot H_\mu} B_\nu - \overline{B}_\nu D \cdot H_\mu - \overline{D_\mu H} B_\nu + \overline{B}_\nu D_\mu H \Big) \Big\}$$
$$\frac{-ie}{m^2} F_{\mu\nu} \Big(\frac{m^2}{2} \overline{H}_{\mu\alpha} H_{\nu\alpha} + 2m^2 \overline{B}_\mu B_\mu \Big)$$

$\Rightarrow$ root-Kerr amplitude!

Massive Gauge Symmetry

Massive Ward identities

Massive spin-1:

$$\delta W_\mu = \partial_\mu \lambda, \delta\varphi = m\lambda$$

$$\left(i p_1 \cdot \frac{\partial}{\partial \varepsilon_1} \mathcal{A}_3(WWA) + m\mathcal{A}_3(\varphi WA) \right) \Big|_{(2,3)} = 0, \ \ (i,j) \text{ on-shell}$$

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Massive spin-2:

$$\delta H_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + m g_{\mu\nu} \lambda,$$
$$\delta B_\mu = \partial_\mu \lambda + m \xi_\mu, \delta\varphi = m\lambda$$

$$\left(i p_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(HHA) + m \mathcal{A}_3(BHA) \right) \Big|_{(2,3)} = 0$$
$$\left(i p_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(BHA) + m \mathcal{A}_3(\varphi HA) + \frac{m}{2} \frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(HHA) \right) \Big|_{(2,3)} = 0$$

Massive Gauge Symmetry

Arbitrary Spin

Arbitrary spin:

$$\langle \xi^k \bar{\Phi}^s A^h \rangle \equiv \frac{1}{k+1} p_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{L}(\Phi^{k+1} \bar{\Phi}^s A^h) + m \alpha_k \mathcal{L}(\Phi^k \bar{\Phi}^s A^h)$$
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Massive Gauge Symmetry
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- Lowest-derivative solution:

$$\mathcal{L}(\Phi^{s_1} \bar{\Phi}^{s_2} A^h) \sim \partial^{s_1 + s_2 - h}, \quad \mathcal{L}(\Phi^{s_1} \bar{\Phi}^{s_2 < s_1 - h} A^h) = 0$$

Massive Gauge Symmetry
Arbitrary Spin

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Outcome: Kerr and root-Kerr 3pt amplitude **unique** for any spin!
Checked up to $s = 6$

Massive Gauge Symmetry

Compton

Four-point Ward identities (spin-2 example):

$$ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_{\Phi\bar{\Phi}AA} + m\mathcal{A}_{B\bar{\Phi}AA} = 0$$

$$ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_{B\bar{\Phi}AA} + m\mathcal{A}_{\varphi\bar{\Phi}AA} + \frac{m}{2}\left(\frac{\partial}{\partial \epsilon_1}\right)^2 \mathcal{A}_{\Phi\bar{\Phi}AA} = 0$$

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Gauge theory Compton

$$A(\Phi_1^s \Phi_2^s A_3^- A_4^+) = \frac{\langle 3|1|4]^2 (T+S)^{2s}}{m^{4s} t_{13} t_{14}} + \frac{\langle 3|1|4] \langle \mathbf{1} 3 \rangle [\mathbf{2} 4] P_{2s}}{m^{4s} t_{13}} + \langle \mathbf{1} 3 \rangle \langle 3 \mathbf{2} \rangle [\mathbf{1} 4][4 \mathbf{2}] \frac{P_{2s-1}}{m^{4s}} + C_s,$$

Massive Gauge Symmetry
Compton

Four-point Ward identities (spin-2 example):

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Gauge theory Compton

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Contact term $C_{s\leq 1}=0$,

$$C_2=\frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^6}\left\{x(\langle\mathbf{12}\rangle+[\mathbf{12}])^2+y(\langle\mathbf{12}\rangle-[\mathbf{12}])^2\right\}+\mathcal{O}(\hbar)$$

Summary and Outlook

- Complete understanding of root-Kerr and Kerr cubic theory
- On-shell realisation of massive gauge symmetry $\Rightarrow$ 4pt and higher!

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! Complete understanding of root-Kerr and Kerr cubic theory

! On-shell realisation of massive gauge symmetry $\Rightarrow$ 4pt and higher!

? More constraints? e.g. 4pt current constraint, off-shell gauge invariance

? Higher-derivative terms $\rightarrow$ neutron stars?

? Apply methods to other massive spinning QFTs?

? Simpler description of massive gauge invariance? (in progress)

Thank you!