Gravitational bremsstrahlung from a heavy-mass effective field theory

To appear with-
Andreas Brandhuber, Gang Chen, Stefano De Angelis, Joshua Gowdy and Gabriele Travaglini.

Graham R. Brown

Queen Mary University of London

13.12.2022-QCD Meets Gravity

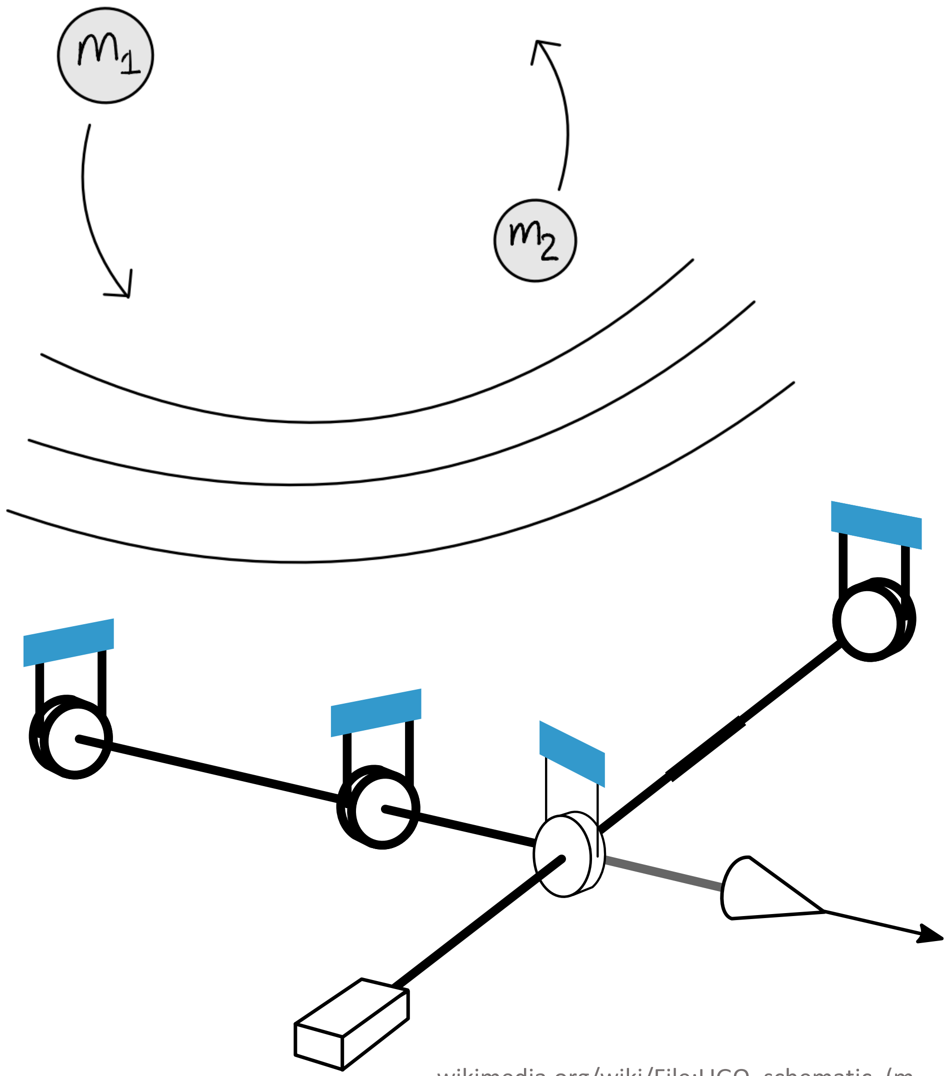
Talk Outline:

- Motivation- gravitational waves.

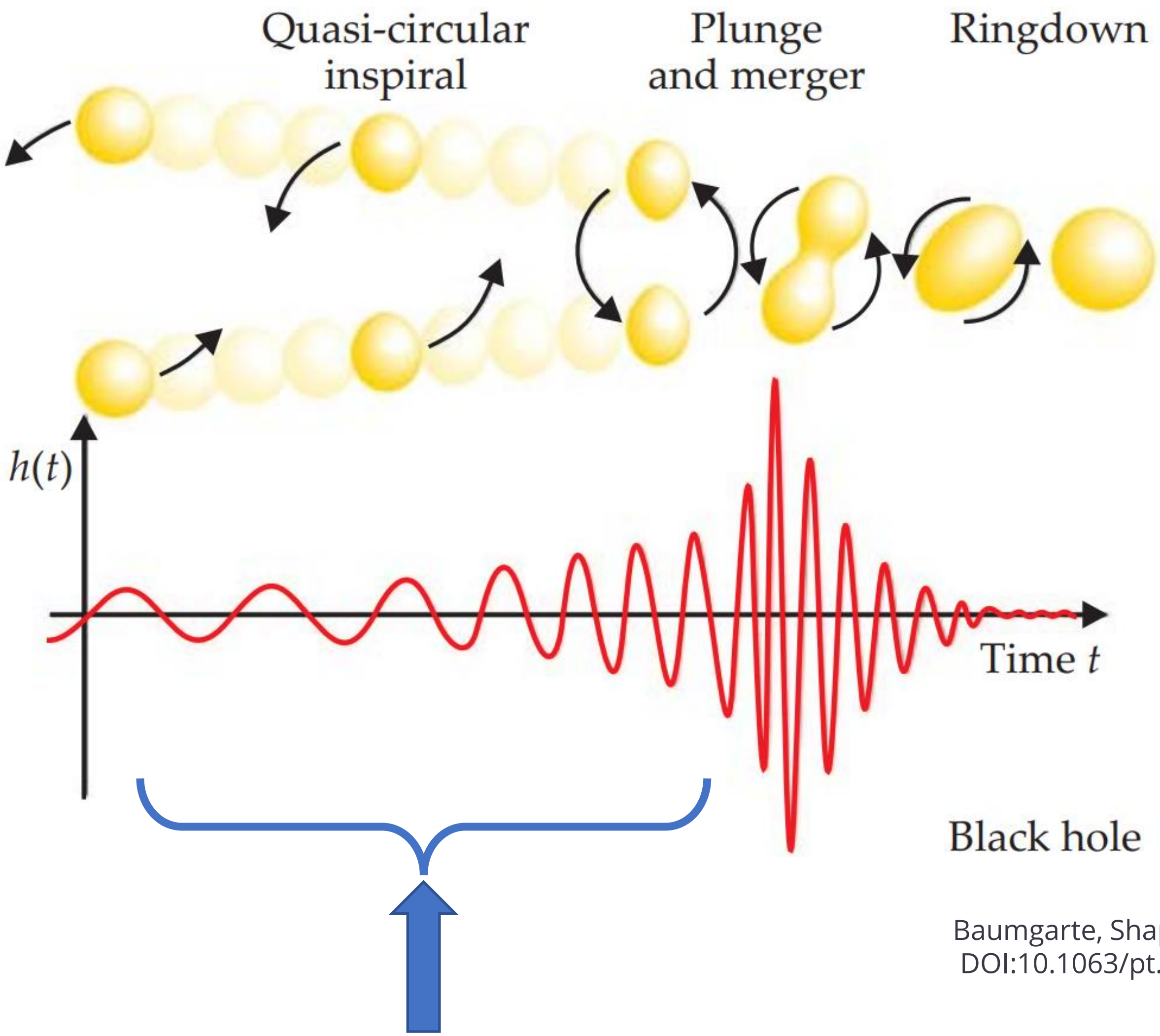
- Everything you always wanted to know about HEFT, but were afraid to ask.

- Using the HEFT to build at 4 and 5-point amplitudes.

Compulsory slide in any GW talk



wikimedia.org/wiki/File:LIGO_schematic_(multilang).svg



Baumgarte, Shapiro, DOI:10.1063/pt.3.1294

We focus on the inspiral.

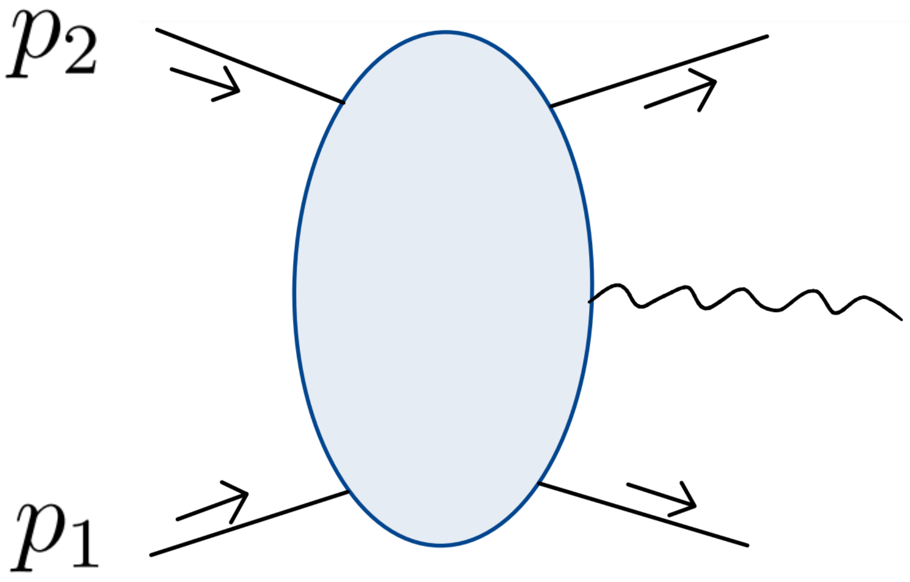
How to approach perturbation theory?

Solve perturbatively in G_N .

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Vs.

Construct amplitudes perturbatively in G_N .



(See talks today and Friday)

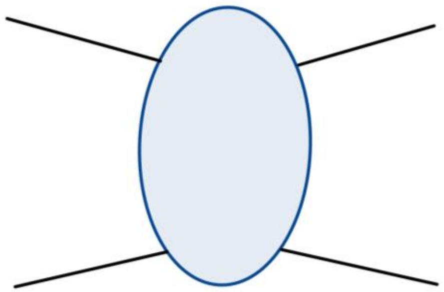
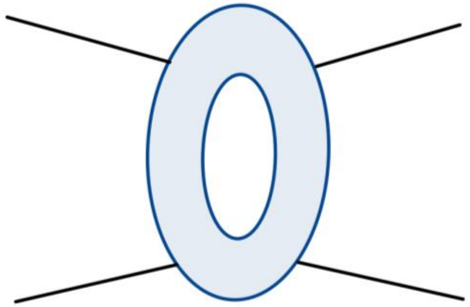
Basic Idea: Non-spinning black holes only have one parameter, their mass m . So describe them by a scalar field:

$$\mathcal{L} = \int d^D x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \sum_{i=1,2} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - m_i^2 \phi_i^2 \right) \right)$$

Then calculate amplitudes:

Loops contain classical information

Iwasaki 1971

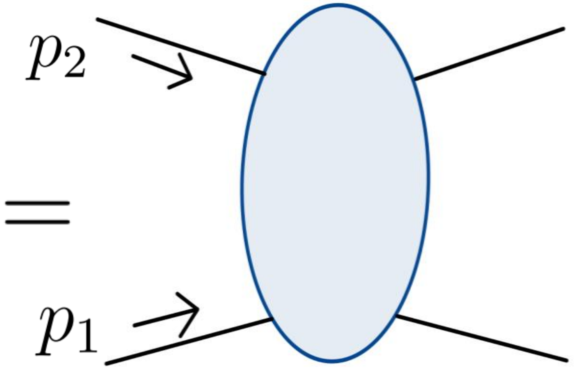
$\mathcal{A} =$  $+$  $+ \quad \ldots$

G_N $\qquad$ G_N^2

From quantum amplitudes to Classical Observables

Full amplitude

$$\mathcal{A} =$$



Extract classical part

Expand as $\hbar \to 0$

$$\mathcal{A}_{Cl}$$

Match to a Hamiltonian:

$$H(p,r) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} - \frac{G_N m_1 m_2}{r} + \mathcal{O}(G_N^2)$$

Find an Eikonal-like phase:

$$\mathcal{A}_{Cl} = e^{i\delta_0 + i\delta_1 + \ldots} - 1$$

KMOC formalism:

Kosower, Maybee, O'Connell 2018

⋮

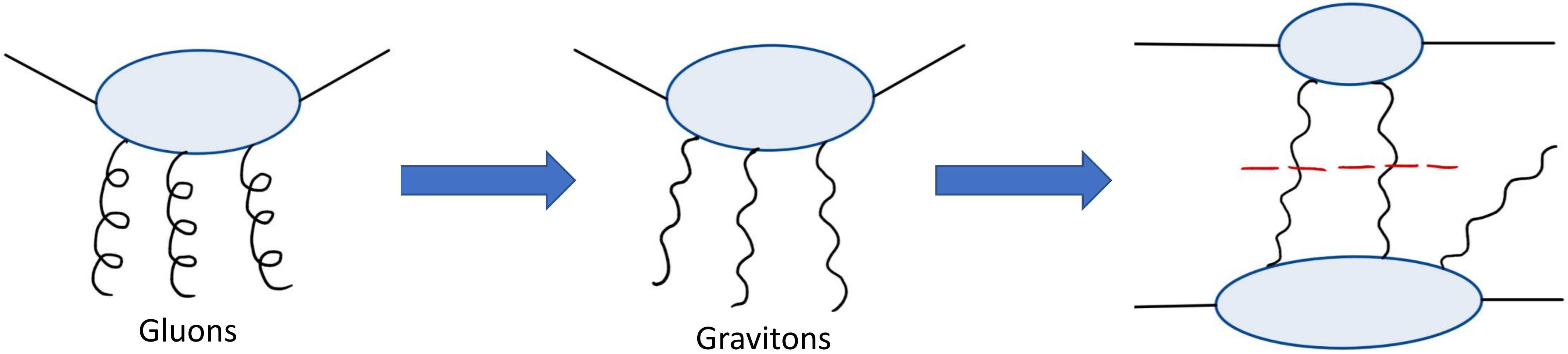
How to calculate Gravity amplitudes?

The Mantra: Recycle previous results

1. Find Tree level Amplitudes in Yang-Mills.

2. Double copy to tree level gravity amplitudes.

3. Glue tree level amplitudes together using unitarity cuts.

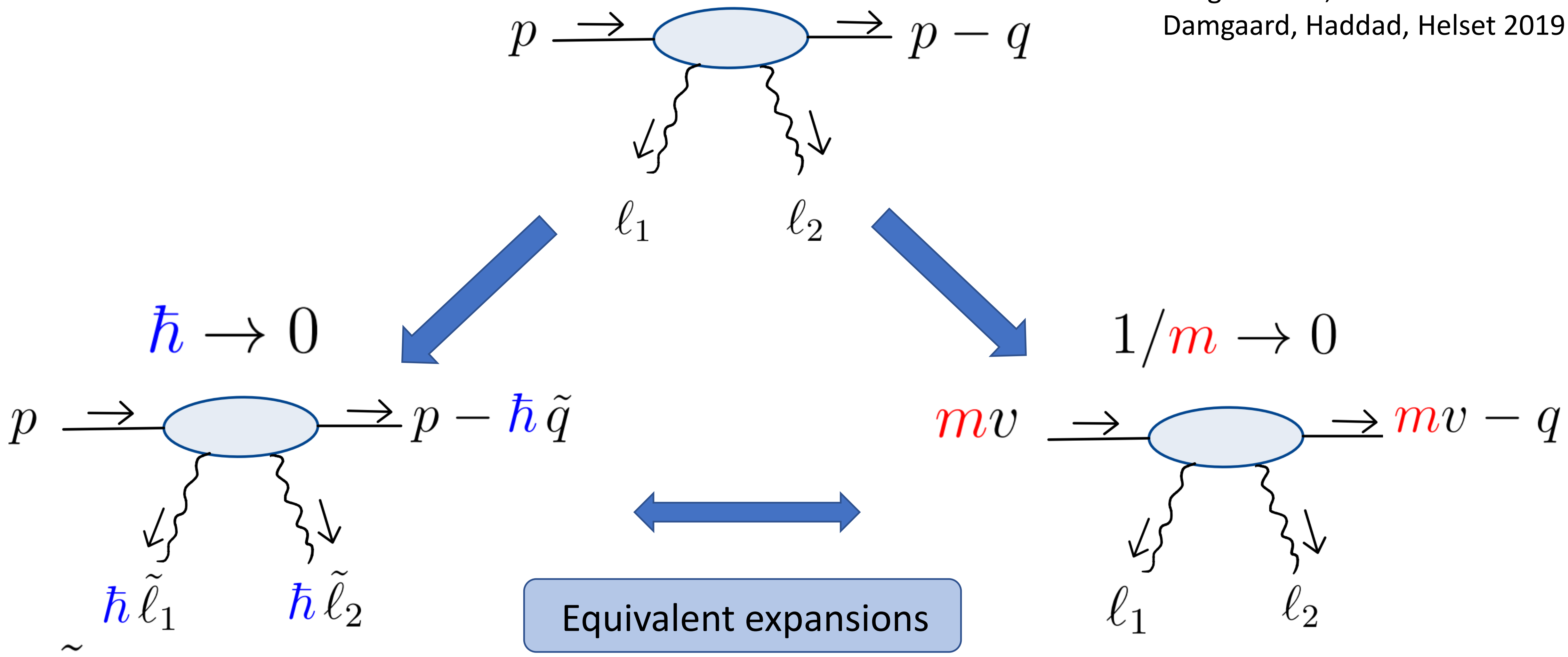


HEFT makes these steps easy!

Enter the “Heavy-mass Effective Field Theory” (HEFT)

Brandhuber, Chen, Travaglini, Wen 2021 x2 +Johansson 2021

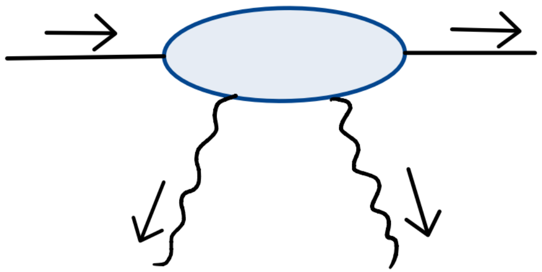
Gregori 1990,
Damgaard, Haddad, Helset 2019



$\tilde{\ell}_i$ = Wavenumber

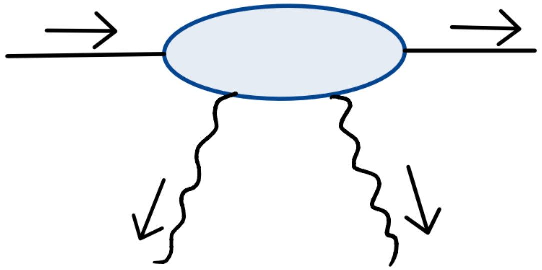
A technical point… I lied

The HEFT is actually an expansion as $1/\bar{m}_i \to 0\,,\quad \bar{m} = \sqrt{m^2 - q^2/4} = m + \mathcal{O}(m^{-1})$

mv  $mv - q$

$$v^2 = 1$$

$$v \cdot q = \frac{q^2}{2m} = \mathcal{O}(m^{-1})$$

$\bar{m}\bar{v} + \frac{q}{2}$  $\bar{m}\bar{v} - \frac{q}{2}$

$$\bar{v}^2 = 1$$

Very useful!

$$\bar{v} \cdot q = 0$$

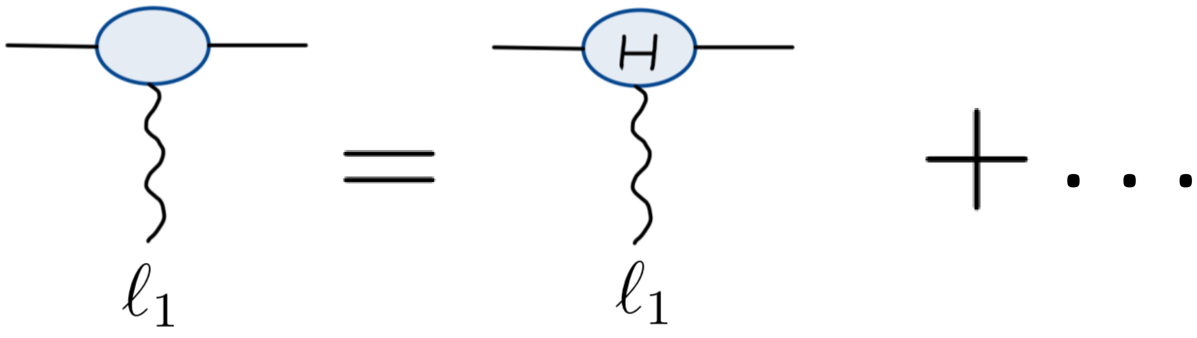
In the very end the power counting is the same.

$\bar{m} \to m$, $\bar{v} \to v$ as $m \to \infty$

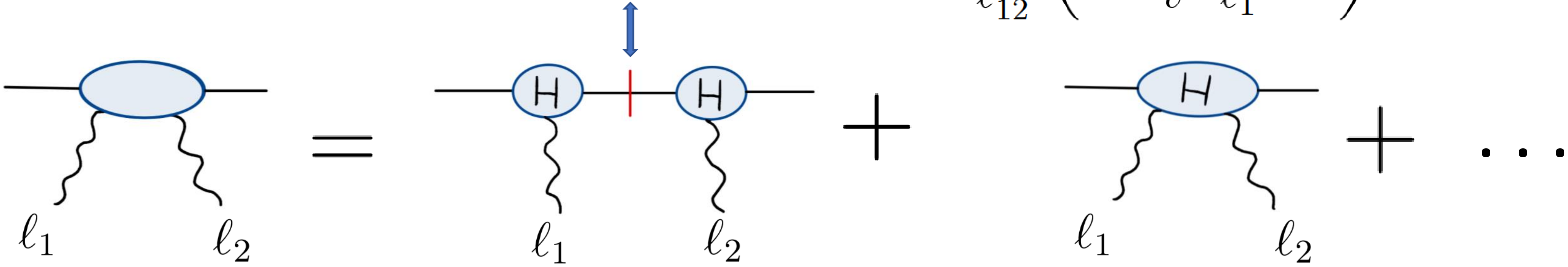
Some HEFT examples:

HEFT Amplitudes

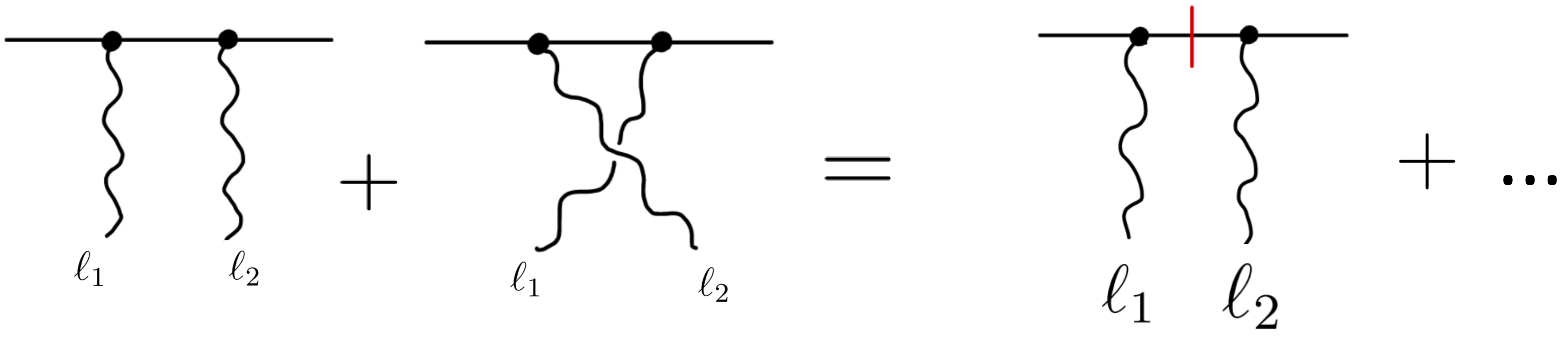
$$A_3^{\mathrm{GR}} = \bar{m}^2 (\bar{v} \cdot \varepsilon_1)^2 + \ldots$$



$$A_4^{\mathrm{GR}} = -(i\pi)\bar{m}^3 (\bar{v} \cdot \varepsilon_1)^2 \delta(\bar{v} \cdot \ell_1)(\bar{v} \cdot \varepsilon_2)^2 + \frac{\bar{m}^2}{\ell_{12}^2} \left(\frac{\bar{v} \cdot F_1 \cdot F_2 \cdot \bar{v}}{\bar{v} \cdot \ell_1} \right)^2 + \ldots$$



HEFT cuts

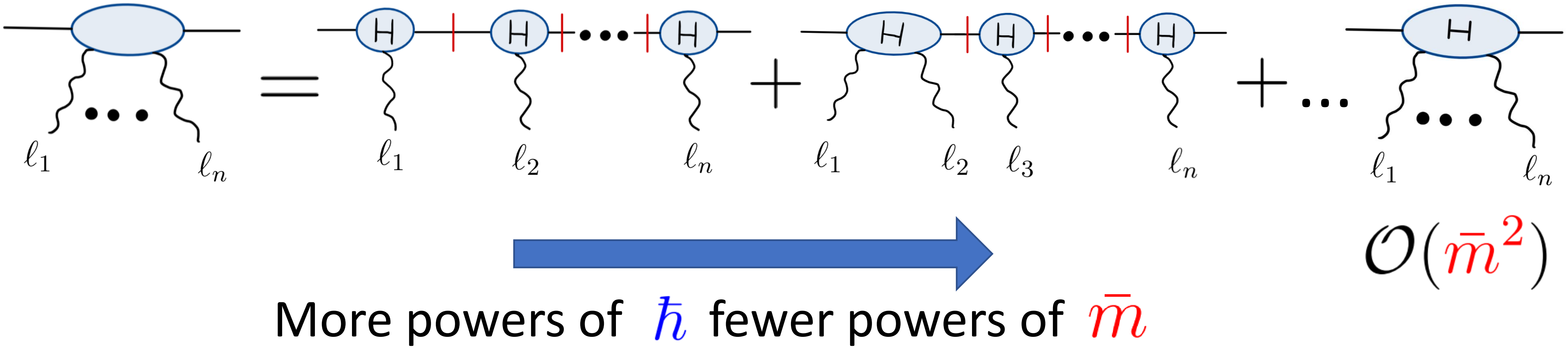


$$\frac{1}{-2\bar{m}\bar{v} - q\cdot\ell_1 + i\epsilon} + \frac{1}{2\bar{m}\bar{v} - q\cdot\ell_1 + i\epsilon} = -(i\pi)\delta(\bar{m}\bar{v}\cdot\ell_1) + \ldots$$

$$\frac{1}{x+i\epsilon} - \frac{1}{x-i\epsilon} = -2\pi i\delta(x)$$

$i\epsilon$ matters even at tree level!

The general structure:

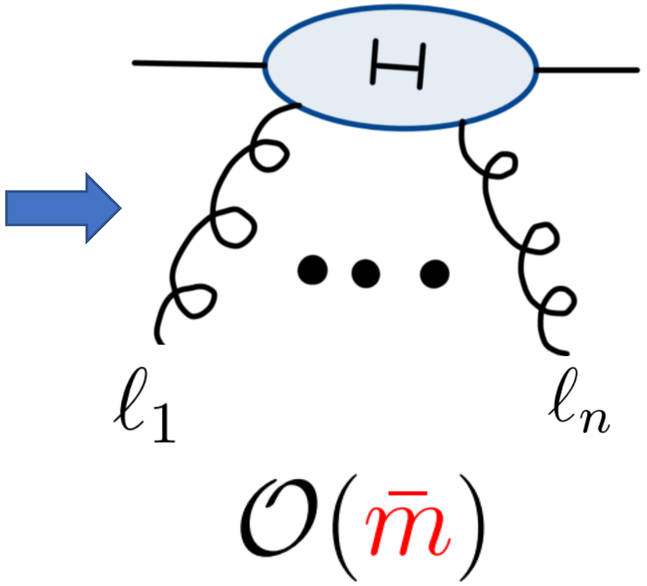


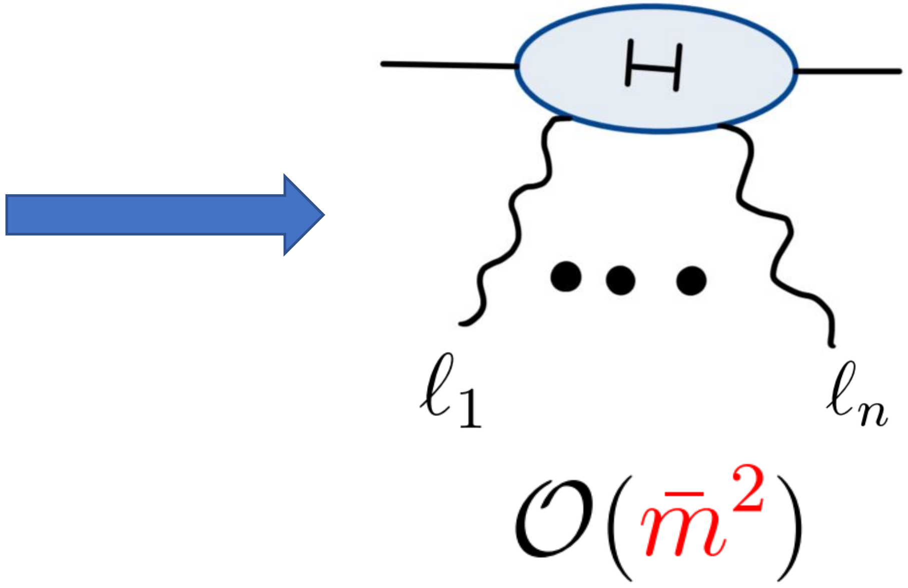
How do we build HEFT amplitudes?

1. Build YM HEFT amplitudes using algebra.

2. Double copy YM HEFT amplitudes to GR.

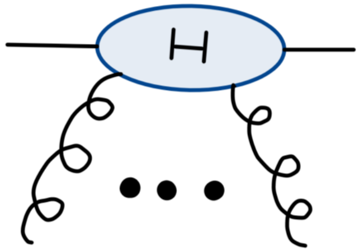
$$T_{(1)} \star T_{(2)} \star \cdots \star T_{(n)}$$



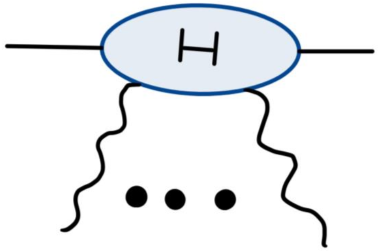


The double copy form of HEFT amplitudes

Brandhuber, Chen, Travaglini, Wen 2021 +Johansson 2021

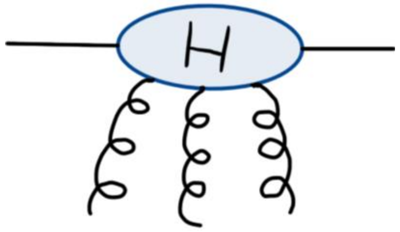


$$= \sum_{\Gamma \in \rho} \frac{\mathcal{N}(\Gamma, \bar{v})}{d_\Gamma}$$



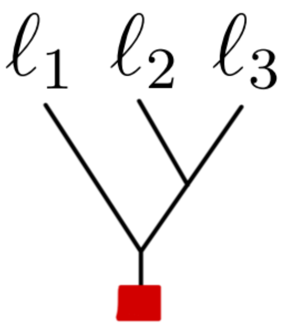
$$= \sum_{\Gamma \in \tilde{\rho}} \frac{[\mathcal{N}(\Gamma, \bar{v})]^2}{d_\Gamma}$$

$\rho(\tilde{\rho})$ Are the set of (un)ordered commutators of massless legs with leftmost entry fixed to 1.



$\ell_1 \quad \ell_2 \quad \ell_3$

$=$



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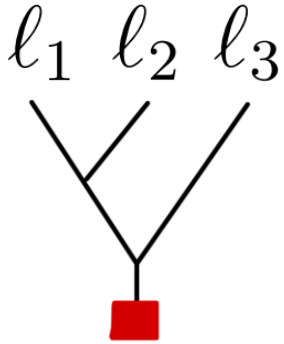


$$\frac{\mathcal{N}([1,[2,3]],\bar{v})}{\ell_{23}\ell_{123}} + \frac{\mathcal{N}([[1,2],3]],\bar{v})}{\ell_{12}\ell_{123}}$$

BCJ numerators from quasi-shuffles

Brandhuber, Chen, Johansson, Travaglini, Wen 2021

$$T_{(1)} \star T_{(2)} \star \cdots \star T_{(n)} \quad \Longrightarrow \quad \mathcal{N}(\Gamma, \bar{v})$$

For 2-gluons: $T_{(1)} \star T_{(2)} = T_{(1),(2)} + T_{(2),(1)} - T_{(12)}$

$$\mathcal{N}([1,2], \bar{v}) = 2\langle T_{(1)} \star T_{(2)} \rangle = \frac{\bar{v} \cdot F_1 \cdot F_2 \cdot \bar{v}}{\bar{v} \cdot \ell_1}$$

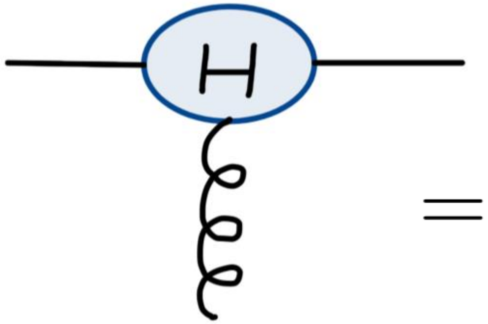
Key result:

An *n-point formula exists* for all HEFT BCJ numerators and hence amplitudes.

Some examples:

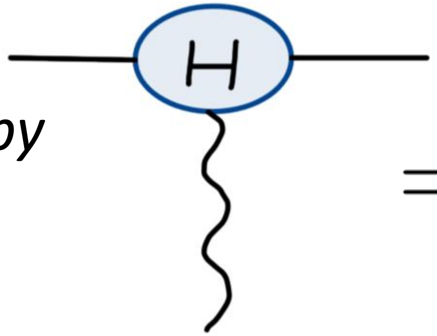
Yang-Mills

Gravity

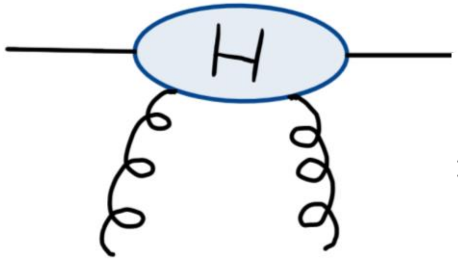


$$= \bar{m}(\bar{v} \cdot \varepsilon_1)$$

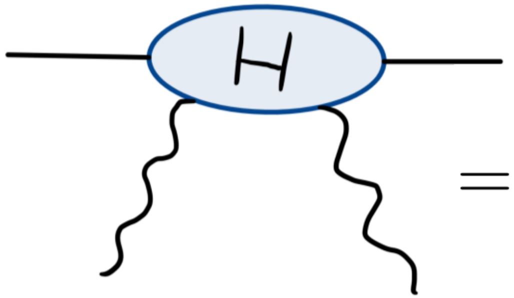
HEFT Double-Copy →



$$= \bar{m}^2(\bar{v} \cdot \varepsilon_1)^2$$



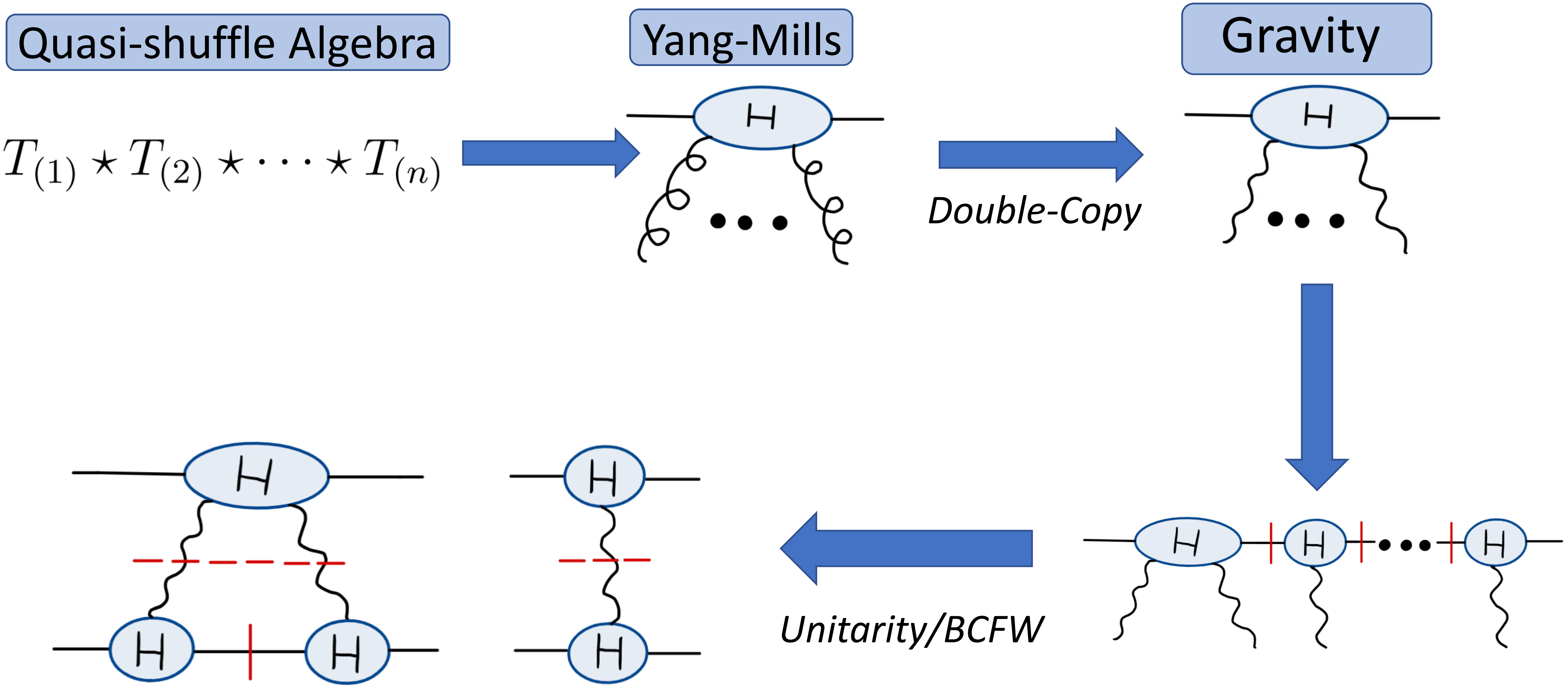
$$= \frac{\bar{m}}{\ell_{12}^2}\left(\frac{\bar{v} \cdot F_1 \cdot F_2 \cdot \bar{v}}{\bar{v} \cdot \ell_1}\right)$$



$$= \frac{\bar{m}^2}{\ell_{12}^2}\left(\frac{\bar{v} \cdot F_1 \cdot F_2 \cdot \bar{v}}{\bar{v} \cdot \ell_1}\right)^2$$

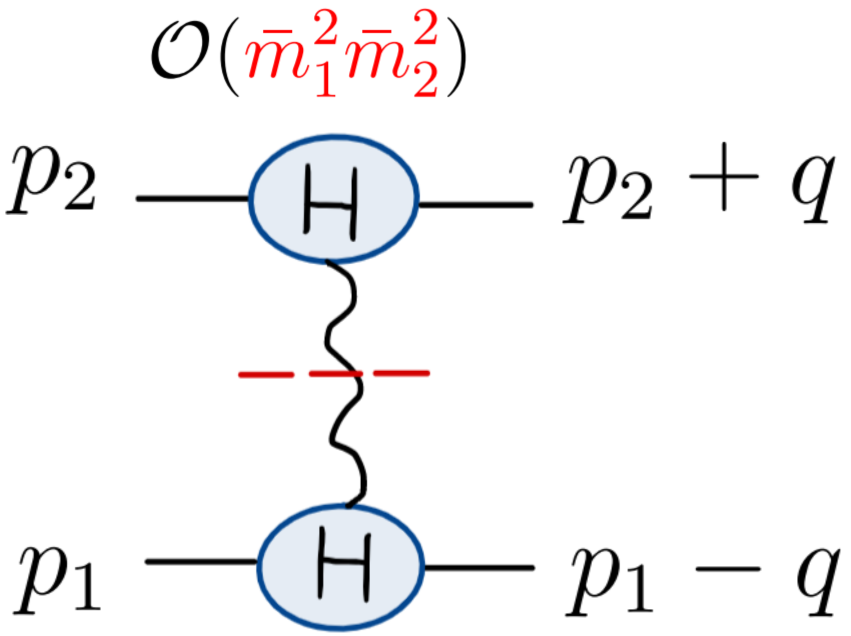
Manifestly gauge invariant: $F_i^{\mu\nu} = l_i^\mu \varepsilon_i^\nu - l_i^\nu \varepsilon_i^\mu$

The HEFT pipeline



Building amplitudes: four-point tree-level

$$\mathcal{A}_4^{\text{Tree}} =$$



A strange massive BCFW shift for HEFT:

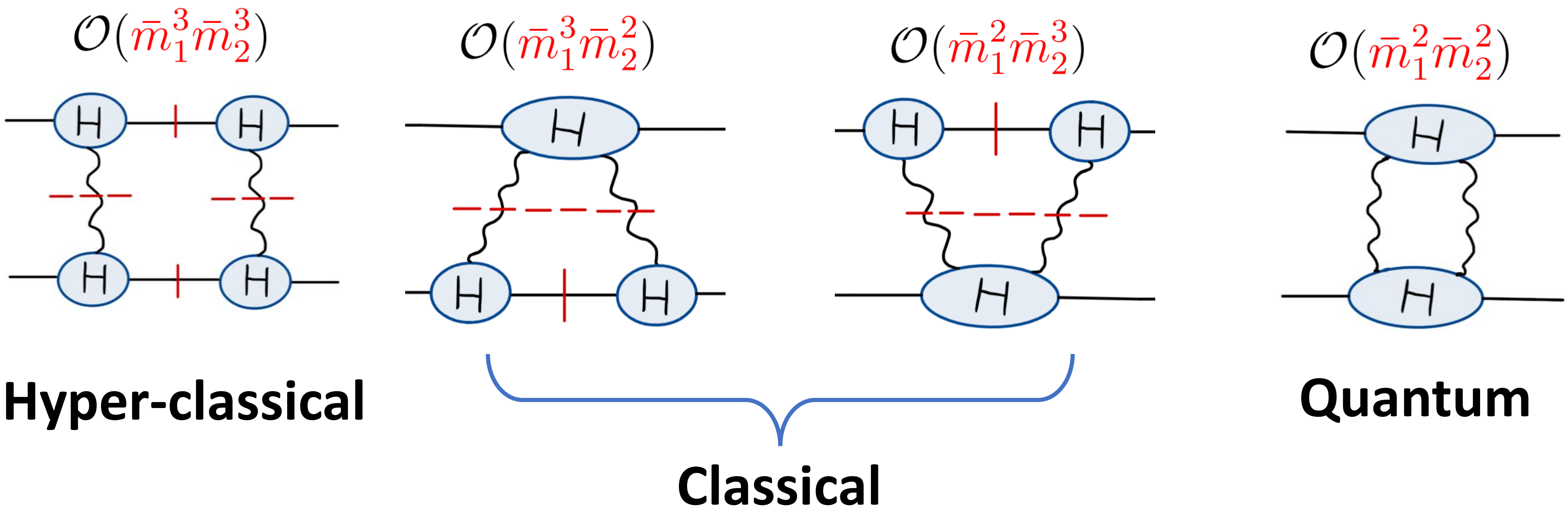
$$\hat{q}^\mu = q^\mu + z r^\mu$$

$$\bar{v}_i \cdot r = 0$$

$$r^2 = 0$$

Building amplitudes: four-point one-loop

Brandhuber, Chen, Travaglini, Wen 2021

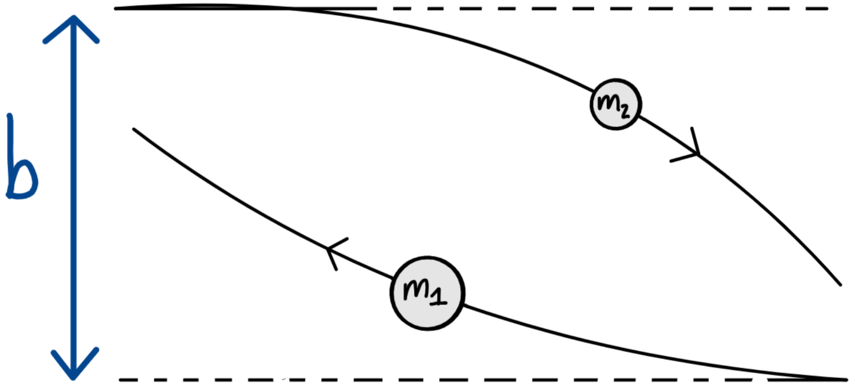


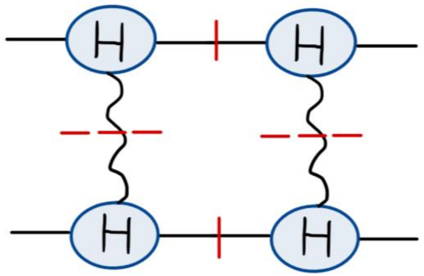
Quantum we can ignore but what about hyper-classical?

The four-point amplitude exponentiates in impact parameter space (Glauber; Levi & Sucher; ….; Amati, Ciafaloni & Veneziano; Kabat & Ortiz)

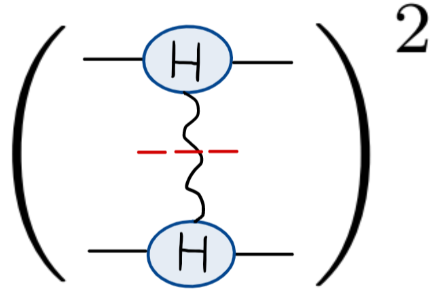
$$\tilde{\mathcal{A}}(b) = \int \frac{d^D q}{(2\pi)^{D-2}} \delta(\bar{v}_1 \cdot q)\delta(\bar{v}_2 \cdot q) e^{i\vec{q}\cdot\vec{b}} \mathcal{A}(q)$$

$$\tilde{S} = 1 + i\tilde{\mathcal{A}} = e^{i\delta_0^{\mathrm{HEFT}} + i\delta_1^{\mathrm{HEFT}} + \ldots}$$





Impact parameter space

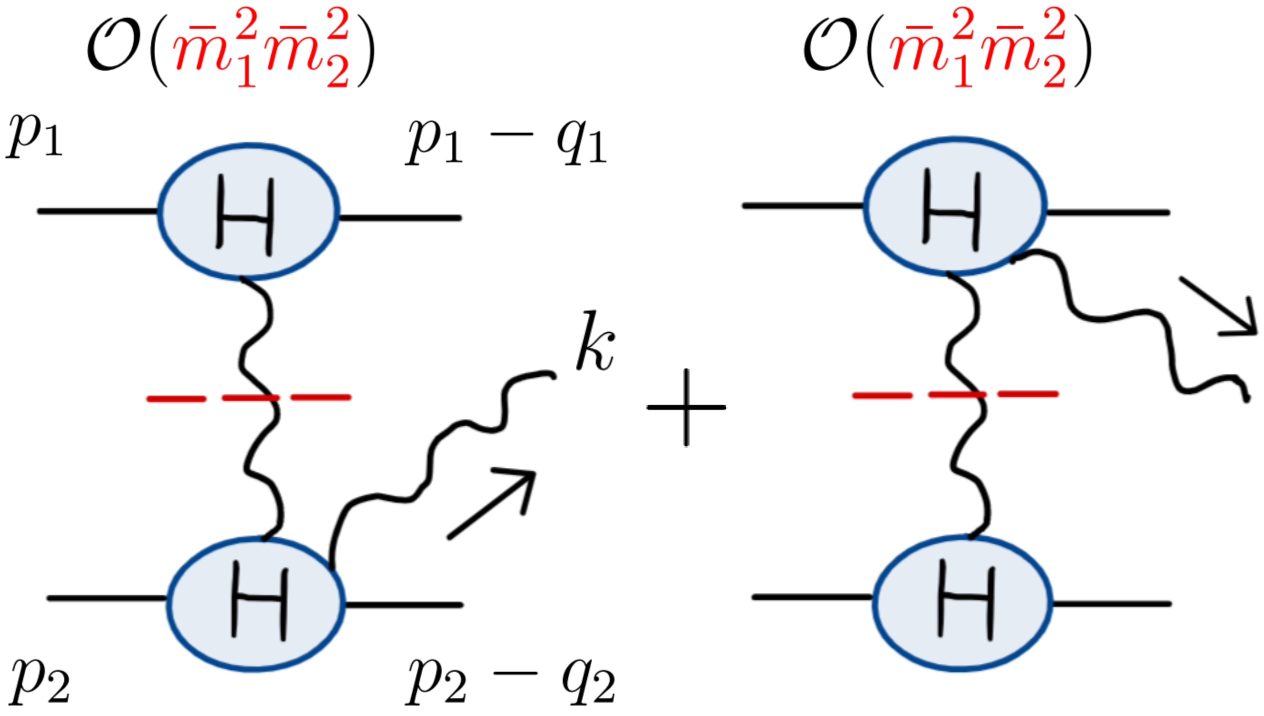


$$\int \frac{d^D l}{(2\pi)^{D-2}} \delta(\bar{v}_1 \cdot l)\delta(\bar{v}_2 \cdot l) \mathcal{A}_{\mathrm{L}}(l)\mathcal{A}_{\mathrm{R}}(l-q)$$

$$\tilde{\mathcal{A}}_{\mathrm{L}}(b)\tilde{\mathcal{A}}_{\mathrm{R}}(b)$$

Building amplitudes: five-point tree-level

$$\mathcal{A}_5^{\text{Tree}} =$$



Agreement with: Luna, Nicholson, O'Connell, White 2017

$$q_1 + q_2 = k$$

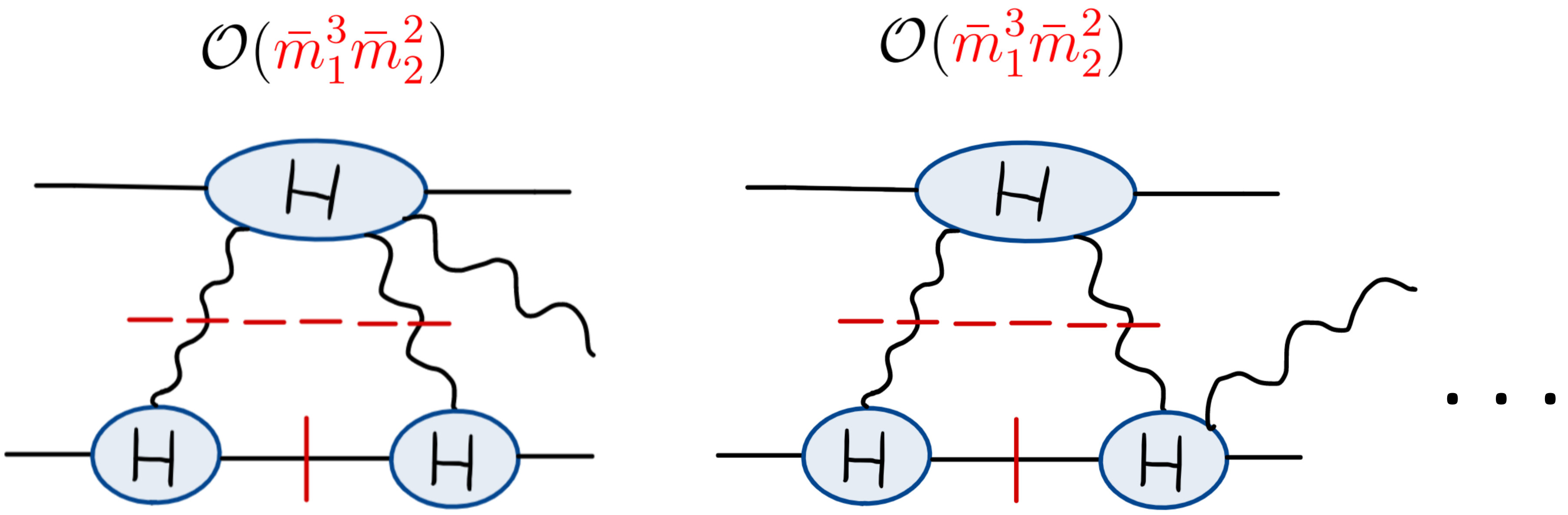
D-dim BCFW shift:

$$\hat{q}_1^\mu = q_1^\mu + z r^\mu$$
$$\hat{q}_2^\mu = q_2^\mu - z r^\mu$$
$$r \cdot k = 0$$
$$r \cdot \varepsilon_k = 0$$
$$r \cdot \bar{v}_i = 0$$
$$r^2 = 0$$

c.f. Britto, Gonzo, Jehu 2021

Building amplitudes: five-point one-loop

$$\mathcal{A}_5^{\text{1-loop}} =$$



- Agreement with: Herderschee, Roiban, Teng -To appear
- We reproduce Weinberg's universal prediction for IR divergences (after a HEFT expansion):

$$\text{IR Div}\left(\mathcal{A}_5^{\text{1-loop}}\right) \sim \frac{G_N}{\epsilon}(\bar{v}_1 \cdot k + \bar{v}_2 \cdot k)\mathcal{A}_5^{\text{Tree}}$$

In dimensional regularisation:

$$D = 4 - 2\epsilon$$

Summary:

- The HEFT provides an efficient way of computing classical pieces of Gravitational scattering amplitudes.
- HEFT amplitudes are manifestly gauge invariant, D-dimensional, and have principle value massive poles.
- The amplitudes themselves are generated via a quasi-shuffle algebra and a closed form exists for any number of gravitons .

For the future:

- Find the origin of the quasi-shuffle algebra.
- Check BCFW recursion at 6-point, and prove the large z behaviour.
- Inclusion of spin, see Gang's talk yesterday.
- Building waveforms from the 5-point amplitudes.

$\vdots$