Kerr binary dynamics from minimal coupling and double copy

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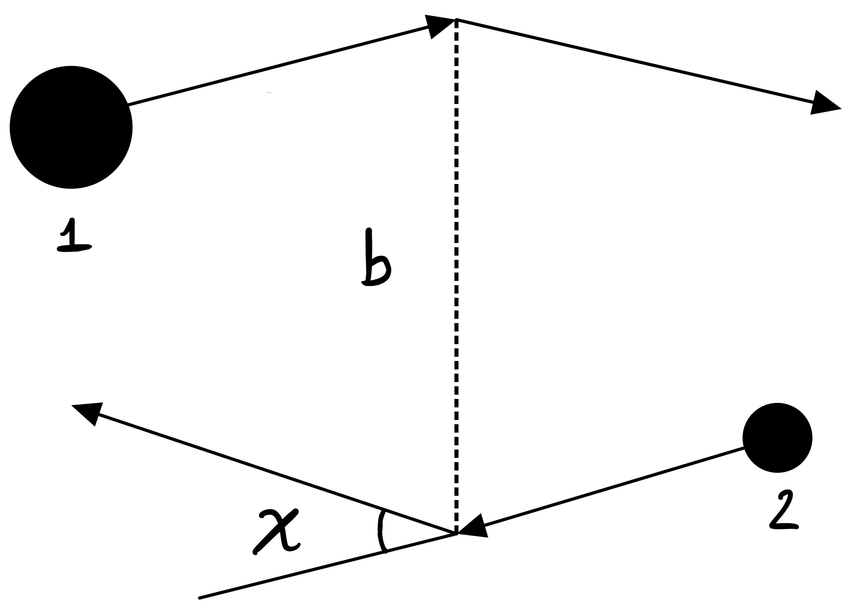
Black-hole scattering in the PM regime:

$r_s = 2 G_N m$



PM regime: $b \gg r_s \gg \frac{\hbar}{m}$ classical limit

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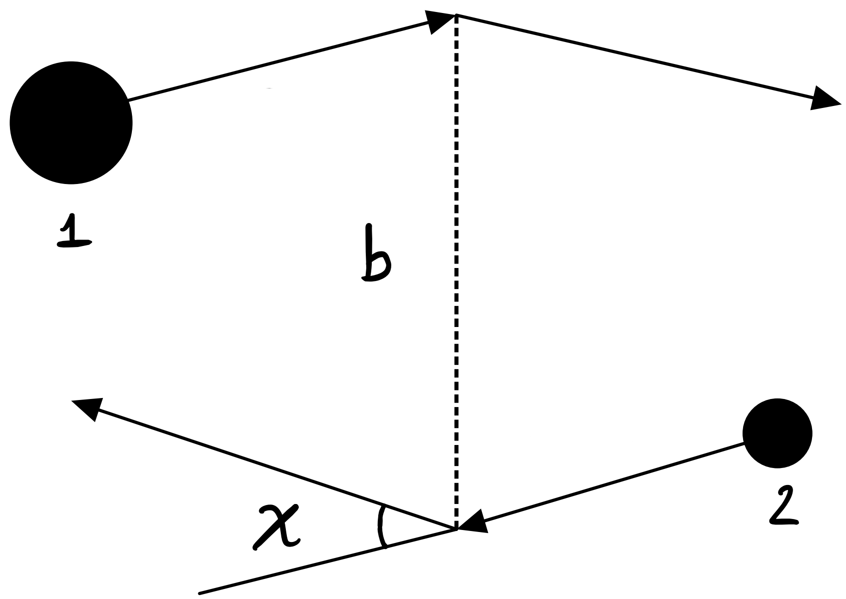


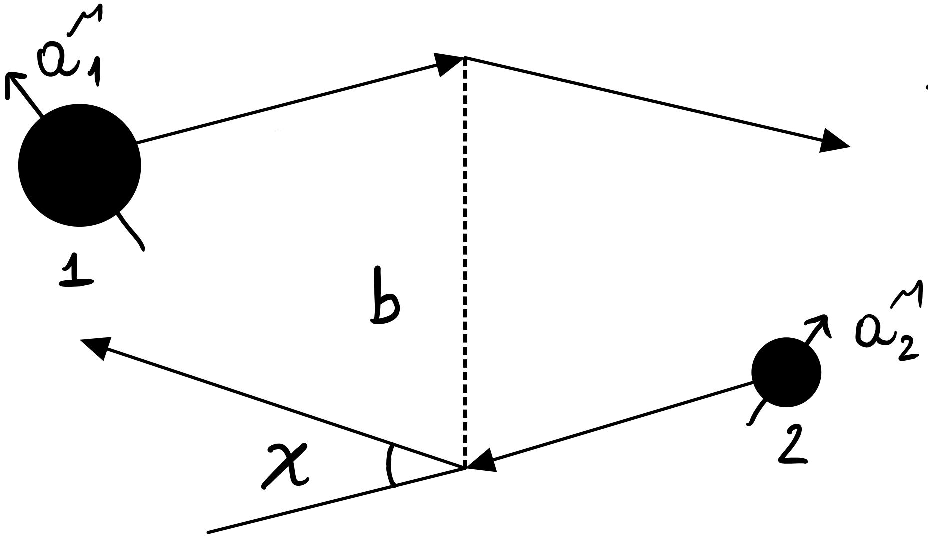
PM regime: $\quad b \gg r_s \gg \dfrac{\hbar}{m}$ classical limit

- Black holes are treated as classical point-particles:

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2\,\kappa_N^2} - \frac{1}{2}\sum_{\alpha=1,2}\left(g^{\mu\nu}\partial_\mu\phi_\alpha\,\partial_\nu\phi_\alpha + m_\alpha^2\,\phi_\alpha^2\right)\right]$$

[Bjerrum-Bohr, Donoghue, Vanhove, Damgaard, Holstein, Festuccia, Planté, Bern, Cheung, Roiban, Shen, Solon, Zeng, Herman, Parra-Martinez, Di Vecchia, Heissenberg, Russo, Veneziano, Collado, Cristofoli, Kosower, Maybe, O'Connell, ...]

Kerr black-hole scattering in the PM regime:

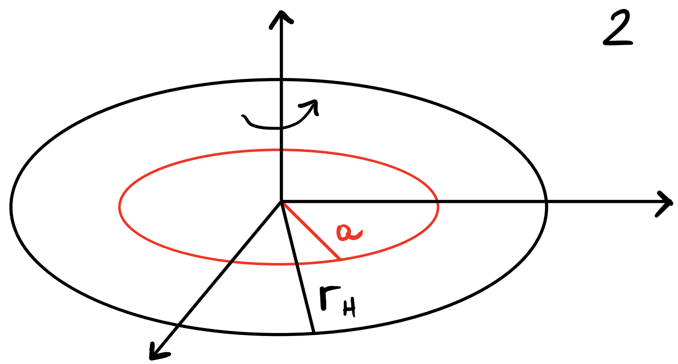


ring radius:

$$a^\mu = \frac{S^\mu}{m}$$

event horizon:

$$r_H = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$$



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$\downarrow$

Spin multipoles: $\quad b \gg a \gg \frac{\hbar}{m}$ $\quad$ classical limit

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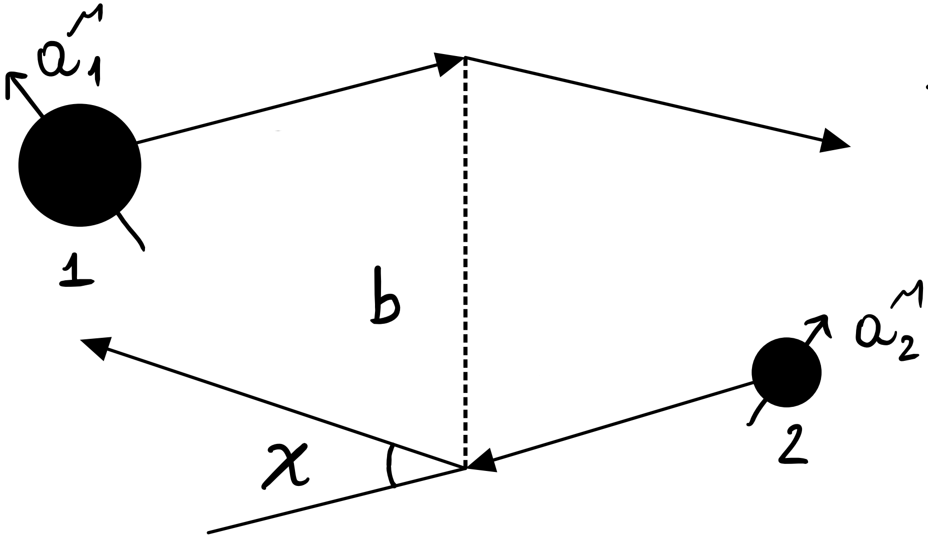

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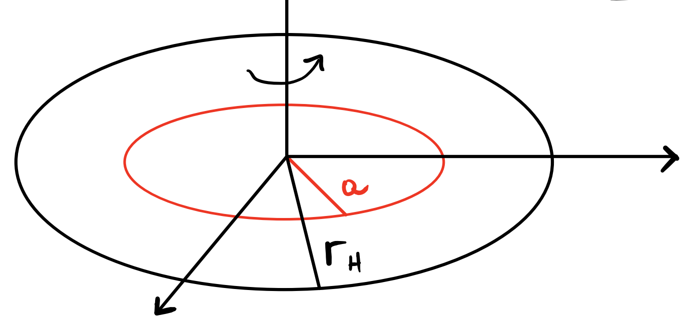

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- From quantum mechanics: $S^2 = S_\mu S^\mu = \underset{\to 0}{\hbar^2}\, \underset{\to\infty}{s}(s+1)$
- Kerr black holes: $\lim_{s\to\infty}$ (massive s fields)

[Arkani-Hamed, Huang, Huang, Bern, Luna, Roiban, Sen, Zeng, Guevara, Ochirov, Vines, Aoude, Haddad, Helset, Chung, Kim, Chen, Chiodaroli, Pichini, Johansson, Jakobsen, Mogull, Plefka, Steinhoff, ...]

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Black holes $\longleftrightarrow$ elementary particles

- Schwarzschild
- Kerr

- minimally coupled scalars
- minimal coupling?

What can GR teach us about QFT?

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M: A simple Lagrangian would allow to increase the precision in analytical computations (e.g. nPM $n>1$)

Kerr - Schild coordinates, 1965

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi K_\mu K_\nu \quad \text{with} \quad \eta_{\mu\nu} K^\mu K^\nu = 0$$

Choose $K^\mu = (1,-1,0,0)$

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1) x^i spherical coordinates • $x^2 + y^2 + z^2 = r^2$

$$\phi + r\,\partial_r \phi = 0 \implies \phi^{(s)} = \frac{r_s}{r} \quad \text{Schwarzschild (1916)}$$

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2) x^i oblate spheroidal coordinates • $\dfrac{x^2+y^2}{r^2+a^2} + \dfrac{z^2}{r^2} = 1$

$$\frac{r^2 - a^2\cos^2\theta}{r^2 + a^2\cos^2\theta}\phi + r\partial_r\phi = 0 \implies \phi^{(K)} = \frac{r\, r_s}{r^2 + a^2\cos^2\theta} \quad \text{Kerr (1963)}$$

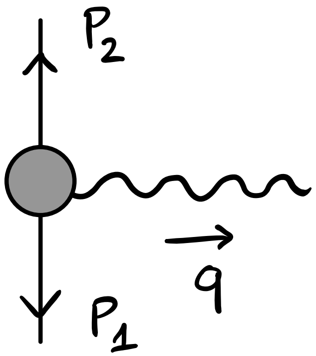
Newman - Janis shift:

$$\phi^{(K)} = \frac{1}{2}\left(\phi^{(s)} + \phi^{(s)*}\right)_{r \to r + ia\cos\theta}$$

3-point amplitudes

- $P = p_1 - p_2$
- $\Lambda^{\rho}{}_{\mu}(a,q) = \exp\left\{ i \epsilon^{\rho}{}_{\mu\alpha\beta}\, a^{\alpha} q^{\beta} \right\}$

$a^{\mu} = 0$



$$= \begin{cases} i g T^{b} p^{\mu} & \text{Gauge theory (scalar QCD)} \\[2em] \kappa_N \, p^{(\mu} p^{\nu)} & \text{Gravity (Schwarzschild)} \end{cases}$$

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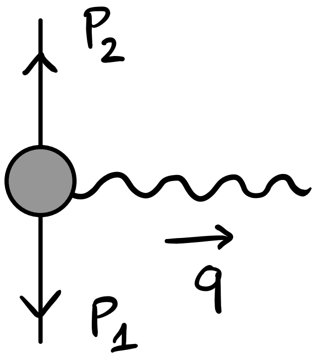

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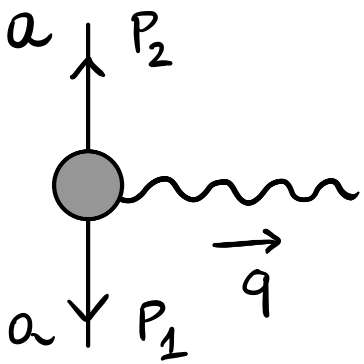


$$= \begin{cases} i g T^{b} \Lambda^{\mu\sigma}(a,q) P_{\sigma} & \text{Gauge theory } (\sqrt{\text{Kerr}}) \\ \\ \kappa_N P_{\sigma} \Lambda^{\sigma(\mu}(a,q) P^{\nu)} & \text{Gravity (Kerr)} \end{cases}$$

[Vines, 2017]

"Minimal coupling" [Arkani-Hamed, Huang, Huang, 2017]

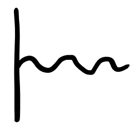
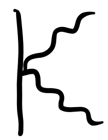
Minimal coupling without spin

$$\mathcal{L}_{free} = -\frac{1}{4} F^{b}_{\mu\nu} F_{b}^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^\dagger (\partial^\mu \phi_\alpha) - m^2_\alpha \phi^\dagger_\alpha \phi_\alpha$$

- Minimal coupling (QFT-like): Define a covariant derivative

$$D_\mu = \partial_\mu - i g A_\mu$$

It automatically provides interactions:

$$\mathcal{L} = \mathcal{L}_{free}(\partial_\mu \to D_\mu) = \mathcal{L}_{free} +$$  $+$ 

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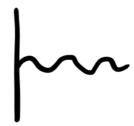
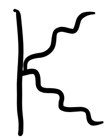
$$\mathcal{L} = \mathcal{L}_{\text{free}}(\partial_\mu \to D_\mu) = \mathcal{L}_{\text{free}} +$$  $+$ 

in a gauge invariant way: $\mathcal{U} = \exp\{i \epsilon^a(x) T^a\} \in SU(N)$

$$\begin{cases} A \to \mathcal{U} A \mathcal{U}^{-1} + \frac{i}{g} \mathcal{U} \partial_\mu \mathcal{U}^{-1} \\ \phi_\alpha \to \mathcal{U} \phi_\alpha \end{cases} \Longrightarrow D_\mu \phi_\alpha \longrightarrow \mathcal{U} D_\mu \phi_\alpha$$

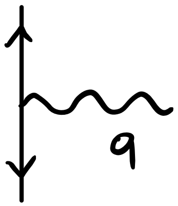
- Message: covariant derivative $\Longrightarrow$ 3-point amplitude + c.t.

$$\overset{?}{\Longleftarrow}$$

Minimal coupling with classical spin

$$\mathcal{L}_{free} = -\frac{1}{4} F^{b}_{\mu\nu} F_{b}^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^{\dagger} (\partial^\mu \phi_\alpha) - m^2_\alpha \phi^{\dagger}_\alpha \phi_\alpha$$

- Assuming minimal coupling (QFT-like)



$$= igT^{b} \Lambda^{\mu\sigma}(a,q) P_\sigma \Longrightarrow D^{(a)}_\mu = \partial_\mu - ig \exp\left\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\right\} A^\sigma$$

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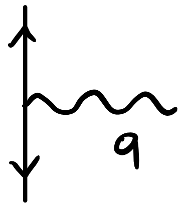
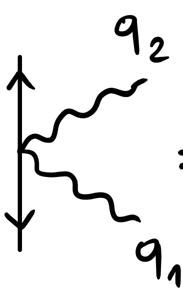


$$= i g T^b \Lambda^{\mu\sigma}(a,q) P_\sigma \Longrightarrow D_\mu^{(a)} = \partial_\mu - i g \exp\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\} A^\sigma$$

- It is a "good" covariant derivative: $D_\mu^{(a)} \phi_\alpha \longrightarrow \mathcal{U} D_\mu^{(a)} \phi_\alpha$!!!

$D_\mu^{(a)} \equiv$ classical spin connection $\rightarrow \mathcal{L}^{(a)} = \mathcal{L}_{free}(\partial_\mu \longrightarrow D_\mu^{(a)})$



$$= i \frac{g^2}{2} \Lambda^\nu{}_\rho (a, q_1) \Lambda^{\mu\rho}(a, q_2)(T^a T^b + T^b T^a)$$

Contact terms are generated by the same mechanism that selects them in the spinless case. Schwarzschild is minimal.

Q : Are Kerr black-holes minimal ?

Feynman rules for minimally coupled $\sqrt{\text{Kerr}}$

- In momentum space: $A^{\mu}_{(h)} \longrightarrow \varepsilon^{\mu}_{(h)}(q)$; $\quad h = \pm 1$

$$\Lambda^{\mu}{}_{\nu}(a,q)\,\varepsilon^{\nu}_{(h)}(q) \overset{\varepsilon^2 = \varepsilon \cdot q = q^2 = 0}{=} e^{h\, a\cdot q}\,\varepsilon^{\mu}_{(h)}(q) + \text{pure gauge}$$

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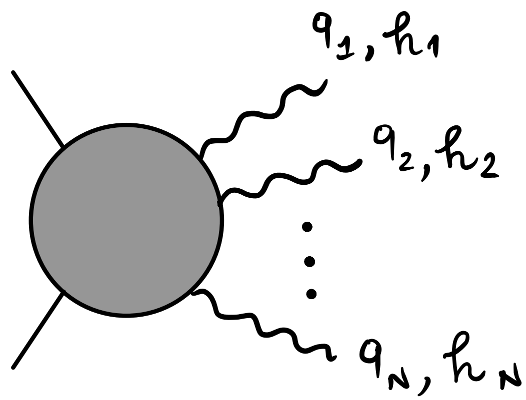
$$A^{(a)}_{(h_1,\dots,h_N)} =$$



$$= \prod_{i=1}^{N} e^{h_i\, a\cdot q_i}\, A^{(a=0)}_{(h_1,\dots,h_N)}$$

- Attach to each massless line with momentum q a Lorentz matrix $\Lambda^{\mu}{}_{\nu}(a,q)$

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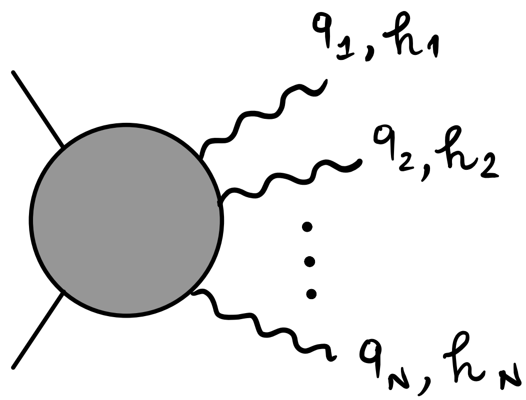
$$A^{(a)}_{(h_1,\dots,h_N)} =$$  $$= \prod_{i=1}^{N} e^{h_i\, a\cdot q_i}\, A^{(a=0)}_{(h_1,\dots,h_N)}$$

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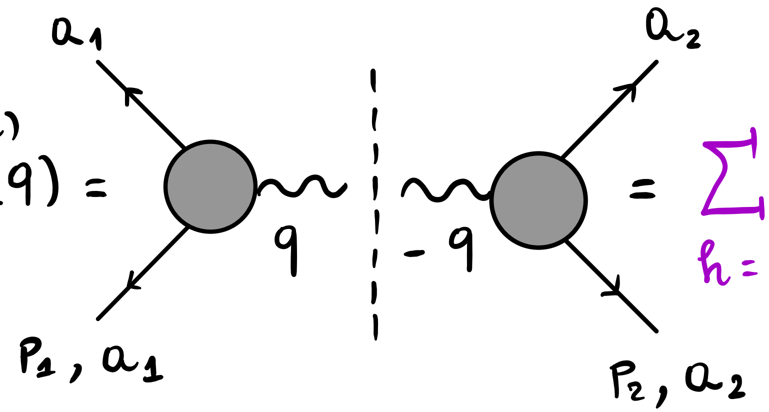
- change the field strength: $F_{\mu\nu} = \frac{i}{g}\left[D^{(a)}_{\mu}, D^{(a)}_{\nu}\right]$ (?)

Kerr three-point and tree-level (1PM)

$$A^{(a)}(q) =$$ 

$$= \sum_{h=\pm} A^3_\mu \, \epsilon'^{\mu}_{(h)}(q) \, \epsilon'^{\nu}_{(-h)}(-q) \, A^3_\nu \equiv A^3_\mu \, \Pi^{\mu\nu}_{(a)} \, A^3_\nu$$

- Double copy: $\text{Kerr}(a) = \sqrt{\text{Kerr}}(a) \otimes \sqrt{\text{Kerr}}(a=0)$

$$\tilde{\mathcal{M}}(q) \sim q^2 A^2(q) = \mathcal{M}(q) + \mathcal{M}_a(q) + \mathcal{M}_\phi(q)$$

$$\mathcal{M}(q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

[Guevara, Ochirov, Vines, 2018]

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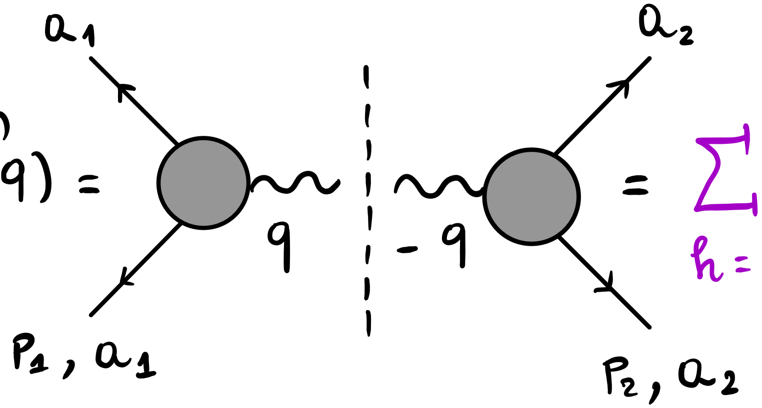
$A^{(a)}(q) =$ 

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[Guevara, Ochirov, Vines, 2018]

- Observables → eikonal

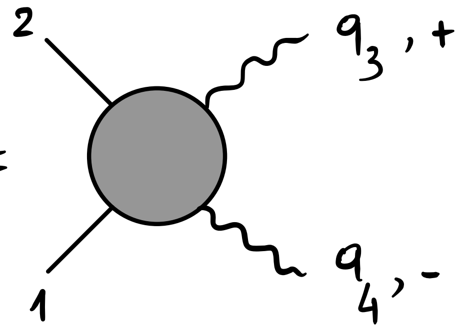
$$\tilde{\mathcal{M}}(b) = \frac{1}{4E|p|} \int \frac{d^{2-2\epsilon} q}{(2\pi)^{2-2\epsilon}} \mathcal{M}(q) \, e^{i q \cdot b} = i \left(1 - e^{2i\delta(b)} \right)$$

$$\propto \sum_{\pm} \frac{1}{|b \pm i a|^{-2\epsilon}}$$

Newman-Janis shift

Kerr Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}^{(a)}_{(+,-)} =$$



$$= e^{a\cdot(q_3 - q_4)}\, \mathcal{M}^{(a=0)}_{(+,-)}$$

It satisfies the "shift" symmetry presented in [2203.06197]

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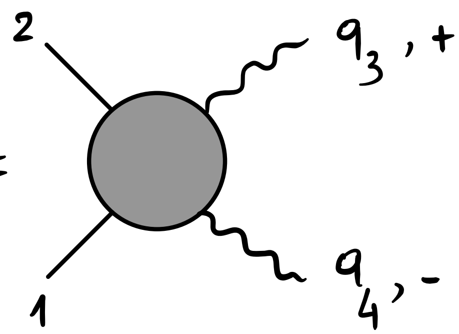

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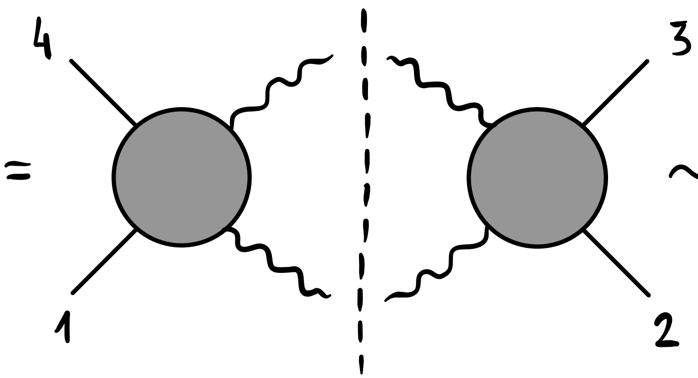
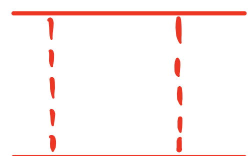
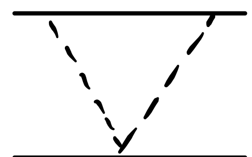


$$\sim$$



$$+$$



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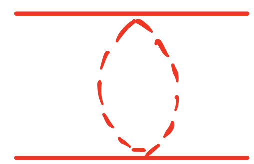

$$= \kappa_N^4\left[c_{\triangleright}\, I_{\triangleright}(a) + (c_{\triangleright})_{\mu\nu}\, I^{\mu\nu}_{\triangleright}(a) + (d_{\triangleright})_{\mu}\, J^{\mu}_{\triangleright}(a) + (m_1 \leftrightarrow m_2)\right]$$

$$I_{\triangleright}(a) = \int \frac{d^4\ell}{(2\pi)^4}\, \frac{\cosh(2a\cdot\ell + a\cdot q)}{\ell^2\,(\ell+q)^2\,[(\ell+P_1)^2 - m_1^2]} \qquad J^{\mu}_{\triangleright}(a) = \int \frac{d^4\ell}{(2\pi)^4}\, \frac{\ell^{\mu}\sinh(2a\cdot\ell + a\cdot q)}{\ell^2\,(\ell+q)^2\,[(\ell+P_1)^2 - m_1^2]}$$

Radiation reaction (RR) effects

- At 3PM the eikonal develops an imaginary part:

$$2\delta_2(b) = \mathrm{Re}\, 2\delta_2(b) + i\, \mathrm{Im}\, 2\delta_2(b)$$

  - $\mathrm{Re}\, 2\delta_2(b) = 2\delta_2^{\mathrm{con}}(b) + 2\delta_2^{\mathrm{rr}}(b)$

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  - $\mathrm{Re}\, 2\delta_2(b) = 2\delta_2^{\mathrm{con}}(b) + 2\delta_2^{rr}(b)$ [Bern et al, 1908.01493]

  - $\mathrm{Im}\, 2\delta_2(b) = -\frac{1}{\pi\epsilon} 2\delta_2^{rr}(b) + O(\epsilon^0)$ [Di Vecchia et al, 2008.12743]

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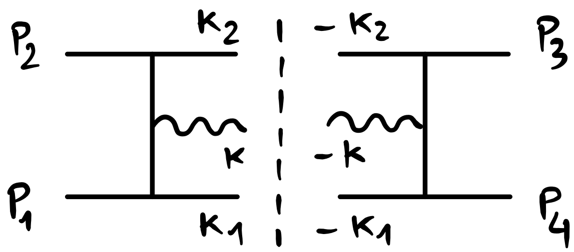
- This is a consequence of real analyticity and crossing symmetry
- Unitarity in "b-space" requires $\mathrm{Im}\, 2\delta_2(b) \neq 0$

$\mathrm{Im}\, 2\delta_2(b) =$ 

$$= \frac{1}{2}\int_{k,k_1,k_2} \left|\tilde{M}_5\right|^2 \overset{\epsilon\to 0}{=} \frac{1}{2}\int_{k,k_1,k_2} \left| k_\nu \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i\cdot k}\, \tilde{M}\right|^2$$

Radiation reaction (RR) effects

- At 3PM the eikonal develops an imaginary part:

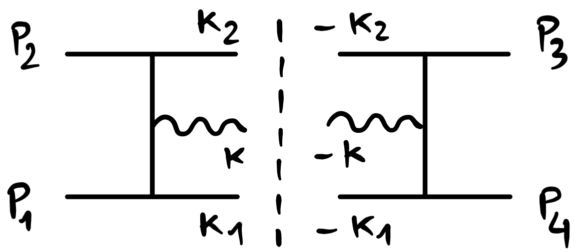
$$2\delta_2(b) = \mathrm{Re}\, 2\delta_2(b) + i\, \mathrm{Im}\, 2\delta_2(b)$$

  - $\mathrm{Re}\, 2\delta_2(b) = 2\delta_2^{\mathrm{con}}(b) + 2\delta_2^{\mathrm{rr}}(b)$

  - $\mathrm{Im}\, 2\delta_2(b) = -\frac{1}{\pi\epsilon}\, 2\delta_2^{\mathrm{rr}}(b) + O(\epsilon^0)$

[Bern et al, 1908.01493]
[Damour, 2010.01641]
[Di Vecchia et al, 2101.05772]

- This is a consequence of real analyticity and crossing symmetry
- Unitarity in "b-space" requires $\mathrm{Im}\, 2\delta_2(b) \neq 0$

$\mathrm{Im}\, 2\delta_2(b) =$ 

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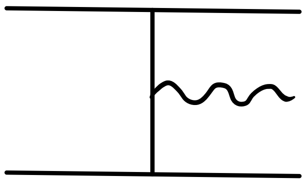
- By knowing the 1PM amplitude we can compute $2\delta_2^{\mathrm{rr}}(b) \Rightarrow \chi_2^{\mathrm{rr}}(b)$

$$\chi_2^{\mathrm{rr}}(b) = -\frac{1}{p}\frac{\partial\, \mathrm{Re}\, 2\delta_2^{\mathrm{rr}}(b)}{\partial b}$$

Radiation reaction (RR) effects for Kerr

1) Use Weinberg soft graviton theorem:

$$\mathcal{M}_5^{\mu\nu}(q,k) =$$



$$\overset{k\to 0}{=} \kappa_N \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i \cdot k} \mathcal{M}(q)$$

with:

$$\mathcal{M}(q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \, \frac{\epsilon_{\mu\nu\rho\sigma} \, p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

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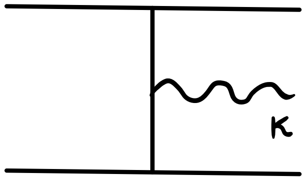

$$\overset{k\to 0}{=} \kappa_N \sum_{i=1}^{4} \frac{p_i^{\mu} p_i^{\nu}}{p_i \cdot k}\, \mathcal{M}(q)$$

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2) Fourier transform:

$$\tilde{\mathcal{M}}_5^{\mu\nu}(b,k) \overset{k\to 0}{=} \sum_{\pm} \int \frac{d^{2-2\epsilon} q}{(2\pi)^{2-2\epsilon}} \left(\dots \right)^{\mu\nu} \frac{e^{i q \cdot \underbrace{(b \pm i a)}_{b_{\pm}}}}{q^2}$$

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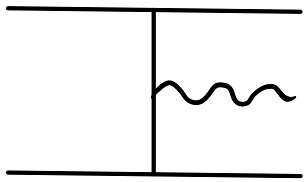

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3) Real analyticity, crossing symmetry and unitarity:

$$2\delta_2^{rr}(b) = -\frac{\pi}{2} \lim_{\epsilon \to 0} \epsilon \int_{k,k_1,k_2} |\tilde{\mathcal{M}}_5|^2 = \left. 2\delta_2^{rr}(b) \right|_{a=0} f^2 , \quad f^\mu = \frac{\sigma^2 b}{2(2\sigma^2 - 1)} \sum_{\pm} \frac{(1 \pm v)^2 b_\pm^\mu}{b_\pm^2}$$

Radiation reaction observables

- $$\chi_2^{rr}(b) = \chi_2^{rr}(b)\Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b}\right)\left[1 + \frac{4\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2\right]}{\left[1-\frac{a}{b}\right]^3}$$

[F.A. Di Vecchia 2203.13272]

$$= \chi_2^{rr}(b)\Big|_{a=0}\left[1 + \frac{6\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2 + O(a^3)\right]$$

[Jakobsen, Mogull 2201.07778]

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[Jakobsen, Mogull 2201.07778]

- Using Bini-Damour formula:

$$J_1^{loss}(b) = J_1^{loss}(b)\Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b}\right)}{1 - \left(\frac{a}{b}\right)^2}$$

Conclusions:

- Using a notion of minimal coupling that incorzoporates classical spin it is possible to construct a consistent field theory that describes the dynamics of classically spinning compact objects and it can be used to perform precision computations.

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- Using a notion of minimal coupling that incorporates classical spin it is possible to construct a consistent field theory that describes the dynamics of classically spinning compact objects and it can be used to perform precision computations.

Outlook:

- Compare and understand differences w.r.t. other 2PM analysis;
- If not Kerr black holes, what is this theory describing? Add corrections? Neutron stars?
- Worldline description;
- Beyond 2PM in the conservative sector.