

Kerr binary dynamics from minimal coupling and double copy

Francesco Alesio

2301.XXXX with H. Johansson

2203.13272 with P. Di Vecchia

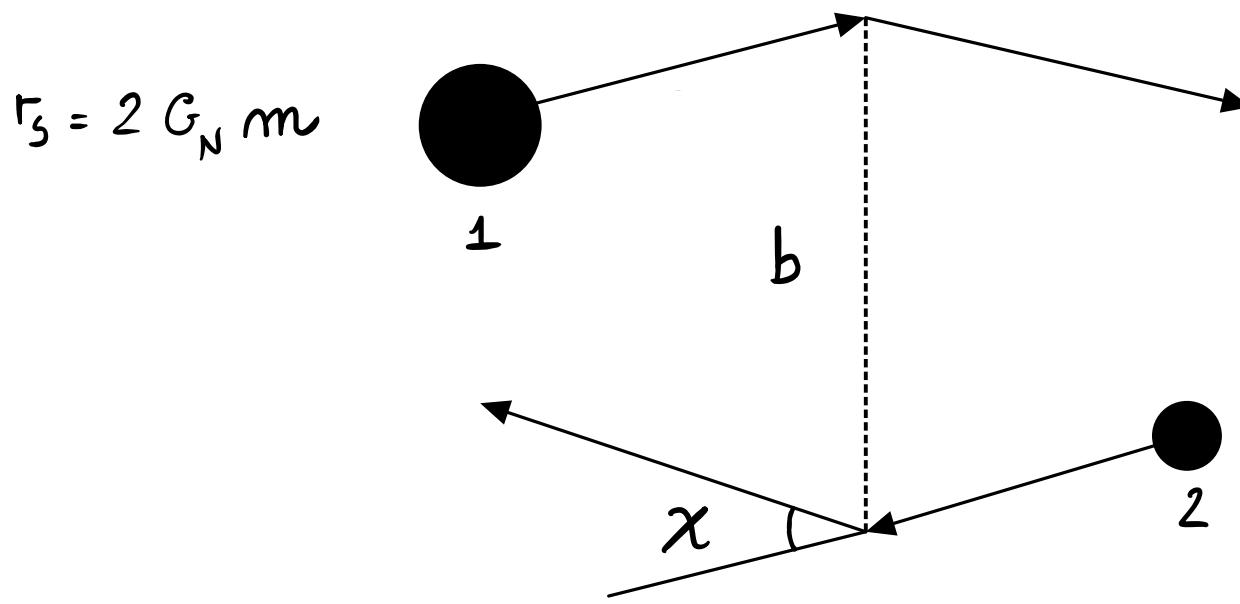


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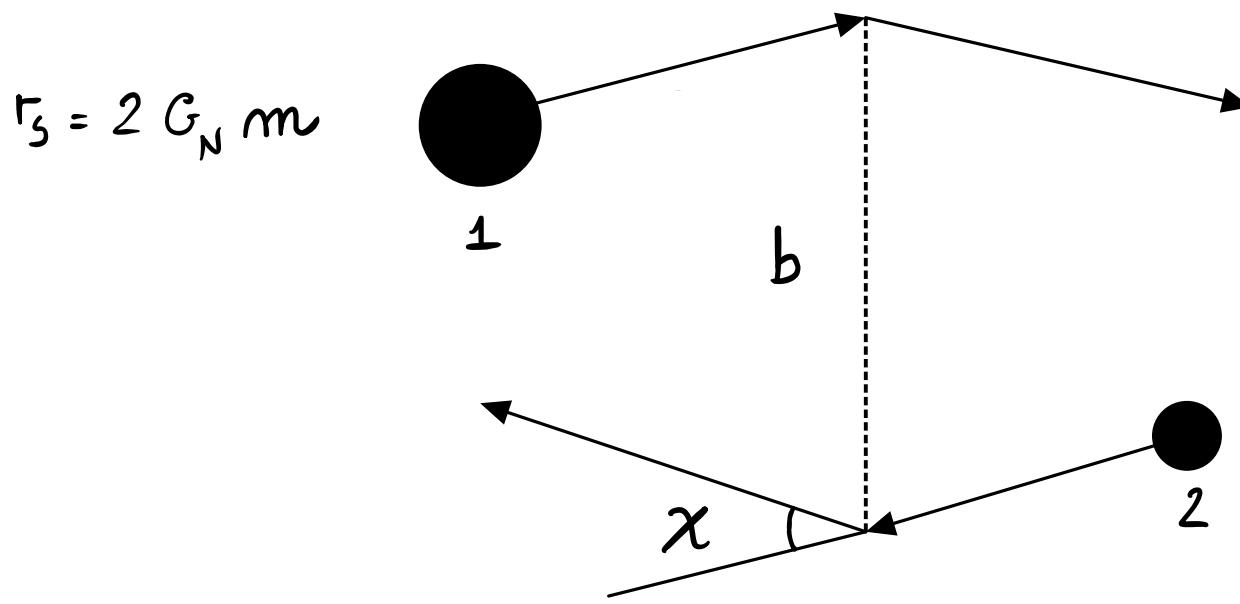
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Black-hole scattering in the PM regime:



PM regime : $b \gg r_s \gg \frac{\hbar}{m}$ classical limit

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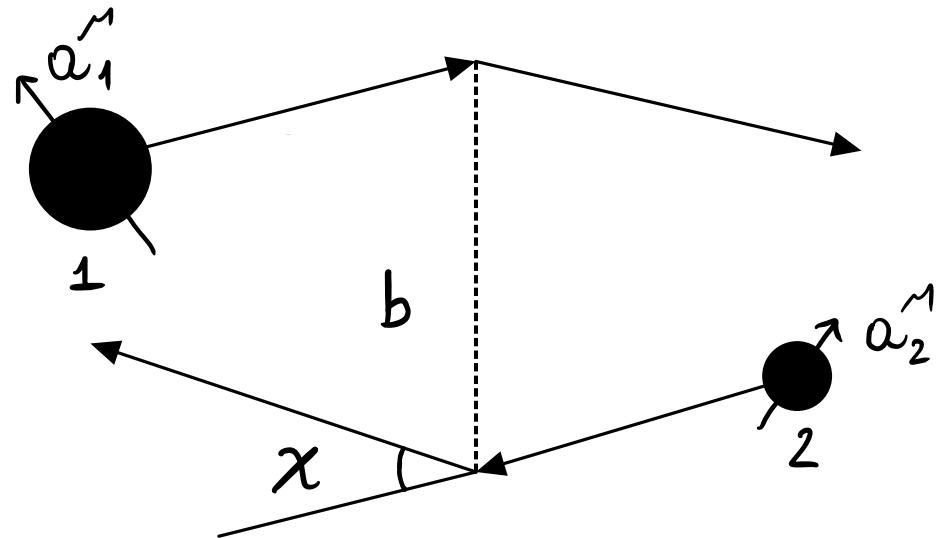
PM regime : $b \gg r_s \gg \frac{\hbar}{m}$ classical limit

- Black holes are treated as **classical point-particles**:

$$\mathcal{L} = \sqrt{-g} \left[\frac{R}{2 K_N^2} - \frac{1}{2} \sum_{\alpha=1,2} \left(g^{\mu\nu} \partial_\mu \phi_\alpha \partial_\nu \phi_\alpha + m_\alpha^2 \phi_\alpha^2 \right) \right]$$

[Bjerrum-Bohr, Donoghue, Vanhove, Damgaard, Holstein, Festuccia, Pante, Bern, Cheung, Roiban, Strom, Solon, Zeng, Hermon Parra-Martinez, Di Vecchia, Heissenberg, Russo, Veneziano, Collado, Cristofoli, Kosower, Maybe, O'Connell, ...]

Kerr black-hole scattering in the PM regime:

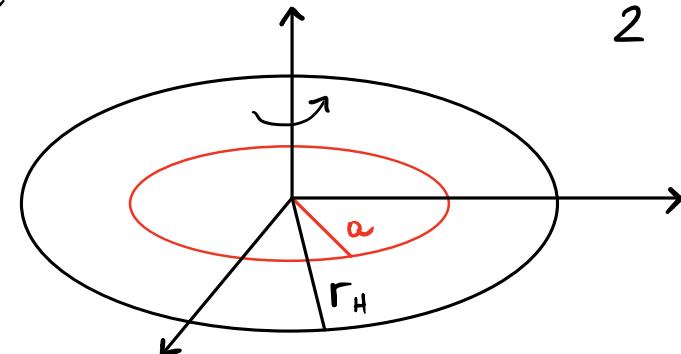


ring radius:

$$a'' = \frac{S''}{m}$$

event horizon:

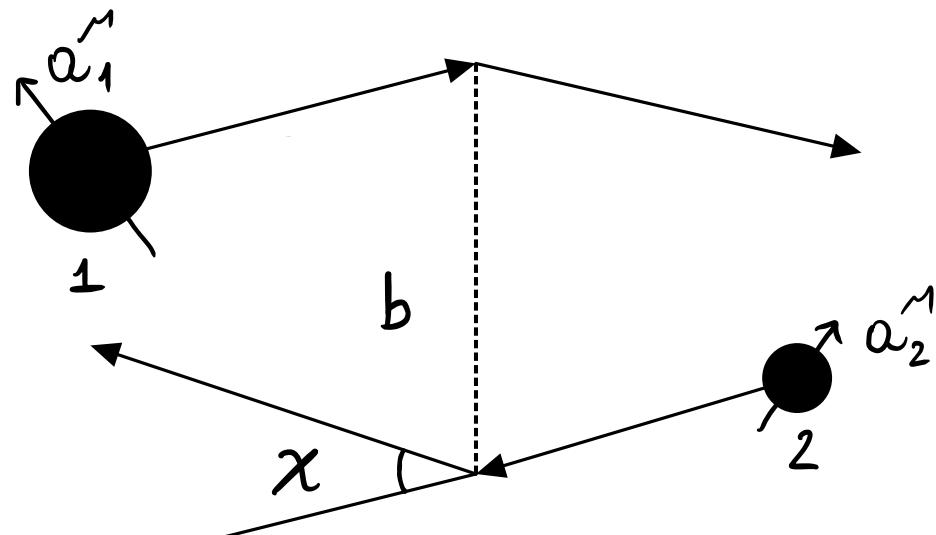
$$r_H = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$$



PM regime: $b \gg r_H \gg \frac{\hbar}{m}$ classical limit

Spin multipoles: $b \gg a \gg \frac{\hbar}{m}$ classical limit

Kerr black-hole scattering in the PM regime:

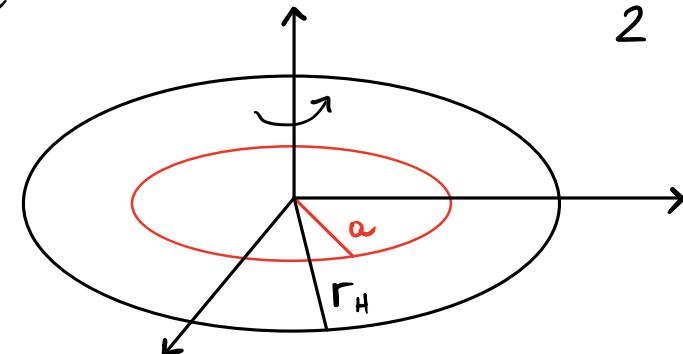


ring radius:

$$a^m = \frac{S^m}{m}$$

event horizon:

$$r_H = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$$



PM regime: $b \gg r_H \gg \frac{\hbar}{m}$ classical limit



Spin multipoles: $b \gg a \gg \frac{\hbar}{m}$ classical limit

- From quantum mechanics: $S^2 = S_M S^M = \frac{\hbar^2}{m} s(s+1)$

$$\begin{matrix} & \\ \downarrow & \downarrow \\ 0 & \infty \end{matrix}$$

- Kerr black holes: $\lim_{s \rightarrow \infty}$ (massive s fields)

[Arkani-Hamed, Huang, Huang, Bern, Luna, Roiban, Sen, Zeng, Guerone, Ochirov, Vines, Aronde, Haddad, Helset, Chung, Kim, Chen, Chiodaroli, Pichini Johansson, Jakobsson, Mogull, Plefka, Steinhoff, ...]

Q: Is it possible to describe Kerr black holes with an effective Lagrangian depending on the classical spin a and therefore carrying, as a built-in feature, an infinite spin-multipole expansion?

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M: Purely theoretical interest. Simplicity of BHs :

Black holes \longleftrightarrow elementary particles

- Schwarzschild
 - minimally coupled scalars
 - minimal coupling?
- Kerr

What can GR teach us about QFT?

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What can GR teach us about QFT?

M: A simple Lagrangian would allow to increase the precision in analytical computations (e.g. nPM $n > 1$)

Kerr - Schild coordinates, 1965

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi K_\mu K_\nu \quad \text{with} \quad \eta_{\mu\nu} K^\mu K^\nu = 0$$

Choose $K^M = (1, -1, 0, 0)$

Kerr - Schild coordinates, 1965

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1) x^i spherical coordinates • $x^2 + y^2 + z^2 = r^2$

$$\phi + r \partial_r \phi = 0 \Rightarrow \overset{(s)}{\phi} = \frac{r_s}{r} \quad \text{Schwarzschild (1916)}$$

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1) x^i spherical coordinates • $x^2 + y^2 + z^2 = r^2$

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2) x^i oblate spheroidal coordinates • $\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$

$$\frac{r^2 - a^2 \cos^2 \theta}{r^2 + a^2 \cos^2 \theta} \phi + r \partial_r \phi = 0 \Rightarrow \phi^{(K)} = \frac{r r_s}{r^2 + a^2 \cos^2 \theta} \quad \text{Kerr (1963)}$$

Newman - Janis shift:

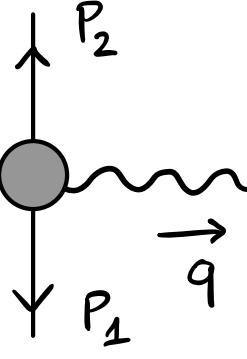
$$\phi^{(K)} = \frac{1}{2} \left(\phi^{(s)} + \phi^{(s)*} \right)_{r \rightarrow r + ia \cos \theta}$$

3-point amplitudes

$$\bullet P = P_1 - P_2$$

$$\bullet \Lambda^\rho_{\mu}(a, q) = \exp \left\{ i \epsilon^\rho_{\mu\alpha\beta} a^\alpha q^\beta \right\}$$

$$a^M = 0$$


$$= \begin{cases} ig T^b p^M & \text{Gauge theory (scalar QCD)} \\ \kappa_N p^{(M} p^N) & \text{Gravity (Schwarzschild)} \end{cases}$$

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$a^M \neq 0$

$$= \begin{cases} ig T^b \Lambda^{M\sigma}(a, q) P_\sigma & \text{Gauge theory } (\sqrt{\text{Kerr}}) \\ K_N P_\sigma \Lambda^\sigma{}^{(M} (a, q) P^N) & \text{Gravity (Kerr)} \end{cases}$$

[Vines, 2017]

"Minimal coupling" [Arkani-Hamed, Huang, Huang, 2017]

Minimal coupling without spin

$$\mathcal{L}_{\text{free}} = -\frac{1}{4} F_{\mu\nu}^b F_b^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^+ (\partial^\mu \phi_\alpha) - m_\alpha^2 \phi_\alpha^+ \phi_\alpha^-$$

- Minimal coupling (QFT-like): Define a covariant derivative

$$D_\mu = \partial_\mu - ig A_\mu$$

It automatically provides interactions:

$$\mathcal{L} = \mathcal{L}_{\text{free}} (\partial_\mu \rightarrow D_\mu) = \mathcal{L}_{\text{free}} + \text{[curly bracket]} + \text{[curly bracket]}$$

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in a gauge invariant way: $U = \exp \{ i \epsilon^a(x) T^a \} \in SU(N)$

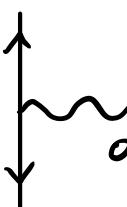
$$\begin{cases} A \rightarrow UAU^{-1} + \frac{i}{g} U \partial_\mu U^{-1} \\ \phi_\alpha \rightarrow U \phi_\alpha \end{cases} \Rightarrow D_\mu \phi_\alpha \longrightarrow U D_\mu U^{-1} \phi_\alpha$$

- Message: covariant derivative $\xrightarrow{?} \xleftarrow{?}$ 3-point amplitude + c.t.

Minimal coupling with classical spin

$$\mathcal{L}_{\text{free}} = -\frac{1}{4} F_{\mu\nu}^b F_b^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^+ (\partial^\mu \phi_\alpha) - m_\alpha^2 \phi_\alpha^+ \phi_\alpha^-$$

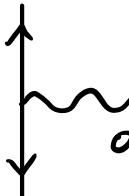
- Assuming minimal coupling (QFT-like)


$$q = ig T^b \Lambda^{\mu\sigma}(\alpha, q) P_\sigma \Rightarrow D_M^{(a)} = \partial_M - ig \exp \left\{ \epsilon_{\mu\sigma\alpha\beta} \alpha^\sigma \partial^\beta \right\} A^\sigma$$

Minimal coupling with classical spin

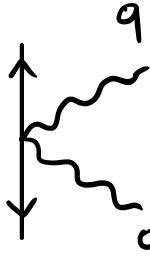
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- Assuming minimal coupling (QFT-like)

 $= ig T^b \Lambda^{\mu\sigma}(a, q) P_\sigma \Rightarrow D_M^{(a)} = \partial_M - ig \exp \left\{ \epsilon_{\mu\sigma\lambda\rho} a^\lambda \partial^\rho \right\} A^\sigma$

- It is a "good" covariant derivative: $D_M^{(a)} \phi_\alpha \rightarrow u D_M^{(a)} \phi_\alpha !!!$

$D_M^{(a)} \equiv \text{classical spin connection} \rightarrow \mathcal{L}^{(a)} = \mathcal{L}_{\text{free}} (\partial_M \rightarrow D_M^{(a)})$

 $= i \frac{g^2}{2} \Lambda^\nu{}_\rho(a, q_1) \Lambda^{\mu\rho}(a, q_2) (T^\alpha T^\beta + T^\beta T^\alpha)$

Contact terms are generated by some mechanism that selects them in the spinless case. Schwarzschild is minimal.

Q : Are Kerr black-holes minimal?

Feynman rules for minimally coupled $\sqrt{\text{Kerr}}$

- In momentum space: $A_{(h)}^{\mu} \rightarrow \mathcal{E}_{(h)}^{\mu}(q); \quad h = \pm 1$

$$\hat{\Lambda}_v^{\mu}(a, q) \mathcal{E}_{(h)}^{\nu}(q) = e^{h a \cdot q} \mathcal{E}_{(h)}^{\mu}(q) + \text{pure gauge}$$

$\mathcal{E}^2 = \mathcal{E} \cdot q = q^2 = 0$

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$$A_{(h_1, \dots, h_N)}^{(a)} = \text{Diagram} = \prod_{i=1}^N e^{h_i a \cdot q_i} A_{(h_1, \dots, h_N)}^{(a=0)}$$

- Attach to each massless line with momentum q a Lorentz matrix $\Lambda_{\nu}^{\mu}(a, q)$

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- Attach to each massless line with momentum q a Lorentz matrix $\Lambda_{\nu}^{\mu}(a, q)$
- change the field strength: $F_{\mu\nu} = \frac{i}{g} [D_M^{(a)}, D_V^{(a)}] (?)$

Kerr three-point and tree-level (1PM)

$$A^{(a)}(q) = \sum_{h=\pm} A_m^3 \epsilon_{(h)}^{\mu}(q) \epsilon_{(-h)}^{\nu}(-q) A_{\nu}^3 = A_m^3 \Pi_{(a)}^{\mu\nu} A_{\nu}^3$$

- Double copy: $\text{Kerr}(a) = \sqrt{\text{Kerr}}(a) \otimes \sqrt{\text{Kerr}}(a=0)$

$$\tilde{\mathcal{M}}(q) \sim q^2 A^2(q) = \mathcal{M}(q) + \mathcal{M}_a(q) + \mathcal{M}_{\phi}(q)$$

$$\mathcal{M}(q) = \frac{K_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm \nu)^2 \exp \left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} P_1^\mu P_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma \nu} \right\}$$

[Guevara, Ochirov, Vines, 2018]

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- Observables \rightarrow eikonal [Guevara, Ochirov, Vines, 2018]

$$\begin{aligned} \tilde{\mathcal{M}}(b) &= \frac{1}{4|E|P|} \int \frac{d^2 q}{(2\pi)^{2-2\epsilon}} \mathcal{M}(q) e^{i q \cdot b} = i \left(1 - e^{2i\delta(b)} \right) \\ &\propto \sum_{\pm} \frac{1}{|b \pm ia|^{-2\epsilon}} \xrightarrow{\text{Newman-Janis shift}} \end{aligned}$$

Kerr Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}_{(+,-)}^{(a)} = \text{Diagram} = e^{a \cdot (q_3 - q_4)} \mathcal{M}_{(+,-)}^{(a=0)}$$

The diagram shows a central gray circle representing an interaction vertex. Four wavy lines extend from it: one line labeled '2' goes upwards and to the left; another line labeled '1' goes downwards and to the left; a third line labeled 'q₃, +' goes upwards and to the right; and a fourth line labeled 'q₄, -' goes downwards and to the right.

It satisfies the "shift" symmetry presented in [2203.06197]

Kerr Compton amplitude & 1-loop amplitude (2PM)

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The diagram shows a central gray circle with four wavy lines extending from it. Two lines are labeled with indices: line 1 is labeled $q_4, -$ and line 2 is labeled $q_3, +$.

It satisfies the "shift" symmetry presented in [2203.06197]

$$\begin{aligned} \mathcal{M}_{\text{1-loop}} &= \text{Diagram} \sim \text{Diagram} \\ &= \kappa_N^4 \left[C_D I_D(a) + (C_D)_{\mu\nu} \tilde{I}_D^{\mu\nu}(a) + (d_D)_\mu \tilde{J}_D^\mu(a) + (m_1 \leftrightarrow m_2) \right] \end{aligned}$$

The diagram for the 1-loop amplitude shows two gray circles connected by a vertical dashed line. The left circle has lines 1 and 4, and the right circle has lines 2 and 3. Below this, three red Feynman-like diagrams are shown with a plus sign between them: a vertical rectangle, a V-shape, and a circle.

$$I_D(a) = \int \frac{d^4 l}{(2\pi)^4} \frac{\cosh(2a \cdot l + a \cdot q)}{l^2 (l+q)^2 [(l+p_1)^2 - m_1^2]}$$

$$\tilde{J}_D^\mu(a) = \int \frac{d^4 l}{(2\pi)^4} \frac{l^\mu \sinh(2a \cdot l + a \cdot q)}{l^2 (l+q)^2 [(l+p_1)^2 - m_1^2]}$$

Radiation reaction (RR) effects

- At 3PM the eikonal develops an imaginary part:

$$2\delta_2(b) = \operatorname{Re} 2\delta_2(b) + i \operatorname{Im} 2\delta_2(b)$$

$$\bullet \operatorname{Re} 2\delta_2(b) = 2\delta_2^{\text{con}}(b) + 2\delta_2^{\pi\pi}(b)$$

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$$\bullet \text{Im } 2\delta_2(b) = -\frac{1}{\pi\epsilon} 2\delta_2^{\pi\pi}(b) + \mathcal{O}(\epsilon^\circ) \quad [\text{Di Vecchia et al, 2008.12743}]$$

- This is a consequence of real analyticity and crossing symmetry

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- Unitarity in "b-space" requires $\text{Im } 2\delta_2(b) \neq 0$

$$\text{Im } 2\delta_2(b) = \begin{array}{c} P_2 \\ \text{---} \\ | \quad | \\ \text{---} \\ P_1 \end{array} \begin{array}{c} \overset{\kappa_2}{\text{---}} \\ | \quad | \\ \text{---} \\ \kappa \\ | \quad | \\ \text{---} \\ \overset{-\kappa_2}{\text{---}} \end{array} \begin{array}{c} P_3 \\ \text{---} \\ | \quad | \\ \text{---} \\ -\kappa \\ | \quad | \\ \text{---} \\ \overset{-\kappa_1}{\text{---}} \end{array} \begin{array}{c} P_4 \\ \text{---} \\ | \quad | \\ \text{---} \\ -\kappa_1 \end{array} = \frac{1}{2} \int_{K, K_1, K_2} \left| \tilde{M}_5 \right|^2 \xrightarrow{\epsilon \rightarrow 0} \frac{1}{2} \int_{K, K_1, K_2} \left| \kappa \sum_{i=1}^4 \frac{P_i^M P_i^N}{P_i \cdot K} \tilde{M} \right|^2$$

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- $\text{Im } 2\delta_2(b) = -\frac{1}{\pi\epsilon} 2\delta_2^{\text{rr}}(b) + O(\epsilon^\circ)$

[Damour, 2010.01641]

[Di Vecchia et al, 2101.05772]

- This is a consequence of real analyticity and crossing symmetry
- Unitarity in "b-space" requires $\text{Im } 2\delta_2(b) \neq 0$

$$\text{Im } 2\delta_2(b) = \begin{array}{c} P_2 \\ \text{---} \\ | \quad | \\ \text{---} \\ K_2 \end{array} \mid \begin{array}{c} -K_2 \\ \text{---} \\ | \quad | \\ \text{---} \\ -K \end{array} \mid \begin{array}{c} P_3 \\ \text{---} \\ | \quad | \\ \text{---} \\ -K_1 \end{array} = \frac{1}{2} \int_{K, K_1, K_2} \left| \tilde{M}_5 \right|^2 \xrightarrow{\epsilon \rightarrow 0} \frac{1}{2} \int_{K, K_1, K_2} \left| K \sum_{i=1}^4 \frac{P_i^M P_i^N}{P_i \cdot K} \tilde{M} \right|^2$$

- By knowing the 1PM amplitude we can compute $2\delta_2^{\text{rr}}(b) \Rightarrow \chi_2^{\text{rr}}(b)$

$$\chi_2^{\text{rr}}(b) = -\frac{1}{P} \frac{\partial \text{Re } 2\delta_2^{\text{rr}}(b)}{\partial b}$$

Radiation reaction (RR) effects for Kerr

1) Use Weinberg soft graviton theorem:

$$M_5^{\mu\nu}(q, \kappa) = \overline{\text{Feynman diagram}} \stackrel{\kappa \rightarrow 0}{=} \kappa_N \sum_{i=1}^4 \frac{P_i^\mu P_i^\nu}{P_i \cdot K} M(q)$$

with :

$$M(q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm \nu)^2 \exp \left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} P_1^\mu P_2^\nu \alpha^{\rho} q^{\sigma}}{m_1 m_2 \sigma \nu} \right\}$$

Radiation reaction (RR) effects

1) Use Weinberg soft graviton theorem:

$$\mathcal{M}_5^{μν}(q, k) = \overline{\text{Feynman diagram}} \xrightarrow{k \rightarrow 0} = k_N \sum_{i=1}^4 \frac{P_i^μ P_i^ν}{P_i \cdot K} \mathcal{M}(q)$$

with:

$$\mathcal{M}(q) = \frac{k_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{±} (1 ± v)^2 \exp \left\{ ± i \frac{E_{μνρσ} P_1^μ P_2^ν a^ρ q^σ}{m_1 m_2 \sigma v} \right\}$$

2) Fourier transform:

$$\tilde{\mathcal{M}}_5^{μν}(b, k) \xrightarrow{k \rightarrow 0} \sum_{±} \int \frac{d^2 q}{(2\pi)^{2-2ε}} (\dots)^{μν} \frac{i q \cdot (b \pm ia)}{q^2}$$

Radiation reaction (RR) effects

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$$\tilde{\mathcal{M}}_5^{\mu\nu}(b, \kappa) \stackrel{\kappa \rightarrow 0}{=} \sum_{\pm} \int \frac{d^2 q}{(2\pi)^{2-2\varepsilon}} (\dots)^{\mu\nu} \frac{i q \cdot (b \pm ia)}{q^2}$$

3) Real analyticity, crossing symmetry and unitarity:

$$2\delta_2^{rr}(b) = -\frac{\pi}{2} \lim_{\epsilon \rightarrow 0} \epsilon \int_{K, K_1, K_2} \left| \tilde{\mathcal{M}}_5 \right|^2 = 2\delta_2^{rr}(b) \Big|_{a=0} f^2, \quad f^{\mu} = \frac{\sigma^2 b}{2(2\sigma^2 - 1)} \sum_{\pm} \frac{(1 \pm v)^2 b_{\pm}^{\mu}}{b_{\pm}^2}$$

Radiation reaction observables

$$\bullet X_2^{rr}(b) = X_2^{rr}(b) \Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b}\right) \left[1 + \frac{4\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b} + \left(\frac{a}{b}\right)^2\right]}{\left[1 - \frac{a}{b}\right]^3}$$

[F.A. Di Vecchia 2203.13272]

$$= X_2^{rr}(b) \Big|_{a=0} \left[1 + \frac{6\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b} + \left(\frac{a}{b}\right)^2 + O(a^3)\right]$$

[Jakobsen, Mogull 2201.07778]

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[Jakobsen, Mogull 2201.07778]

• Using Bini-Damour formula:

$$J_1^{loss}(b) = J_1^{loss}(b) \Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b}\right)}{1 - \left(\frac{a}{b}\right)^2}$$

Conclusions:

- Using a notion of minimal coupling that incorporates classical spin it is possible to construct a consistent field theory that describes the dynamics of classically spinning compact objects and it can be used to perform precision computations.

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Outlook:

- Compare and understand differences w.r.t. other 2PM analysis;
- If not Kerr black holes, what is this theory describing?
Add corrections? Neutron stars?
- Worldline description;
- Beyond 2PM in the conservative sector.