QCD meets gravity

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Gravitational radiation contributions to the scattering angle (4PM & 5PN)

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5PN binary dynamics

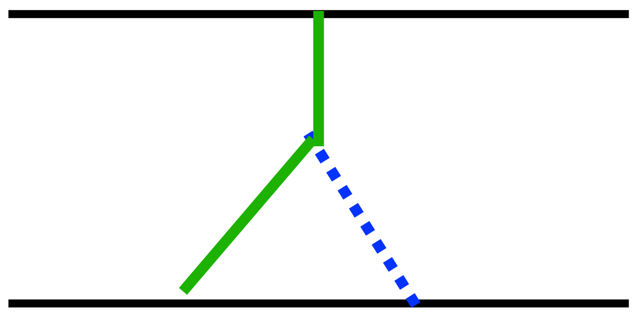
EFT “complete” calculation of **conservative** sector

Blumlein Maier Marquard Schaefer

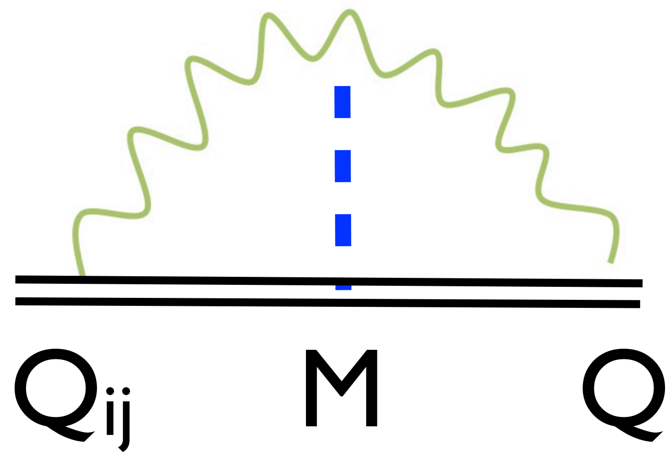
SF Sturani
SF Mastrolia Sturani Sturm Torres

+ several PM partial cross-checks

$$\mathcal{L}_{5PN} = \text{potential modes} + \text{radiative contributions (tails and memory)}$$



$$\frac{1}{\mathbf{p}^2 - (p_0 + i\varepsilon)^2} \simeq \frac{1}{\mathbf{p}^2}\left(1 + \frac{{p_0}^2}{\mathbf{p}^2} + \dots\right)$$

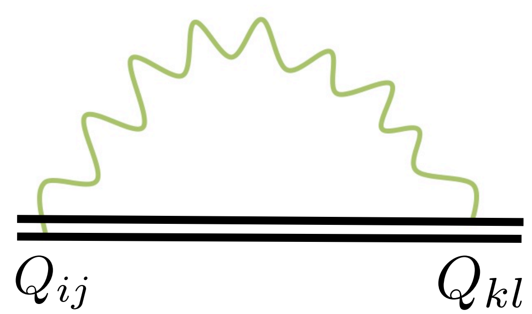


Two reasons to include **dissipative** effects

1) because they are there

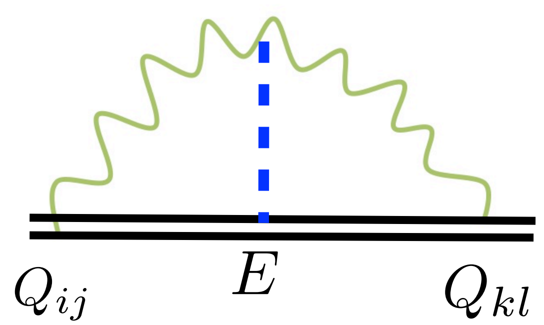
and difficult to separate from conservative ones:

Simple radiation reaction (2.5PN, 3.5PN …)



purely **dissipative**
odd under time reversal

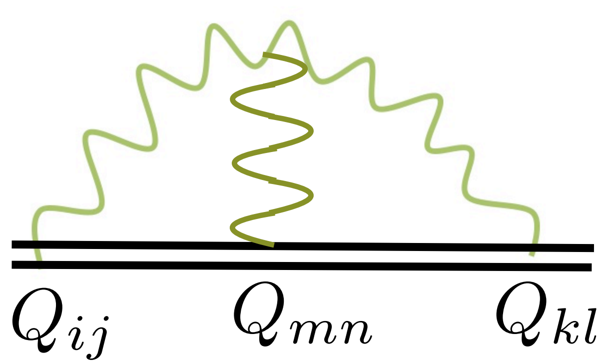
Tails (from 4PN)



conservative (local and nonlocal)
even under time reversal

\+ **dissipative** nonlocal

From 5PN



+…

local terms only,
conservative and **dissipative**
have the same form
$\sim G^4 v^4$ at 5PN

See also recent papers by **Kalin, Neef, Porto**, and **Bini, Damour, Geralico** on this subject

Two reasons to include **dissipative** effects

2) polynomiality of scattering angle

Damour

$$\frac{1}{2}\chi^{\rm cons,tot}(j,\gamma,\nu) = \sum_{n\geq 1} \frac{\chi_n^{\rm cons,tot}(\gamma,\nu)}{j^n}$$

(PM expansion)

$$\chi_4^{cons,tot} = \chi_4^{Schw} + \nu\chi_4^{\nu}$$

$$\nu \equiv \frac{m_1 m_2}{M^2}$$

Bini Damour Geralico

TUTTIFRUTTI

$\mathcal{H}_{5PN}$ → χ_4^{ν}

modulo 2 coefficients

(and even 6PN, modulo 4 coefficients)

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$$\mathcal{H}_{5PN}$$

modulo 2 coefficients

(and even 6PN, modulo 4 coefficients)

Blumlein Maier Marquard Schaefer

OK with EFT

but disagreement with EFT which has $\chi_4^{\nu^2} \neq 0$

while PM calculations by **Bern et al.** **Porto et al.** also hint at $\chi_4^{\nu^2} = 0$

Action plan

1) find out all dissipative terms at 5PN

~~potential modes~~

tails

memory terms

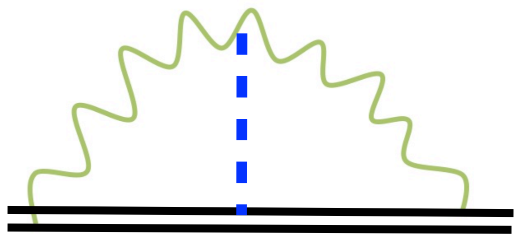
$\left(\begin{matrix}\text{radiation}\\ \text{reaction}\end{matrix}\right)^2$

2) compute **relative acceleration**

without distinction between **conservative** and **dissipative**

3) compute contribution to scattering angle

Tails at 4PN, 5PN….and beyond



Almeida SF Sturani

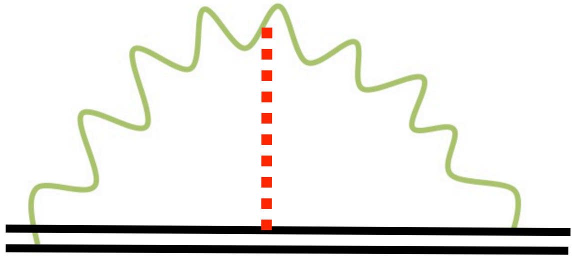
$$\mathcal{S}^{(e,r)}_{tail} \simeq -G_N^2 E \frac{2^{r+2}(r+3)(r+4)}{(r+1)(r+2)(2r+5)!} \int \frac{dp_0}{2\pi} (p_0^2)^{r+3} I^{ijR}(p_0) I^{ijR}(-p_0) \left[\frac{1}{\tilde{\varepsilon}} - \gamma_r^{(e)}\right]$$

$$\gamma_r^{(e)} \equiv \frac{1}{2}\left(H_{r+\frac{5}{2}} - H_{\frac{1}{2}} + 2H_r + 1\right) + \frac{2}{(r+2)(r+3)} = \left\{\frac{41}{30}, \frac{82}{35}, \frac{1819}{630}, \dots\right\}$$

$$\frac{1}{\tilde{\varepsilon}} = \frac{1}{\tilde{\varepsilon}} + i\pi\ \mathrm{sgn}(p_0) + \log\left(\frac{p_0^2 e^{\gamma}}{\pi\mu^2}\right)$$

$$\mathcal{S}^{(m,r)}_{tail} \simeq -G_N^2 E \frac{2^{r+4}(r+2)(r+4)}{(r+1)(r+3)^2(2r+5)!}$$
$$\times \int \frac{dp_0}{2\pi} (p_0^2)^{r+3} J_{l|jRk}(p_0) J_{n|aRk}(-p_0) \left[\delta_{ja}\delta_{ln} + (r+1)\delta_{jn}\delta_{la}\right] \left[\frac{1}{\tilde{\varepsilon}} - \gamma_r^{(m)}\right]$$

$$\gamma_r^{(m)} \equiv \frac{1}{2}\left(H_{r+\frac{5}{2}} - H_{\frac{1}{2}} + 2H_{r+3} + 2\right) - \frac{2r^2+13r+22}{(r+2)(r+3)(r+4)} = \left\{\frac{49}{20}, \frac{22}{7}, \frac{4541}{1260}, \dots\right\}$$



$$S^{(LQ^2)}_{eff} = \frac{8}{15} G_N^2 \int dt\, \overset{....}{Q}_{il} \dddot{Q}_{jl} \epsilon_{ijk} L_k$$

SF Sturani

In-in formalism

(irrelevant for potential modes *and tails*)

$$e^{iS_{eff}[x]} = \int \mathcal{D}[h] e^{iS_{tot}[x,h]}$$

$$S_{tot}[x,h] \to S_{tot}[x_1,h_1] - S_{tot}[x_2,h_2]$$

$$x_- = x_1 - x_2$$
$$x_+ = \frac{1}{2}(x_1 + x_2)$$
$$\langle h_- h_+ \rangle = G_{ret}$$

$$\left.\frac{\delta S_{eff}[x_\pm]}{\delta x_-}\right|_{x_-=0} = 0$$

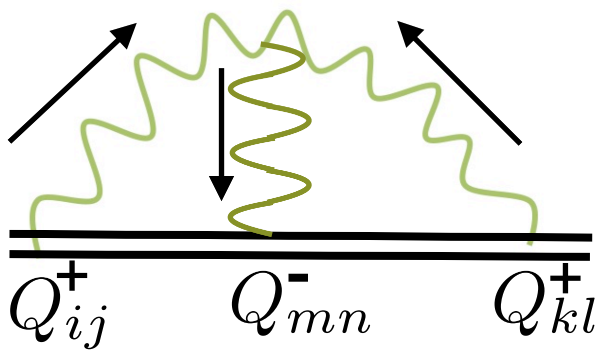
if $S_{eff}[x_\pm] = S_{eff}[x_1] - S_{eff}[x_2]$ the dynamics is conservative

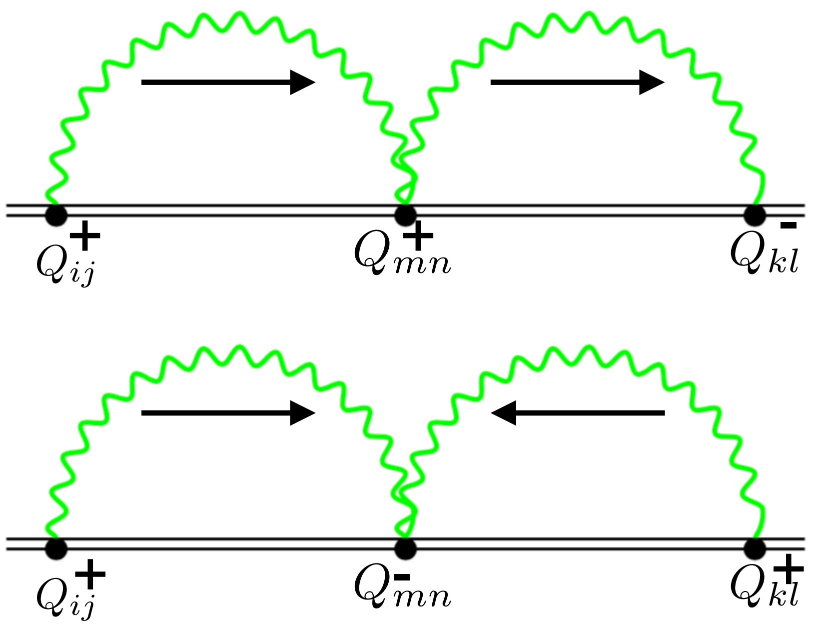
and one can avoid using Keldish variables

otherwise $S_{eff}[x\pm] = S_{eff}[x_1] - S_{eff}[x_2] + A[x_1, x_2]$

but the separation is ambiguous

Memory and double emission





$$\frac{1}{35}\int G^2\left[8\left(\dddot{Q}^+\right)^2\ddot{Q}^- + 7\left(\ddddot{Q}^+\right)^2 Q^- - 12\dddot{Q}^+\ddot{Q}^+\ddot{Q}^- - 14\dddot{Q}^+Q^+\dddot{Q}^-\right]$$

$$\int_t G^2\left[Q_-^{(4)}Q_+^{(4)}Q_+ - \frac{1}{2}Q_+^{(4)}Q_+^{(4)}Q_-\right]$$

Blumlein Maier Marquard Schaefer

Foffa Sturani Almeida

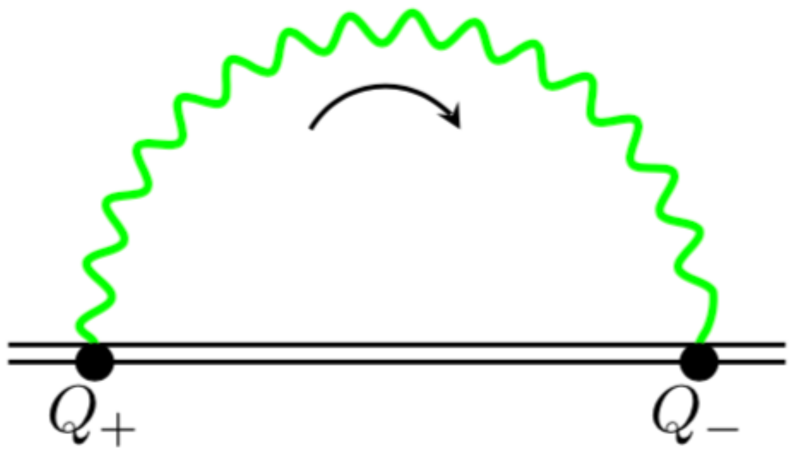
Brunello

$$\left.\frac{\delta S_{eff}[x_\pm]}{\delta x_-}\right|_{x_-=0} = 0$$

$$\mathbf{a}^k = G^2\left(\mathbf{r}^i\delta_{kl} + \mathbf{r}^l\delta_{ki} - \frac{2\delta_{il}}{3}\mathbf{r}^k\right)\sum_{n=0}^{4}\alpha_{8-n}Q_{ij}{}^{(8-n)}Q_{jl}^{(n)}$$

$$\mathbf{a} = \frac{G^4M^4\nu^2}{r^6}\left[\left(c_1v^4 + c_2v^2v_n^2 + c_3v_n^4\right)\mathbf{r} + \left(c_4v^2 + c_5v_n^2\right)v_r\mathbf{v}\right] + \mathcal{O}\left(G^5\right)$$

$\left(\text{radiation reaction}\right)^2$



$$\mathbf{a}^i_{rr} = -\frac{2}{5} G Q^{(5)}_{ij} x^j$$

$$\mathbf{a} = -\frac{GM}{r^3}\mathbf{r} + \cdots + \mathbf{a}_{BT} + \cdots + \mathbf{a}\Big|_{mem,ds-e} + \ldots$$

N

Burke Thorne 2.5PN

5PN

$+\mathbf{a}_{BT^2}$

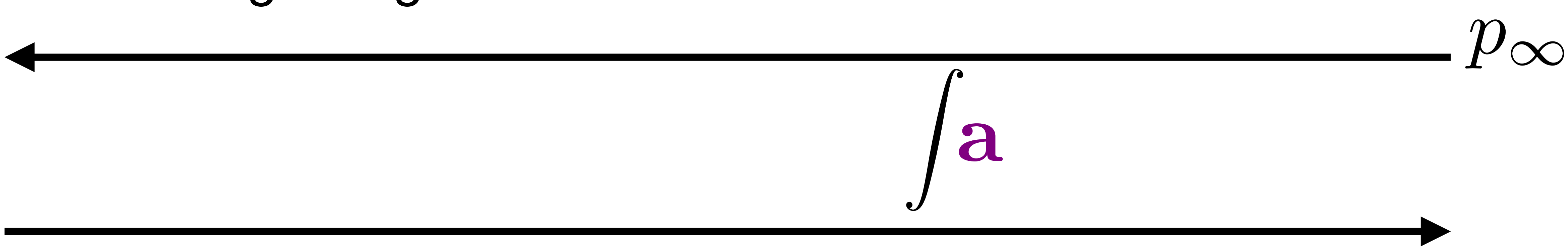
$$\mathbf{a}_{BT^2} = \frac{G^4 M^4 \nu^2}{r^6}\left[\left(c_1 v^4 + c_2 v^2 v_n^2 + c_3 v_n^4\right)\mathbf{r} + \left(c_4 v^2 + c_5 v_n^2\right) v_r \mathbf{v}\right] + \mathcal{O}\left(G^5\right)$$

Scattering angle

$$\chi = \text{arcos}\left(\frac{\mathbf{p}^+ \cdot \mathbf{p}^-}{|\mathbf{p}^+||\mathbf{p}^-|}\right) \qquad \mathbf{p}^+ = \mathbf{p}^- + \mu \int_{-\infty}^{\infty} \mathrm{d}t\ \mathbf{a}$$

perturbative along straight lines…

$$\mathbf{a} = \mathcal{O}(G^4 v^4)$$



$$\chi_4 = \frac{1}{2}\alpha\pi p_\infty^6\,, \quad \text{with} \quad \alpha = -\frac{48c_1 + 8c_2 + 3c_3}{128}\nu^2$$

$$\alpha_{memory} = -\frac{2267}{210}\nu^2$$

$$\alpha_{double\ emission} = \frac{251}{12}\nu^2$$

$$\alpha_{BT^2} = \frac{97}{50}\nu^2$$

Scattering angle

…and on simple 2.5 PN radiation-reaction trajectory

$$\ddot{\mathbf{r}} = \mathbf{a}_{BT} = \mathcal{O}(G^2 v^2)$$

$$\int \mathbf{a}_{BT} \quad \Rightarrow \quad \alpha_{BT+BT} = \frac{479}{25}\nu^2$$

Scattering angle

…and on simple 2.5 PN radiation-reaction trajectory

$$\ddot{\mathbf{r}} = \mathbf{a}_{BT} = \mathcal{O}(G^2 v^2)$$

$$\int \mathbf{a}_{BT} \Rightarrow \alpha_{BT+BT} = \frac{479}{25}\nu^2$$

$$\tilde{\chi}_4 - \chi_4^{Schw} = \pi \left\{ -\frac{15}{4}\nu + \left(\frac{123}{256}\pi^2 - \frac{557}{16}\right)\nu p_\infty^2 + \left[-\frac{6113}{96} + \frac{33601}{16384}\pi^2 - \frac{37}{5}\log\left(\frac{p_\infty}{2}\right)\right]\nu p_\infty^4 \right.$$
$$\left. + \left[\left(-\frac{615581}{19200} + \frac{93031}{32768}\pi^2 - \frac{1357}{280}\log\left(\frac{p_\infty}{2}\right)\right)\nu + \frac{576}{25}\nu^2\right] p_\infty^6 \right\},$$

Conclusions

We followed an **eom-based** approach which allowed us to:

evaluate **altogether** conservative and dissipative effects of memory (and memory-like) contributions to the scattering angle

include (radiation-reaction)^2 terms

Still a residual term

$$\pi \left(\frac{24}{5}\right)^2 \nu^2 p_\infty^6$$

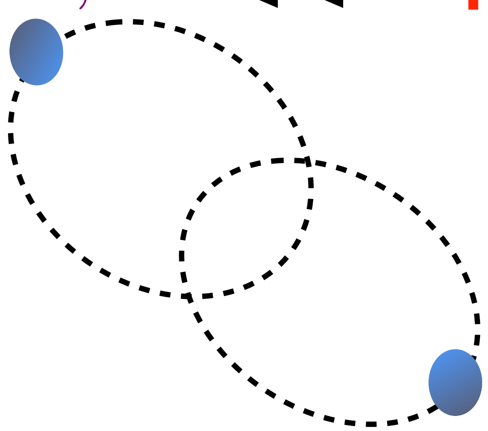
remains in the 4PM, 5PN part of the scattering angle, breaking down the expected polynomiality

Spare slides on EFT

Length scales in a binary system: a double hierarchy

r <<* $\lambda_{GW} \simeq \frac{r}{v}$

$l_{BH,NS}$ <<** r



$$S_{m_a} \simeq S^a_{pp} = -m_a \int \sqrt{-g_{\mu\nu} \mathrm{d}x^\mu_a \mathrm{d}x^\nu_a} = -m_a \int \mathrm{d}t \sqrt{-g_{\mu\nu} \dot{x}^\mu_a \dot{x}^\nu_a}$$

$a = 1, 2$

up to 5PN
no spin

*during the inspiral phase

**during the inspiral phase and for compact objects

Integrating out the gravitational field

$$S_g = \frac{1}{32\pi G}\int \mathrm{d}^{(d+1)}x\sqrt{-g}\left[R - g_{\mu\nu}g^{\alpha\beta}g^{\gamma\delta}\Gamma^{\mu}_{\alpha\beta}\Gamma^{\nu}_{\gamma\delta}\right]$$

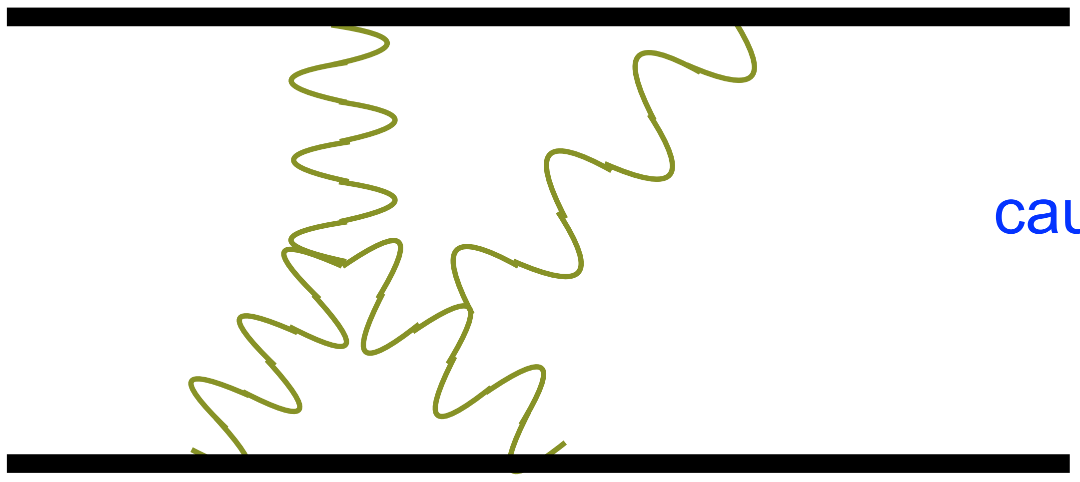
gauge fixing term

$$S_{tot}[x,h] = S_g[h] + S_m[x,h]$$

$$\mathrm{e}^{iS_{eff}[x]} = \int \mathcal{D}[h]\mathrm{e}^{iS_{tot}[x,h]}$$

perturbative expansion in

$$\frac{GM}{r}\ (\simeq v^2)$$



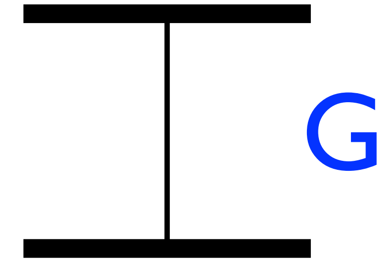
with causal boundary conditions

r $\ll$ $\lambda_{GW} \simeq \frac{r}{v}$

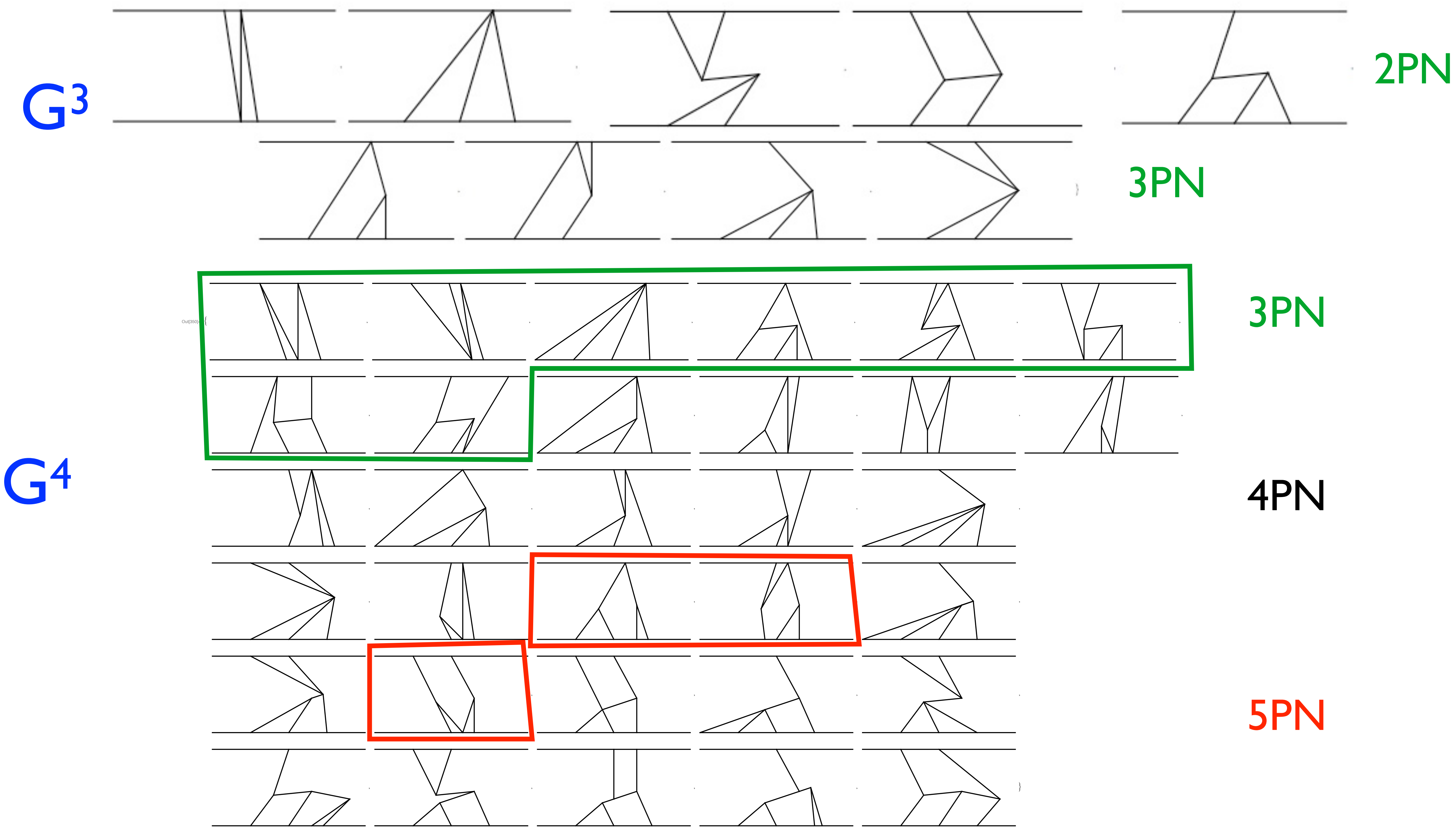
potential (quasi-static) modes $k \simeq \frac{1}{r}$ $\gg$ $\omega_{gw} \simeq \frac{v}{r} \simeq k_{gw}$ radiation modes

Method of regions: near zone

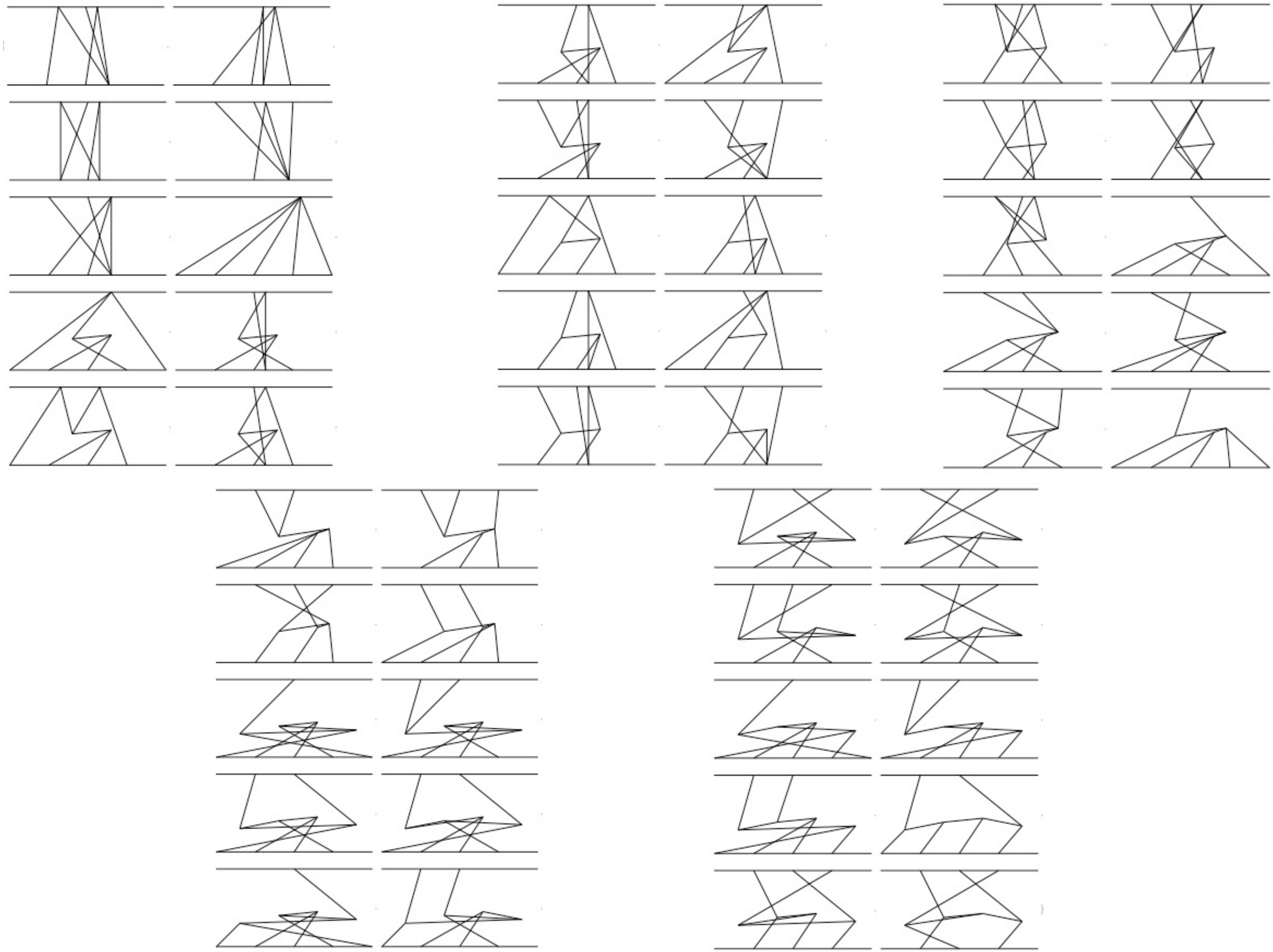
$$\frac{i}{\mathbf{k}^2 - k_0^2} \quad \longrightarrow \quad \frac{i}{\mathbf{k}^2} \sum_{n \geq 0} \left(\frac{k_0^2}{\mathbf{k}^2} \right)^n$$



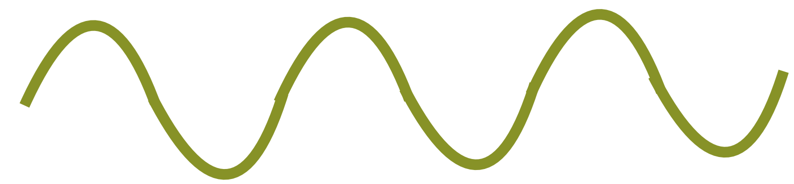
605 diagrams to evaluate at 4PN



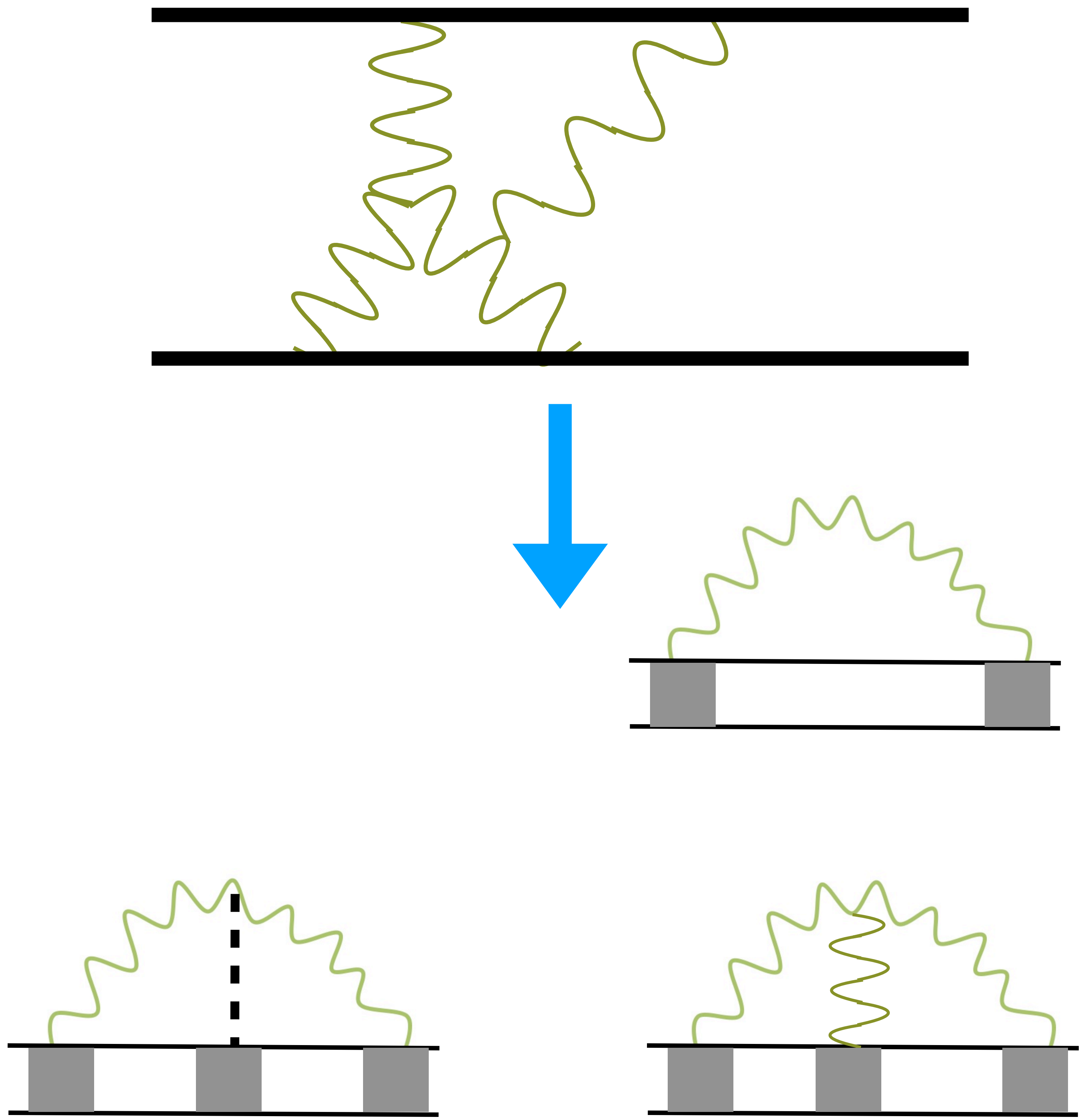
G^5 (50 relevant at 4PN, out of 164)



Method of regions: far zone



$$\frac{i}{\mathbf{k}^2 - k_0^2}$$



$+ \dots$

$$S_m[x,h] = -\int \mathrm{d}\tau \left[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2} I_{ij}E^{ij} + \frac{2}{3} J_{ij}B^{ij} + c_E E^{ij}E_{ij} + \dots \right.$$

multipole expansion and matching

KK variables

$$c_d = \frac{2(d-1)}{d-2}$$

$$m_p \equiv \frac{1}{\sqrt{32\pi G}}$$

$$g_{\mu\nu} = e^{2\phi} \begin{pmatrix} -1 & A_j \\ A_i & e^{-c_d\phi}(\delta_{ij} + \sigma_{ij}) - A_i A_j \end{pmatrix}$$

$$\phi \;\; A_i \;\; \sigma_{ij} \longrightarrow \frac{\phi \;\; A_i \;\; \sigma_{ij}}{m_p}$$

$$S_{pp}^{a} = -m_a \int d\tau_a = -m_a \int dt\; e^{\phi/m_{Pl}} \sqrt{\left(1 - \frac{A_i}{m_{Pl}} v_a^i\right)^2 - e^{-c_d\phi/m_{Pl}}\left(v^2 + \frac{\sigma_{ij}}{m_{Pl}} v_a^i v_a^j\right)}$$

$$S_g \simeq \int \mathrm{d}^{d+1}x \sqrt{-\gamma}\Bigg\{\frac{1}{4}\left[(\vec{\nabla}\sigma)^2 - 2(\vec{\nabla}\sigma_{ij})^2 - \left(\dot{\sigma}^2 - 2(\dot{\sigma}_{ij})^2\right)\mathrm{e}^{\frac{-c_d\phi}{m_{Pl}}}\right] - c_d\left[(\vec{\nabla}\phi)^2 - \dot{\phi}^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right]$$
$$+\left[\frac{F_{ij}^2}{2} + \left(\vec{\nabla}\cdot\vec{A}\right)^2 - \dot{\vec{A}}^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} + 2\frac{\left[F_{ij}A^i\dot{A}^j + \vec{A}\dot{\vec{A}}(\vec{\nabla}\cdot\vec{A})\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} - c_d\dot{\phi}\vec{A}\vec{\nabla}\phi}{m_{Pl}}$$
$$+2c_d\left(\dot{\phi}\vec{\nabla}\cdot\vec{A} - \dot{\vec{A}}\vec{\nabla}\phi\right) + \frac{1}{m_{Pl}}\left[-\dot{\sigma}A_i\hat{\Gamma}^i_{jj} + 2\dot{\sigma}_{ij}\left(A_k\hat{\Gamma}^k_{ij} - A_i\hat{\Gamma}^j_{kk}\right)\right] - c_d\frac{\dot{\phi}^2\vec{A}^2}{m_{Pl}^2}$$
$$+\frac{1}{m_{Pl}}\sigma^{ij}\left(\frac{1}{2}\sigma_{kl,i}\sigma^{kl}_{,j} + \sigma_{ik,l}\sigma_j^{k,l} + \sigma_{ik,l}\sigma_j^{l,k} - \sigma^k_{i,k}\sigma^l_{j,l} + \sigma_{,i}\sigma^k_{j,k} - \frac{1}{2}\sigma_{ij,k}\sigma^{,k} - \sigma_{ik,j}\sigma^{,k} - \frac{1}{4}\sigma_{,i}\sigma_{,j}\right)$$
$$+\frac{1}{2m_{Pl}}\sigma\left(\frac{1}{4}\sigma_k\sigma k + \sigma^{ki}_{,i}\sigma^{,j}_{kj} - \sigma_{ki,j}\sigma^{kj,i} - \frac{1}{2}\sigma_{ki,j}\sigma^{ki,j}\right)\Bigg\} + \mathcal{O}(5PN)$$