A stringy massive double copy

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QCD Meets Gravity
Zürich, December 2022

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Probing the string spectrum

- two universal string parameters: α' and g_S

- infinitely many *physical* states

$$M_N^2 = N \, \frac{1}{\alpha'^2}$$

that are thought of as:

1. mass eigenstates $\Rightarrow$ on–shell mass

2. irreps of $SO(D-1)$ or $SO(D-2)$ $\Rightarrow$ TT

1–particle states à la Bargmann and Wigner ?

The graviton as a field

1. *flat* spacetime:
   - symmetric, massless, TT

$$\Box G_{\mu\nu} = 0 \quad , \quad \partial^\mu G_{\mu\nu} = 0 \quad , \quad [G] = 0$$

   - Lorentz invariance + locality $\overset{\nexists \text{ ghost}}{\Longrightarrow}$ (linearized) diffeos

2. *nonlinear* completion:
   - Einstein's GR $\Rightarrow$ unique kinematics of $g_{\mu\nu}$

Gupta, Kraichnan, Weinberg, Boulware, Deser, ... '50s – '80s

3. on–shell amplitudes: double copy, example:

$$\mathcal{A}_4^{\text{tree}} \sim \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right) \overset{\text{CK Duality}}{\Longrightarrow} \mathcal{M}_4^{\text{tree}} \sim \left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Bern, Carrasco, Johansson '08,'10

2 propagating d.o.f.

The graviton as a string state

- string embedding: *universally* a *bulk* state
  - construction: tensor product of massless transverse brane vectors

Scherk and Schwarz 1974

$$V_G^{(-1,-1)}(z,\bar{z},\varepsilon,q) = V_A^{(-1)}(z,\epsilon,p)\,V_A^{(-1)}(\bar{z},\tilde{\epsilon},p)$$
$$= e^{-\phi(z)-\tilde{\phi}(\bar{z})}\,\varepsilon_{\mu\nu}\,\psi^\mu(z)\tilde{\psi}^\nu(\bar{z})\,e^{iqX(z,\bar{z})}$$

$$\varepsilon_{\mu\nu}q^\mu = \varepsilon_{\mu\nu}q^\nu = 0 \quad , \quad q^2 = 0 \quad , \quad \varepsilon_{\mu\nu} = \varepsilon_{\nu\mu} \quad , \quad \varepsilon_{\mu\nu}\eta^{\mu\nu} = 0$$

eg. Mayr, Stieberger '94

identical structure in 10D and in 4D

  - 3– and 4–point amplitudes ⇒ effective action *is* the EH

Green, Schwarz, Brink 1982
Gross, Sloan 1987

- more generally:
  1. $(\text{open string spectrum})_\mathrm{L} \otimes (\text{open string spectrum})_\mathrm{R} \Rightarrow$ closed
  2. amplitudes: KLT relations

Kawai, Lewellen, Tye 1986
Stieberger '09, ...

$$\xrightarrow{\alpha' \to 0} \text{DC}$$

Massive fields?

1. massive graviton: 5 p.d.o.f.
   - *flat* spacetime: $\mathcal{L}_{\rm FP} = \frac{1}{2} h^{\mu\nu} \varepsilon_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} - \frac{m_{\rm FP}^2}{2}(h^{\mu\nu}h_{\mu\nu} - h^2)$

     Fierz, Pauli 1939
   - *nonlinear* completion: ghost–free massive gravity (dRGT)

     de Rham, Gabadadze, Tolley '10, '11

     massive DC: can work up to 4–points, ∃ spurious poles beyond

     $$\mathcal{A}_4^{\rm tree} = g^2\Big(\frac{n_s c_s}{s-m^2} + \frac{n_t c_t}{t-m^2} + \frac{n_u c_u}{u-m^2}\Big)$$

     Momeni, Rumbutis, Tolley '20
     Johnson, Jones, Paranjape '20
   - supersymmetric massive gravity

     Engelbrecht, Jones, Paranjape '22
   - ghost–free bimetric theory: 7 p.d.o.f.

     $$\mathcal{L}_{\rm HR} = m_g^2 \sqrt{g} R(g) + m_f^2 \sqrt{f} R(f) - 2\, m_g^2 m_f^2 \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \Big[\Big(\sqrt{g^{-1} f}\Big)^{\mu}{}_{\nu}\Big]$$

     Hassan, Rosen '11
2. massive HS: constructed for the first time this year

   Ochirov, Skvortsov '22

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Lightest massive spin–2 string states

- appear in both open and closed string spectra
- look at *universal* massive multiplets of $\mathcal{N} = 1$, $D = 4$ compactifications, 1st level contains $(2, 3/2, 3/2, 1)$

Feng, Lüst, Schlotterer, Stieberger, Taylor '10
Feng, Lüst, Schlotterer '12

- spin–2: *identical* structure in 10D and in 4D

$$V^{(-1)}_{M,\text{open}}(x,\alpha,k) \quad = \quad T^a\, e^{-\phi(x)}\, \alpha_{\mu\nu}\, i\partial X^\mu(x)\psi^\nu(x)\; e^{ikX(x)}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\tfrac{1}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

Koh, Troost, van Proeyen 1987
Bianchi, Guerrieri '15

- spin–1: $U(1)$ current $J(z)$ due to internal fermions

Ferrara, Lüst, Theisen 1989
Dixon, Kaplunovsky, Louis 1989

$$V^{(-1)}_{\boldsymbol{A}}(z,a,p) = T^a\, e^{-\phi(z)}\, a_\mu\, \psi^\mu(z)\mathcal{J}(z)\, e^{ipX(z)}$$

$$p\cdot a = 0 \quad , \quad p^2 = -\tfrac{1}{\alpha'}$$

[normalisations in front of v.o. omitted for clarity]

Lightest massive spin–2 string states

1. brane state, $m^2 = 1/\alpha'$

Ferrara, Kehagias, Lüst '19
Lüst, CM, Mazloumi, Stieberger '21

2. bulk state, $m^2 = 4/\alpha'$: we *construct* this state as the tensor product of massive transverse vectors

$$\begin{aligned} V^{(-1,-1)}_{M,\text{closed}}(z,\overline{z},\alpha,k) &= V^{(-1)}_{\boldsymbol{A}}(z,a,p)\,V^{(-1)}_{\boldsymbol{A}}(\overline{z},\widetilde{a},p) \quad , \quad p^\mu = k^\mu/2 \\ &= e^{-\phi(z)-\phi(\overline{z})}\,\alpha_{\mu\nu}\,\psi^\mu(z)\mathcal{J}(z)\,\widetilde{\psi}^\mu(\overline{z})\widetilde{\mathcal{J}}(\overline{z})\,e^{ikX(z,\overline{z})} \end{aligned}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\frac{4}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

- this *mimics* the massless graviton's structure as a bulk state
  We are naturally in a *bigravity* (even multigravity) setup!

  Lüst, CM, Mazloumi, Stieberger *to appear*

- supersymmetric massive DC: self–interactions of massive spin–2 multiplets $\Rightarrow$ dRGT!

  Engelbrecht, Jones, Paranjape '22

A stringy massive DC

3–point string amplitudes:

$$\mathcal{M}_{GGG} = \varepsilon_{1\mu\rho}\,\varepsilon_{2\nu\sigma}\,\varepsilon_{3\lambda\kappa}\,E_{GGG}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}$$

$$\mathcal{M}_{GGM} = \varepsilon_{1\mu\rho}\,\varepsilon_{2\nu\sigma}\,\alpha_{\lambda\kappa}\,E_{GGM}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_3\rangle\,\langle\widetilde{\mathcal{J}}_3\rangle$$

$$\mathcal{M}_{MMG} = \alpha_{1\mu\rho}\,\alpha_{2\nu\sigma}\,\varepsilon_{\lambda\kappa}\,E_{MMG}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_1\,\mathcal{J}_2\rangle\,\langle\widetilde{\mathcal{J}}_1\,\widetilde{\mathcal{J}}_2\rangle$$

$$\mathcal{M}_{MMM} = \alpha_{1\mu\rho}\,\alpha_{2\nu\sigma}\,\alpha_{3\lambda\kappa}\,E_{MMM}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_1\,\mathcal{J}_2\,\mathcal{J}_3\rangle\,\langle\widetilde{\mathcal{J}}_1\,\widetilde{\mathcal{J}}_2\,\widetilde{\mathcal{J}}_3\rangle$$

$$E = |z_{12}|^{\alpha' p_1\cdot p_2}\,|z_{13}|^{\alpha' p_3\cdot p_3}\,|z_{23}|^{\alpha' p_2\cdot p_3} \quad , \quad \mathcal{B}^{\mu\nu\lambda} = 2\alpha'\,(\eta^{\mu\nu}p_1^{\lambda} + \eta^{\nu\lambda}p_2^{\mu} + \eta^{\lambda\mu}p_3^{\nu})$$

- manifest **DC** structure, e.g. $\mathcal{M}_{MMG} = \overline{\mathcal{A}}_{\boldsymbol{AA}A}\cdot\overline{\mathcal{A}}_{\boldsymbol{AA}A}$
- WS location dependence: Koba–Nielsen factors vs $\mathcal{J}(z)$ correlators
- but $\langle\mathcal{J}\rangle = 0$, $\langle\mathcal{J}_1\mathcal{J}_2\mathcal{J}_3\rangle = 0$!

Lüst, CM, Mazloumi, Stieberger *to appear*

A stringy massive DC

- we thus need a non–abelian current for a non–vanishing $\mathcal{M}_{MMM}$!

  Lüst, CM, Mazloumi, Stieberger *to appear*

- let's look at lightest massive spin–2 multiplet of $\mathcal{N}=2$, $D=4$ compactifications

  Feng, Lüst, Schlotterer '12

  - spin–2: same as for $\mathcal{N}=1$!

$$V^{(-1)}_{M,\text{open}}(x,\alpha,k) \quad = \quad T^a\, e^{-\phi(x)}\, \alpha_{\mu\nu}\, i\partial X^\mu(x)\psi^\nu(x)\; e^{ikX(x)}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\tfrac{1}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

  - spin–1: $SU(2)$ triplet of currents $J^A(z)$ due to internal fermions

    Banks, Dixon 1988
    Ferrara, Lüst, Theisen 1989

$$V^{(-1)}_{\boldsymbol{A}}(z,a,p) = T^a\, e^{-\phi(z)}\, a^A_\mu\, \psi^\mu(z)\mathcal{J}^A(z)\, e^{ipX(z)}$$

$$p\cdot a = 0 \quad , \quad p^2 = -\tfrac{1}{\alpha'}$$

now $\langle \mathcal{J}^A_1 \mathcal{J}^B_2 \mathcal{J}^C_3 \rangle \quad \Rightarrow \quad f^{ABC}$!

[normalisations in front of v.o. omitted for clarity]

Three–point results

- $\mathcal{M}_{GGM} = 0$
  - true for both abelian and non–abelian case
  - extension of Landau–Yang theorem? agreement with

Arkani–Hamed, Huang, Huang '17

- $\mathcal{M}_{MMG}$ same for abelian and non–abelian

$$\begin{aligned}\mathcal{M}_{MMG} \quad = \quad & \Big[- (p_1 \cdot \varepsilon \cdot p_2) \,\mathrm{Tr}(\alpha_1 \cdot \alpha_2) \\ & + (p_2 \cdot \alpha_1 \cdot p_2) \,\mathrm{Tr}(\alpha_2 \cdot \varepsilon) + (p_1 \cdot \alpha_2 \cdot p_1) \,\mathrm{Tr}(\alpha_1 \cdot \varepsilon) \\ & - 2\, p_1 \cdot \alpha_2 \cdot \varepsilon \cdot \alpha_1 \cdot p_2 + 2\, p_1 \cdot \alpha_2 \cdot \alpha_1 \cdot \varepsilon \cdot p_2 + 2\, p_2 \cdot \alpha_1 \cdot \alpha_2 \cdot \varepsilon \cdot p_1 \Big] \, \alpha'^2\end{aligned}$$

- $\mathcal{M}_{MMM} = 0$ for abelian but non–zero for non–abelian:

$$\begin{aligned}\mathcal{M}_{MMM} \quad = \quad & \Big[(k_1 \cdot \alpha_3 \cdot k_1) \,\mathrm{Tr}(\alpha_1 \cdot \alpha_2) + (k_2 \cdot \alpha_1 \cdot k_2) \,\mathrm{Tr}(\alpha_2 \cdot \alpha_3) + (k_3 \cdot \alpha_2 \cdot k_3) \,\mathrm{Tr}(\alpha_3 \cdot \alpha_1) \\ & + 2\, k_2 \cdot \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot k_1 + 2\, k_3 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_1 \cdot k_2 + 2\, k_1 \cdot \alpha_3 \cdot \alpha_1 \cdot \alpha_2 \cdot k_3 \Big] \alpha'^2\end{aligned}$$

$\Rightarrow$ same form as universal 3–graviton string amplitude

Gross, Sloan 1987
Lüst, Theisen, Zoupanos 1988
Stieberger, Taylor '14 – '16

Lüst, CM, Mazloumi, Stieberger *to appear*

[normalisations in front of amplitudes omitted for clarity]

"Effective" Lagrangians

- *finite cubic* amplitudes: can we compare with field theory?
- let's compare with bigravity expanded around Minkowski bkgs:
  $\Rightarrow$ two eigenstates: massless graviton and massive spin-2 field

Hassan, Schmidt–May, von Strauss '12
Babichev, Marzola, Raidal, Schmidt–May, Urban, Veerm, von Strauss '16

$$\mathcal{L}^{\text{eff}}_{\text{G}^3} = g_c\, G^{\mu\nu}\left[\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\right]$$

graviton self–interactions: strictly GR

Boulanger, Damour, Gualtieri, Henneaux 2000

$$\mathcal{L}^{\text{eff}}_{\text{GM}^2} = g_c\left[G^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right) + z\, M^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\right)\right]$$

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} = \frac{g_c}{\alpha'}\left\{y\left[M^3\right] + 2\alpha' M^{\mu\nu}\left[\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - x\,\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right]\right\}$$

$$(x,y,z) = \begin{cases}(2,0,2), & \text{if } M_{\mu\nu} \text{ is a bulk state} \Rightarrow \text{ tune } \beta_1=\beta_3\\ (3,1,1), & \text{if } M_{\mu\nu} \text{ is a brane state}\end{cases}$$

Lüst, CM, Mazloumi, Stieberger '21 and *to appear*

Conclusions

- for the first time, we have [to *cubic* order]
  - extracted a set of "low–energy" interaction vertices of massive spin–2 string states from string amplitudes
  - formulated a stringy massive DC: need
    1. non–abelian Chan–Paton
    2. *nonabelian* internal currents
- the string and bimetric yield the *same* set of terms at 3–points
  1. brane massive spin–2 kinematics not GR–like (ghosts?)
  2. ∃ region in parameter space ($\beta_1 = \beta_3$) for bulk $M_{\mu\nu}$ with a matching

Lüst, CM, Mazloumi, Stieberger '21 and *to appear*

- Future: multiplicity of closed massive spin–2, beyond 3–points, massive HS from twisted strings, …