Climbing Towers & Counting Virtues of the Double-Copy

Color-dual constraints on effective operators & higher-derivatives from flavor and symmetric-structure

Nic H. Pavao

Carrasco, Lewandowski, NHP [2203.03592, 2211.04441],
NHP [2210.12800], Carrasco, NHP [2211.04431, 2212.xxxx]

Northwestern University

Some Effective Questions with Virtuous Answers

Some Effective Questions with Virtuous Answers

Supergravity + DBIVA

from double-copy?

Some Effective Questions with Virtuous Answers

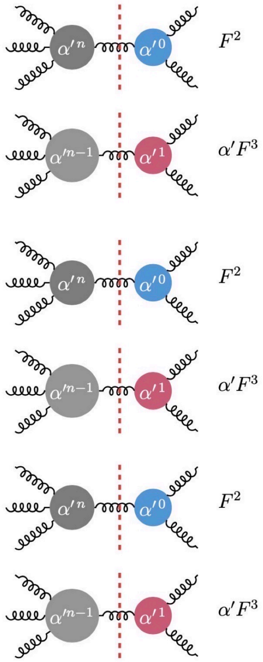
YM + pions   color-dual?

$\updownarrow$

YM + F^3 color-dual?

Carrasco, Lewandowski, NHP
[2203.03592, 2211.04441]

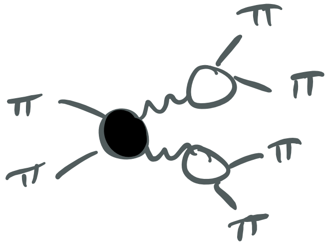
Infinite tower $\Rightarrow$ UV!



Some Effective Questions with Virtuous Answers

YM + pions   color-dual?

spurious poles!

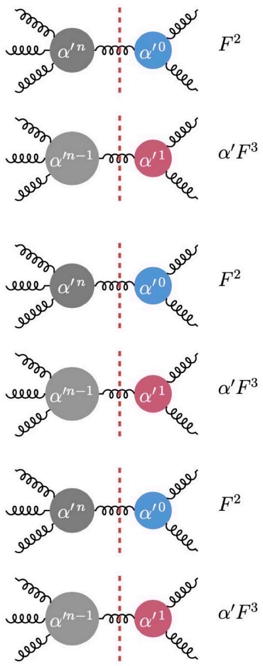


YM + F^3 color-dual?

Infinite tower ⇒ UV!

anomaly cancelling

UV finite $\mathcal{N}=4$ supergravity?



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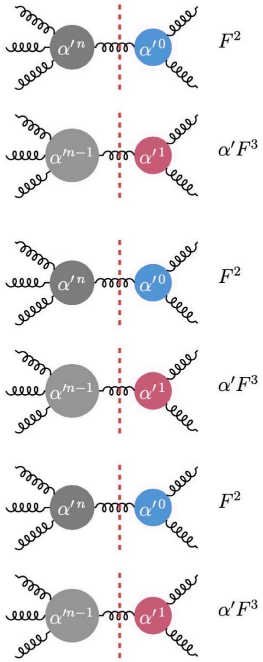

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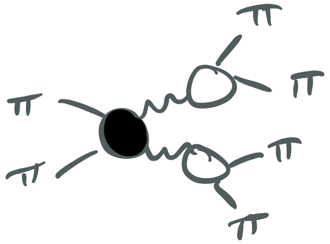
Born-Infeld ~ NLSM ⊗ YM + HD?



Infinite tower ⇒ UV!

$$\sim \mathcal{A}^{\text{BI,1-loop}}_{(++++)} \sim (s^4+t^4+u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\sim \mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim (s^6+t^6+u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

higher-spin ⊗ Adler's zero

Carrasco, NHP [2211.04431, 2212.xxxxx]

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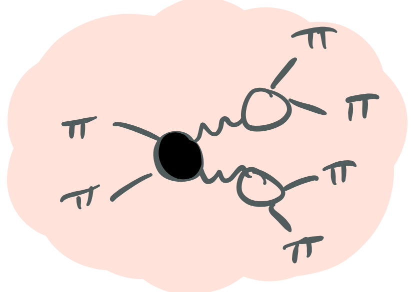

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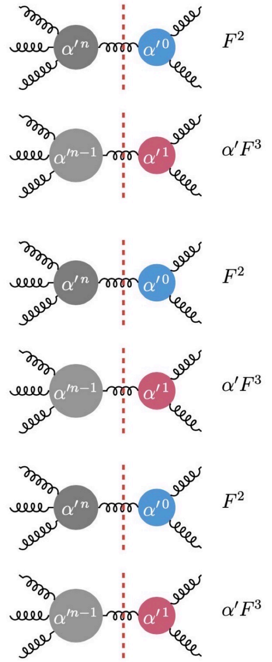

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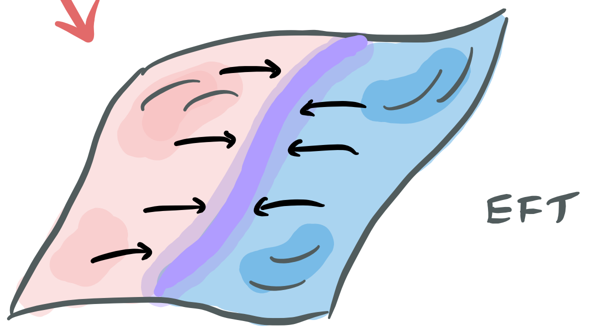

Decompose YM + pions with flavor

NHP [2210.12800]

Carrasco, NHP [2211.04431]

Decompose BI + HD with symmetric-structure

Starting Point

"The model that launches at least one ship": covariantized NLSM

$$\mathcal{L} = -\underbrace{\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]}_{\text{YM}} + \frac{1}{2}\mathrm{Tr}\left[\underbrace{\left(1-\Lambda\pi^2\right)^{-1}\partial_\mu\pi\left(1-\Lambda\pi^2\right)^{-1}\partial^\mu\pi}_{\text{NLSM}}\right]$$

Starting Point

Carrasco, Lewandowski, NHP [2211.04441]

"The model that launches at least one ship": **covariantized NLSM**

$$\mathcal{L}^{\text{cov.}\pi} = -\frac{1}{4}\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2}\text{Tr}\left[\left(1-\Lambda\pi^2\right)^{-1} D_\mu\pi \left(1-\Lambda\pi^2\right)^{-1} D^\mu\pi\right]$$

$$D_\mu\pi = \left(\partial_\mu\pi^a - igf^{abc}A^b_\mu\pi^c\right)T^a$$

Starting Point

Carrasco, Lewandowski, NHP [2211.04441]

"The model that launches at least one ship": covariantized NLSM

$$\mathcal{L}^{\text{cov.}\pi} = -\frac{1}{4}\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2}\text{Tr}\left[\left(1-\Lambda\pi^2\right)^{-1}D_\mu\pi\left(1-\Lambda\pi^2\right)^{-1}D^\mu\pi\right]$$

pheno application

$$\mathcal{A}^{\text{DBIVA+SG}} = \underbrace{\mathcal{A}^{\text{NLSM+YM}}} \otimes \mathcal{A}^{\text{sYM}}$$

→ only works if color-dual!

→ observables for α-attractor models of inflation

Kallosh, Linde, Ferrara, Carrasco

Matt's talk QMG '21

formal interest

$$m^2A^\mu A_\mu \to \frac{m^2}{g^2}(D^\mu U)(D_\mu U^{-1})$$

Stückelberg, $\Lambda = \frac{m^2}{g^2}$

→ massive Yang-Mills color-dual?

↓

massive gravity

See afternoon talks!

de Rahm, Gabadadze, Tolley, Johnson, Jones, Paranjape, Gonzalez, Momeni, Rambutis

Color-dual?

Carrasco, Lewandowski, NHP [2211.04441]

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Nothing new @ 4-point
YM + YMS + NLSM

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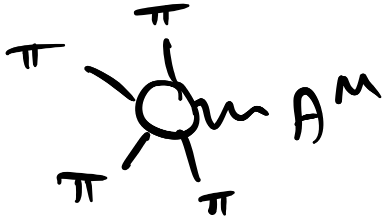
5-point? (radiative corrections)



$\equiv A(1_\pi, 2_\pi, 3_\pi, 4_\pi, 5_A)$

Not color-dual!

↳ $\{A(1,\sigma,4,5)\} \Rightarrow$

$$s_{1|2}A_5^{\text{cov.}\pi}(1_\pi,2_\pi,3_\pi,4_\pi,5_A) + s_{13|2}A_5^{\text{cov.}\pi}(1_\pi,3_\pi,2_\pi,4_\pi,5_A)$$
$$+ s_{134|2}A_5^{\text{cov.}\pi}(1_\pi,3_\pi,4_\pi,2_\pi,5_A) \neq 0,$$

Color-dual?

Carrasco, Lewandowski, NHP [2211.04441]

$$\mathcal{L}^{\text{cov}.\pi} = -\frac{1}{4}\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2}\text{Tr}\left[\left(1-\Lambda\pi^2\right)^{-1} D_\mu\pi \left(1-\Lambda\pi^2\right)^{-1} D^\mu\pi\right]$$

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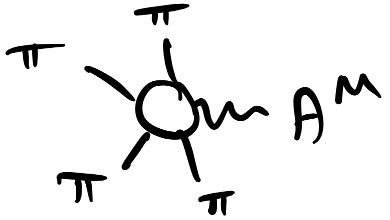

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Ansatz
(mass-dim, little group)
+ constraints

Look for
new operators!

Color-dual?

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$$\mathcal{L}^{\text{cov.}\pi} = -\frac{1}{4}\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2}\text{Tr}\left[\left(1-\Lambda\pi^2\right)^{-1} D_\mu\pi \left(1-\Lambda\pi^2\right)^{-1} D^\mu\pi\right]$$

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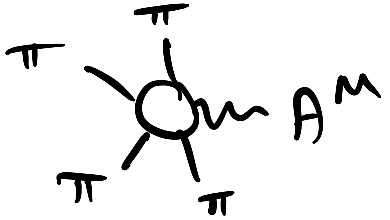

$\equiv A(1_\pi, 2_\pi, 3_\pi, 4_\pi, 5_A)$

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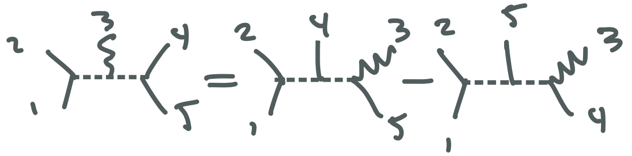
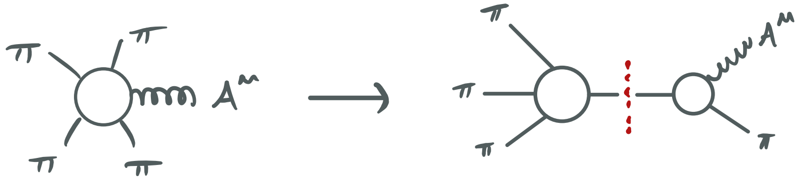
(1) Color-kinematics / BCJ relations



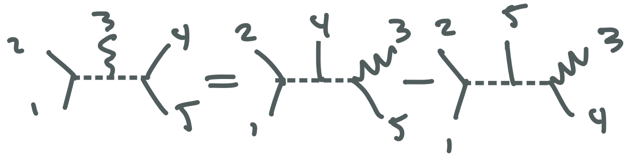
(2) Factorization



(3) Gauge-Invariance

Result of color-dual search

(1) Color-kinematics / BCJ relations



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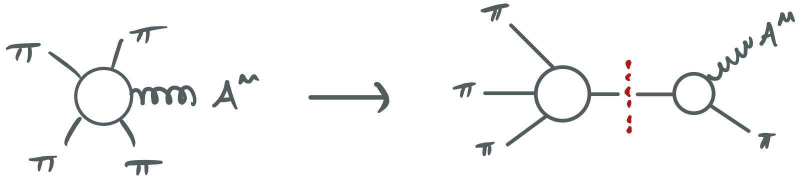

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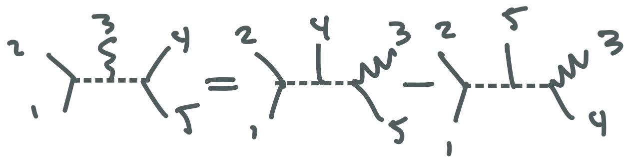

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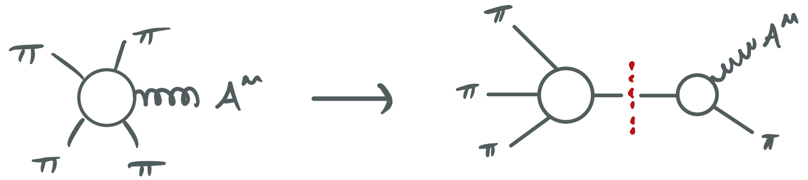

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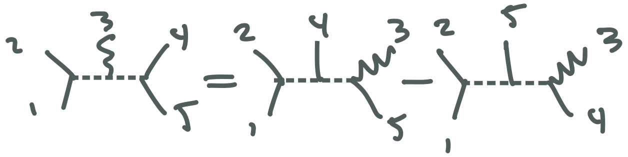
$$A_5(1_\pi,2_\pi,3_\pi,4_\pi,5_A) = g\Lambda\left[\frac{s_{35}\kappa_2^{(5)} - s_{25}\kappa_3^{(5)}}{s_{23}} + \frac{s_{35}s_{25}\kappa_{12}^{(5)}}{s_{12}s_{34}} + \frac{s_{25}\kappa_3^{(5)}}{s_{34}} - \frac{s_{35}\kappa_2^{(5)}}{s_{12}} + 3\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}} - \frac{s_{13}\kappa_4^{(5)}}{s_{45}} + \kappa_{24}^{(5)}\right)\right]$$

Result of color-dual search

(1) Color-Kinematics / BCJ relations



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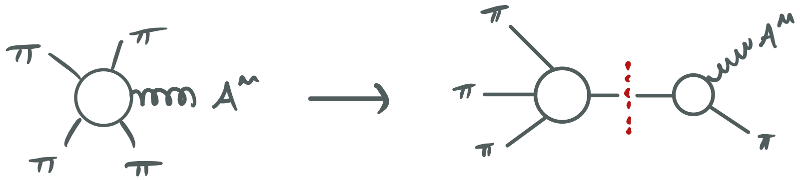


(3) Gauge-Invariance

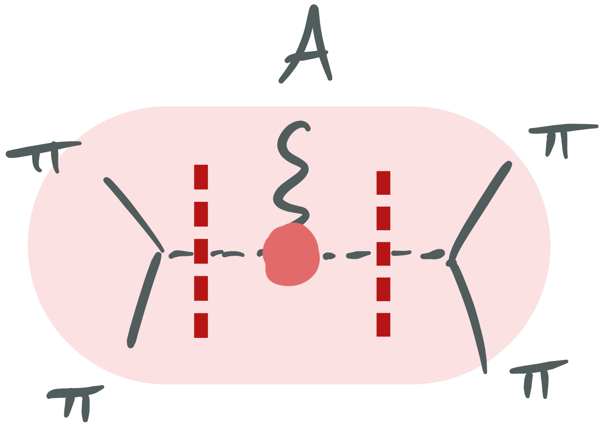
Feynman Rules

$$\underbrace{\mathcal{L}^{\text{cov.}\pi}}$$

$$A_5(1_\pi,2_\pi,3_\pi,4_\pi,5_A) = g\Lambda\Bigg[\frac{s_{35}\kappa_2^{(5)} - s_{25}\kappa_3^{(5)}}{s_{23}} + \frac{s_{35}s_{25}\kappa_{12}^{(5)}}{s_{12}s_{34}} + \frac{s_{25}\kappa_3^{(5)}}{s_{34}} - \frac{s_{35}\kappa_2^{(5)}}{s_{12}} + 3\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}} - \frac{s_{13}\kappa_4^{(5)}}{s_{45}} + \kappa_{24}^{(5)}\right)\Bigg]$$

What was missing?

$$A_5(1_\pi, 2_\pi, 3_\pi, 4_\pi, 5_A) = g\Lambda \left[\frac{s_{35}\kappa_2^{(5)} - s_{25}\kappa_3^{(5)}}{s_{23}} + \frac{s_{35}s_{25}\kappa_{12}^{(5)}}{s_{12}s_{34}} + \frac{s_{25}\kappa_3^{(5)}}{s_{34}} - \frac{s_{35}\kappa_2^{(5)}}{s_{12}} + 3\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}} - \frac{s_{13}\kappa_4^{(5)}}{s_{45}} + \kappa_{24}^{(5)} \right) \right]$$



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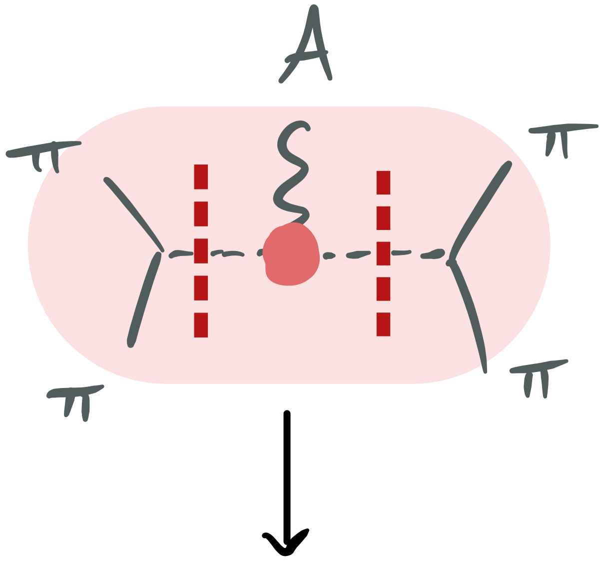

$$\frac{\Lambda}{3g}\text{Tr}\left(F^3\right)$$

$$\mathcal{L}^{\text{cov.}\pi} = -\frac{1}{4}\text{Tr}\left(F^2\right) + \frac{\Lambda}{3g}\text{Tr}(F^3) + \frac{1}{2}\text{Tr}\left[\left(1 - \Lambda\pi^2\right)^{-1} D_\mu\pi\left(1 - \Lambda\pi^2\right)^{-1} D^\mu\pi\right]$$

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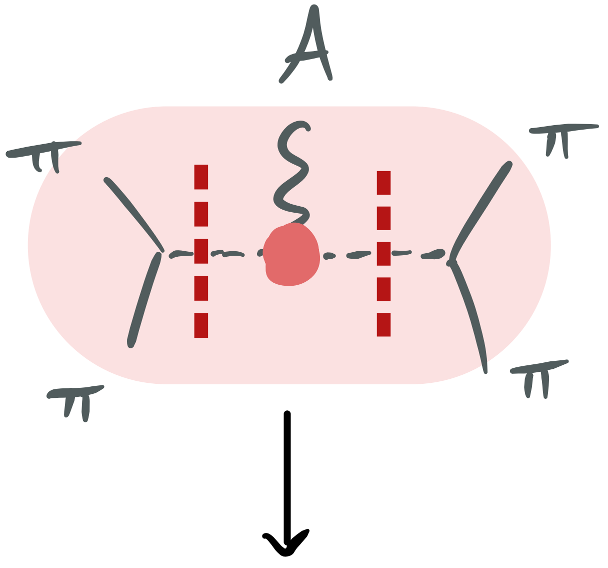

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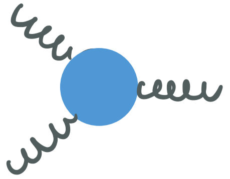
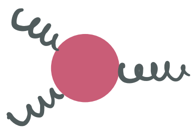
Now that F^3 has forced itself into conversation... α' mismatch @ 3-point

F^2



$$\equiv \quad n_3 = (\varepsilon_1\cdot\varepsilon_2)\left((k_1-k_2)\cdot\varepsilon_3\right) + \text{cyclic}$$

$\alpha' F^3$



$$\equiv \quad n_3^{F^3} = \left((k_1-k_2)\cdot\varepsilon_3\right)\left((k_2-k_3)\cdot\varepsilon_1\right)\left((k_3-k_1)\cdot\varepsilon_2\right)$$

Is this the full story?

$\alpha' \sim \Lambda$

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3$$

Is this the full story?

$$\mathcal{L}^{\mathrm{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \underline{\underline{\alpha'^2 F^4}}$$

Broedel, Dixon

Garozzo, Quiemada, Schlotterer

Is this the full story?

$$\mathcal{L}^{\mathrm{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4$$

If just 4-point $\longrightarrow$

$$+\alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

Carrasco, Rodina, Yin, Zekioglu

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Carrasco, Lewandowski, NHP [2203.03592]

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Double-Copy Consistency:

color-dual @ all multiplicity
tree-level

all orders in α'

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Double-Copy Consistency:

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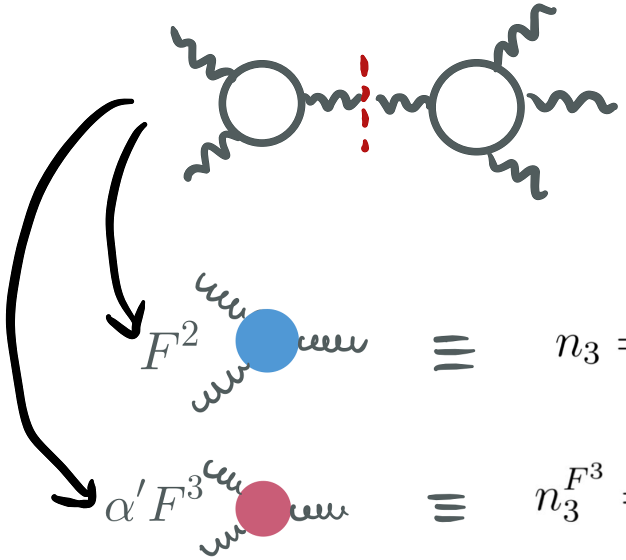
To compose theories check 5-point consistency!



+ BCJ relations

$$\equiv \quad n_3 = (\varepsilon_1 \cdot \varepsilon_2)\left((k_1 - k_2)\cdot \varepsilon_3\right) + \text{cyclic}$$

$$\equiv \quad n_3^{F^3} = \left((k_1 - k_2)\cdot \varepsilon_3\right)\left((k_2 - k_3)\cdot \varepsilon_1\right)\left((k_3 - k_1)\cdot \varepsilon_2\right)$$

Is this the full story?

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If just 4-point $\longrightarrow$ $$+\alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

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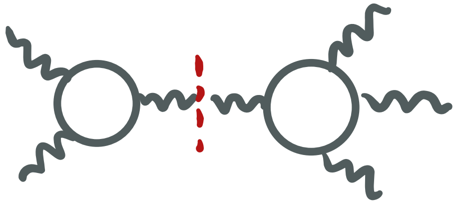
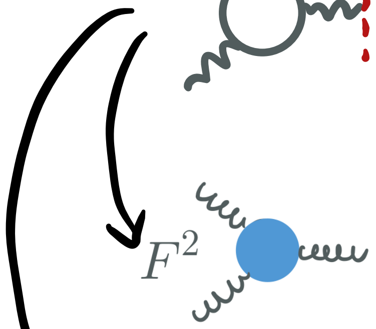
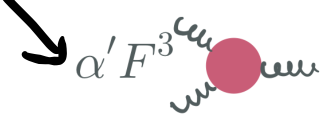


+ BCJ relations



$\equiv$ $n_3 = (\varepsilon_1 \cdot \varepsilon_2)\left((k_1 - k_2)\cdot \varepsilon_3\right) + \text{cyclic}$



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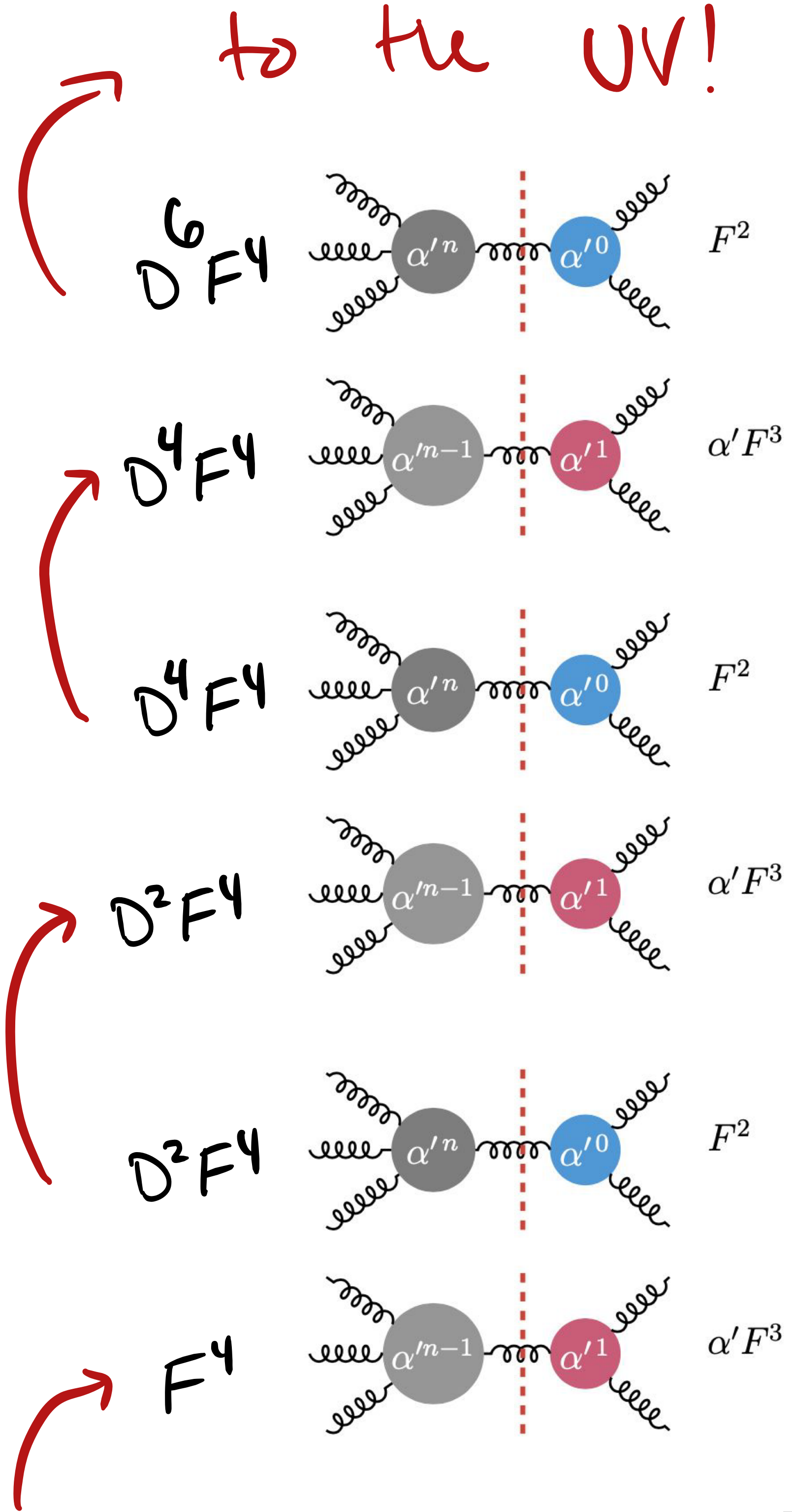
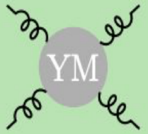
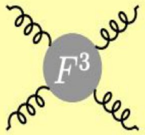
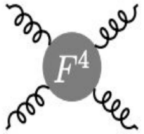


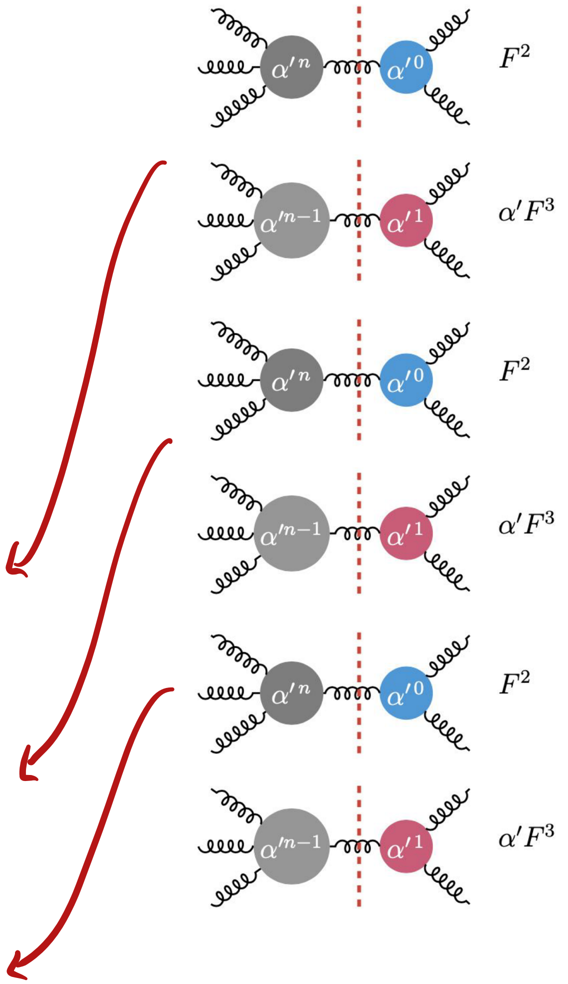
Structure in Towers

Carrasco, Lewandowski, NHP [2203.03592, 2211.04441]

4-vector tower

$$A_4^{(n)}(1_A,2_A,3_A,4_A) = g^2\left(\frac{\Lambda}{g^2}\right)^n u \times \left[\frac{\mathrm{tr}[F_1F_2]\,\mathrm{tr}[F_3F_4]}{s_{12}^2} s_{12}^{n-1} + \mathrm{cyc}(2,3,4)\right]$$

$\mathcal{O}(\Lambda^n)$ \ $\lvert\lvert\pi\rvert\rvert = 2k$	$k=0$
$n=0$	 YM **0**
$n=1$	 F^3 **0**
$n=2$	 F^4 **0**
$n=3$	 D^2F^4 **1**
$n=4$	 D^4F^4 **1**
$n=5$	 D^6F^4 **2**



Structure in Towers

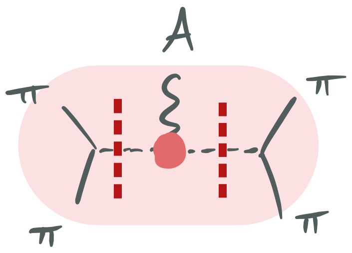
Carrasco, Lewandowski, NHP [2203.03592, 2211.04441]

4-vector tower

$$A_4^{(n)}(1_A,2_A,3_A,4_A) = g^2\left(\frac{\Lambda}{g^2}\right)^n u \times \left[\frac{\mathrm{tr}[F_1F_2]\,\mathrm{tr}[F_3F_4]}{s_{12}^2}\, s_{12}^{n-1} + \mathrm{cyc}(2,3,4)\right]$$

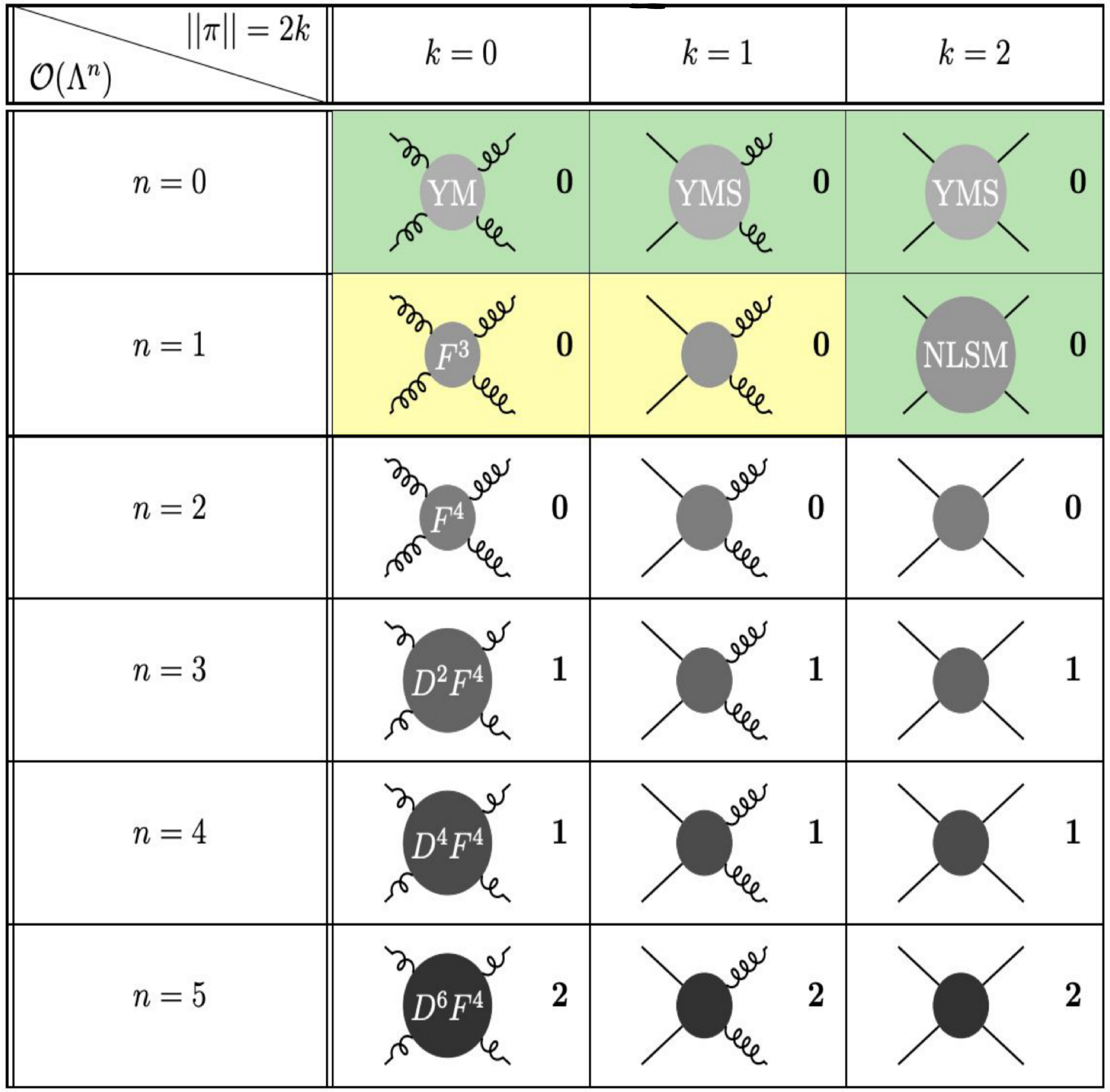
2-vector 2-pion tower

$$A_4^{(n)}(1_\pi,2_\pi,3_A,4_A) = g^2\left(\frac{\Lambda}{g^2}\right)^n u\, s^{n-1}\,\frac{\mathrm{tr}[F_3F_4]}{s}$$



4-pion tower

$$A_4^{(n)}(1_\pi,2_\pi,3_\pi,4_\pi) = g^2\left(\frac{\Lambda}{g^2}\right)^n u\left[s^{n-1}+t^{n-1}+u^{n-1}\right]$$



$\|\pi\| = 2k$ $\mathcal{O}(\Lambda^n)$	$k=0$	$k=1$	$k=2$
$n=0$	YM 0	YMS 0	YMS 0
$n=1$	F^3 0	0	NLSM 0
$n=2$	F^4 0	0	0
$n=3$	D^2F^4 1	1	1
$n=4$	D^4F^4 1	1	1
$n=5$	D^6F^4 2	2	2

Structure in Towers

Carrasco, Lewandowski, NHP [2203.03592, 2211.04441]

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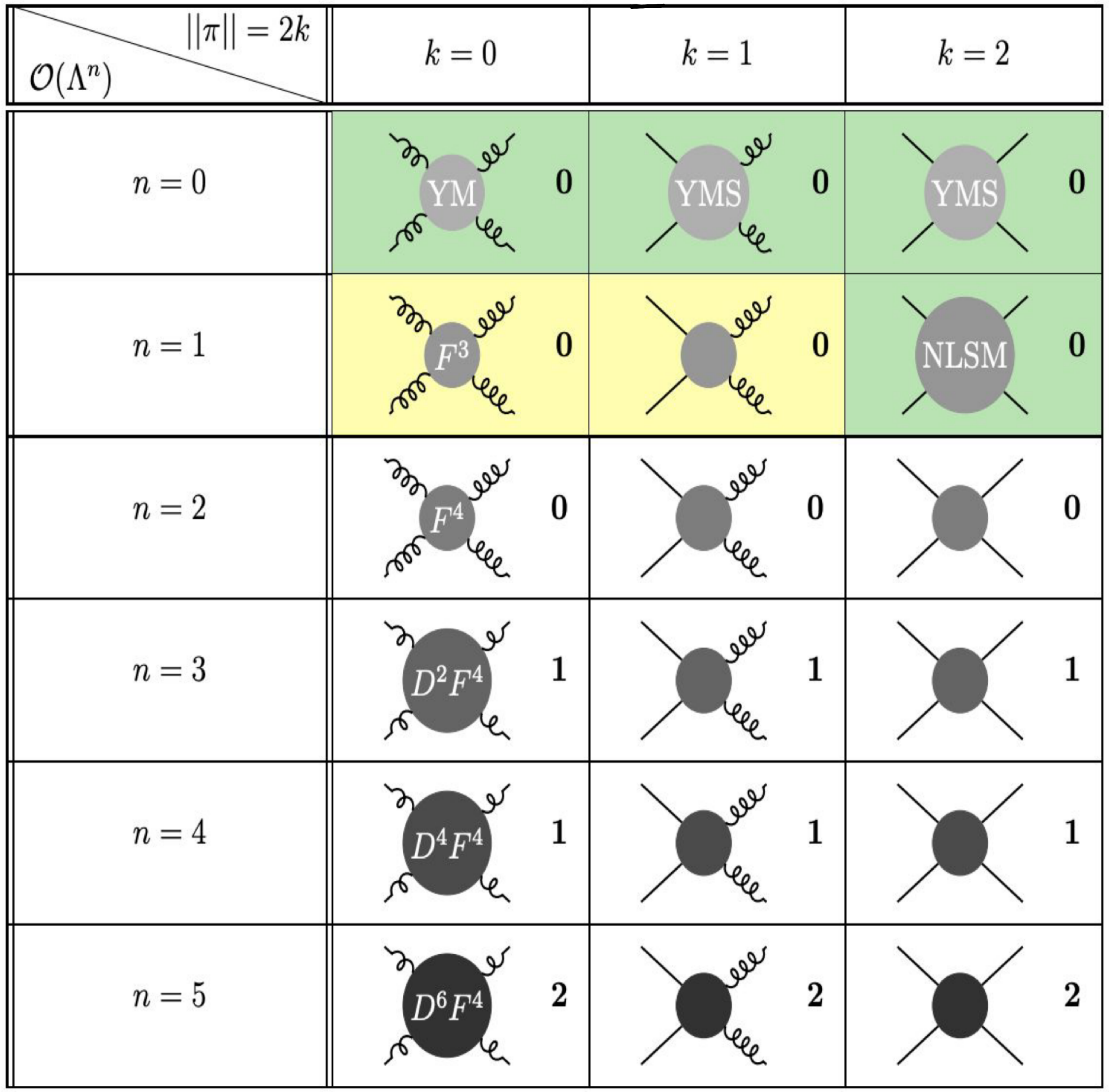
$$A^{\mathrm{full}}_{(1234)} \sim \sum_k A^{(k)}_{(1234)}$$



$\mathcal{O}(\Lambda^n)$ \ $\|\|\pi\|\| = 2k$	$k=0$	$k=1$	$k=2$
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4-point towers can be re-summed!

4-vector tower

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$$A_4^{(n)}(1_\pi, 2_\pi, 3_A, 4_A) = g^2 \left(\frac{\Lambda}{g^2}\right)^n u\, s^{n-1} \frac{\mathrm{tr}[F_3F_4]}{s}$$

$$\sim \frac{1}{1+\alpha' s}$$

4-pion tower

$$A_4^{(n)}(1_\pi, 2_\pi, 3_\pi, 4_\pi) = g^2 \left(\frac{\Lambda}{g^2}\right)^n u \left[s^{n-1} + t^{n-1} + u^{n-1}\right]$$

$$A^{\mathrm{full}}_{(1234)} \sim \sum_k A^{(k)}_{(1234)}$$

$$\mathrm{YM} + F^3$$

$$\Big\downarrow \text{ Resums}$$

$$DF^2 + \mathrm{YM} + \mathrm{H.D.}$$

Johansson, Nohle
Azevedo, Chioraroli, Mogull, Schlotterer, Teng

4-point towers can be re-summed!

4-vector tower

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$$A^{\mathrm{full}}_{(1234)} \sim \sum_k A^{(k)}_{(1234)}$$

YM + F3

↓

DF^2 + YM + H.D.

$$A^{\mathrm{dcc}}_{(1234)} = A^{(DF)^2+\mathrm{YM}}(1234)\left[1+\sum_{x\geq 1,y} c_{(x,y)}\sigma_3^x\sigma_2^y\right]$$

$$\sigma_3 \sim stu \qquad \sigma_2 \sim s^2+t^2+u^2$$

Carrasco, Lewandowski, NHP [2203.03592]

4-point towers can be re-summed!

4-vector tower

$$A_4^{(n)}(1_A,2_A,3_A,4_A) = g^2\left(\frac{\Lambda}{g^2}\right)^n u \times \left[\frac{\mathrm{tr}[F_1F_2]\,\mathrm{tr}[F_3F_4]}{s_{12}^2}\, s_{12}^{n-1} + \mathrm{cyc}(2,3,4)\right]$$

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$$A^{\text{full}}_{(1234)} \sim \sum_k A^{(k)}_{(1234)}$$

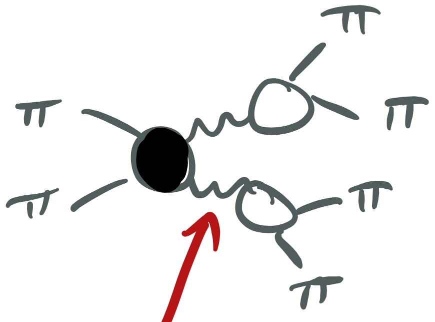
YM + NLSM

↓ Resums

DF^2 + YM + H.D.

(dimensionally reduced)

No Adler zero!



introduces spurious 6-point poles in NLSM

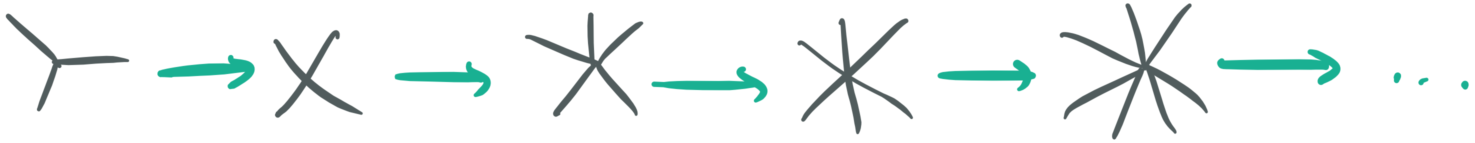
$F^2 + F^3$ = UV Tower

Carrasco, Lewandowski, NHP [2203.03592]

$F^2 + F^3 = $ UV Tower

Carrasco, Lewandowski, NHP [2203.03592]

Historically color-kinematics constrains out



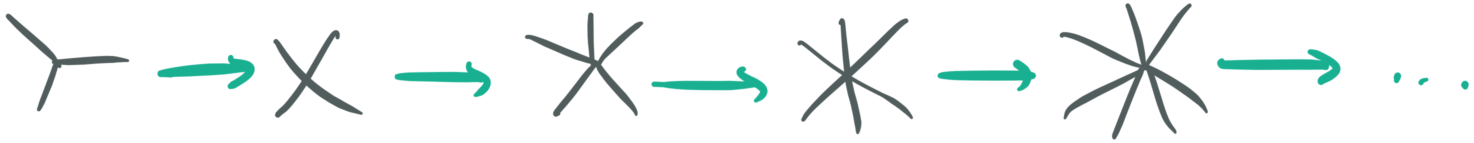
YM, GR

NLSM, BI

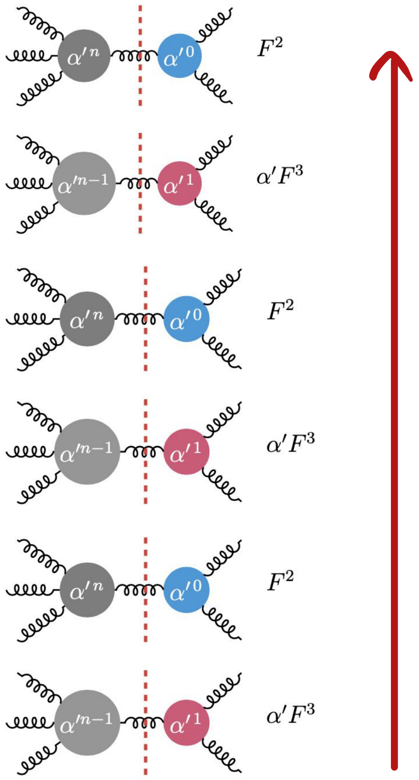
SG, ...

$F^2 + F^3$ = UV Tower

Historically color-kinematics constrains out



Consistency between HD operators constrains UP



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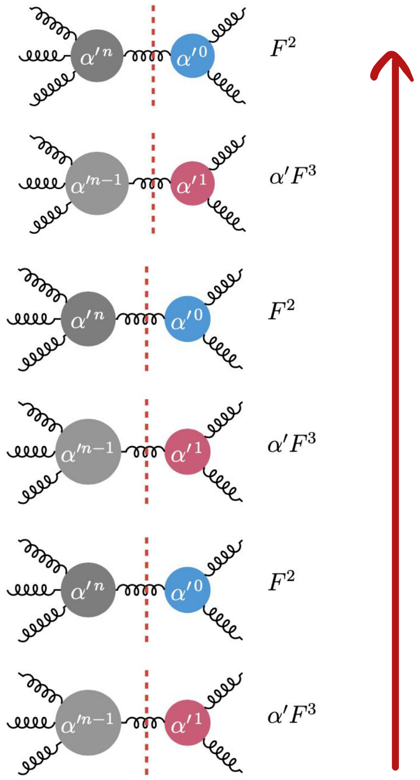

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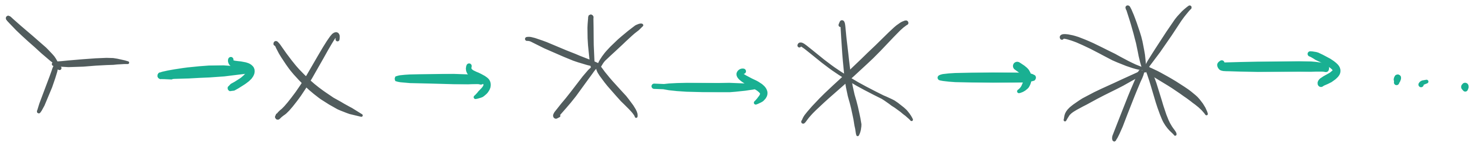


Consistency between HD operators constrains UP

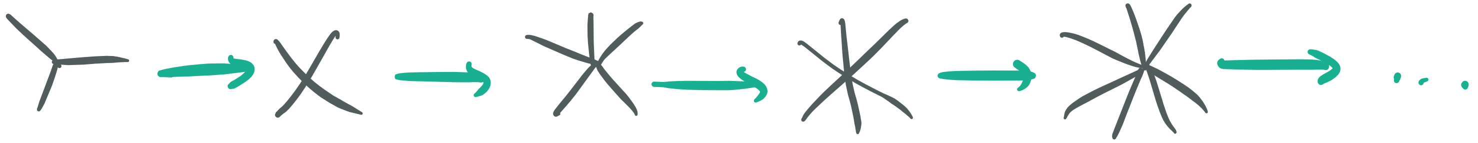
$$A^{YM+F^3} \otimes A^{\mathcal{N}=4}$$

cancels U(1) anomaly in $\mathcal{N}=4$ supergravity

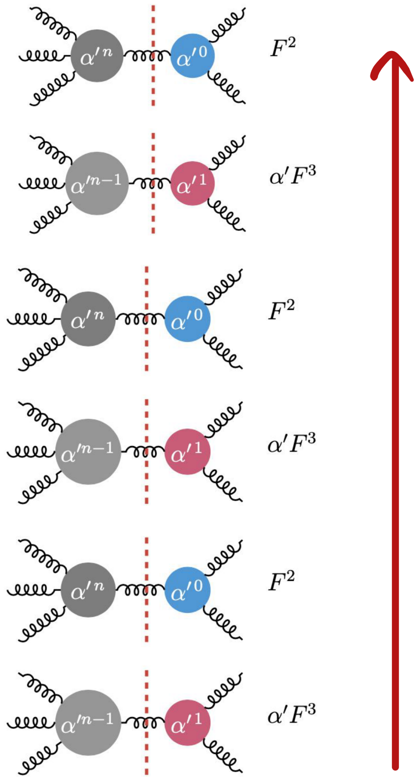
Bern, Edison, Kosower, Parra-Martinez, Roiban

$F^2 + F^3$ = UV Tower

Historically color-kinematics constrains out



Consistency between HD operators constrains UP



$\Downarrow$

Carrasco, Lewandowski, NHP [2203.03592]

$A^{YM+F^3} \otimes A^{\mathcal{N}=4}$

cancels U(1) anomaly
in $\mathcal{N}=4$ supergravity

Bern, Edison, Kosower, Parra-Martinez, Roiban

$$\Rightarrow \underbrace{(DF^2+YM+HD) \otimes (\mathcal{N}=4 \text{ sYM})}_{\substack{\| \\ \text{UV finite } \mathcal{N}=4\,?}}$$

conformal SG / Heterotic String $\longleftarrow$

Hints for other anomalies?

$$\mathcal{A}^{\mathcal{N}=4\ \text{SG}}_{(++++)} = \mathcal{A}^{\mathcal{N}=4\ \text{sYM}} \otimes \mathcal{A}^{\text{YM}\ \text{1-loop}}_{(++++)}$$

$$\mathcal{A}^{\mathcal{N}=4\ \text{SG,1-loop}}_{(h^{++}h^{++}t\bar{t})} \sim st\mathcal{A}^{\mathcal{N}=4\ \text{sYM}}_{(s,t)} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

↑

cancelled by $\mathcal{A}^{N=4} \otimes \mathcal{A}^{YM+F^3}$

Bern, Kosower, Parra-Martinez, Roiban
Carrasco, Kallosh, Roiban, Tseytlin

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Bern, Kosower, Parra-Martinez, Roiban
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↑

cancelled by $\mathcal{A}^{\mathcal{N}=4} \otimes \mathcal{A}^{\text{YM}+F^3}$

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$$\mathcal{A}^{\text{BI,1-loop}}_{(++++)} \sim (s^4 + t^4 + u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Elvang, Hadjiantonis
Jones, Paranjape

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Elvang, Hadjiantonis
Jones, Paranjape

$$\mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim (s^6+t^6+u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Carrasco, NHP
[2211.04431, 2212.xxxxx]

Hints for other anomalies?

$$\mathcal{A}^{\mathcal{N}=4\text{ SG}}_{(++++)} = \mathcal{A}^{\mathcal{N}=4\text{ sYM}} \otimes \mathcal{A}^{\text{YM}\ \text{1-loop}}_{(++++)}$$

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Bern, Kosower, Parra-Martinez, Roiban
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Elvang, Hadjiantonis
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Carrasco, NHP
[2211.04431, 2212.xxxxx]

$$\mathcal{A}^{\text{BI}} = \mathcal{A}^{\text{NLSM}} \otimes \mathcal{A}^{\text{YM}+\text{HD}}??$$

Henriette's talk @ QMG ‘21

Can anomaly be absorbed with double copy?

$$\mathcal{A}^{\text{BI}} = \mathcal{A}^{\text{NLSM}} \otimes \mathcal{A}^{\text{YM}} + \text{HD} \; ??$$

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These can be constructed with BCJ numerators

Carrasco, NHP
[2211.04431]

$$\mathcal{A}^{\rm BI}_{(++++)} \sim \sum_g \frac{n_g^{\rm NLSM} n_g^{\rm CT}}{d_g}$$

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These can be constructed with BCJ numerators

Carrasco, NHP
[2211.04431]

$$n_s^{\text{1-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\,(t^3 - u^3)$$

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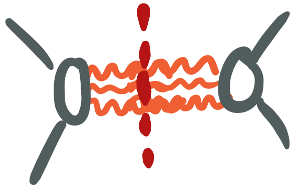
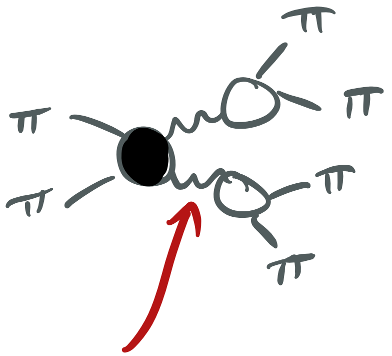
$$\mathcal{A}^{\text{BI}}_{(++++)} \sim \sum_g \frac{n_g^{\text{NLSM}} n_g^{\text{CT}}}{d_g}$$

But with higher-spin states on poles!

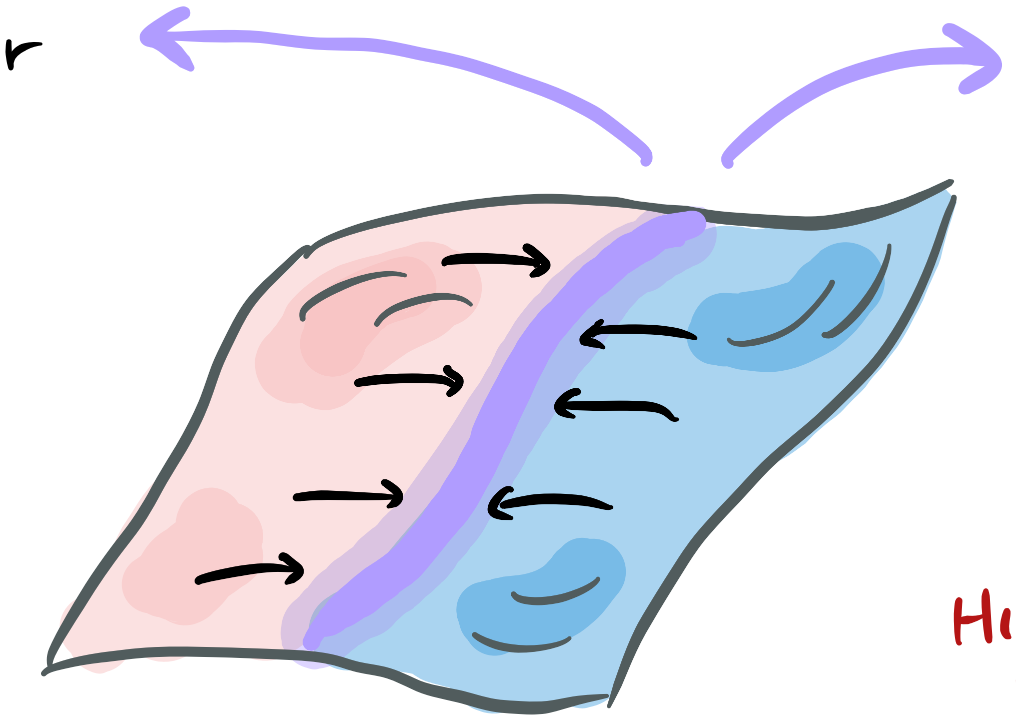


Tensions with Rigidity of Double-Copy consistency.

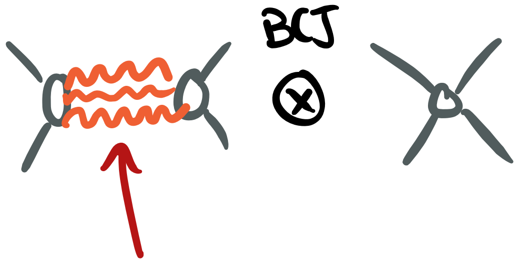
- Rigid HD tower



introduces spurious poles in NLSM



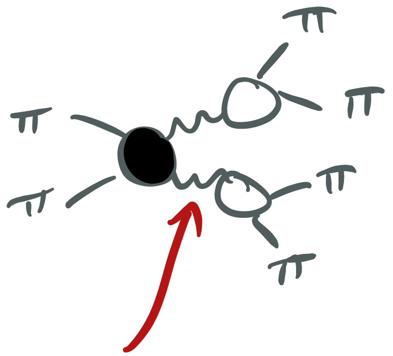
- BI anomaly captured by DC



Higher-spin states in single-copies

Tensions with Rigidity of Double-Copy consistency.

- Rigid HD tower



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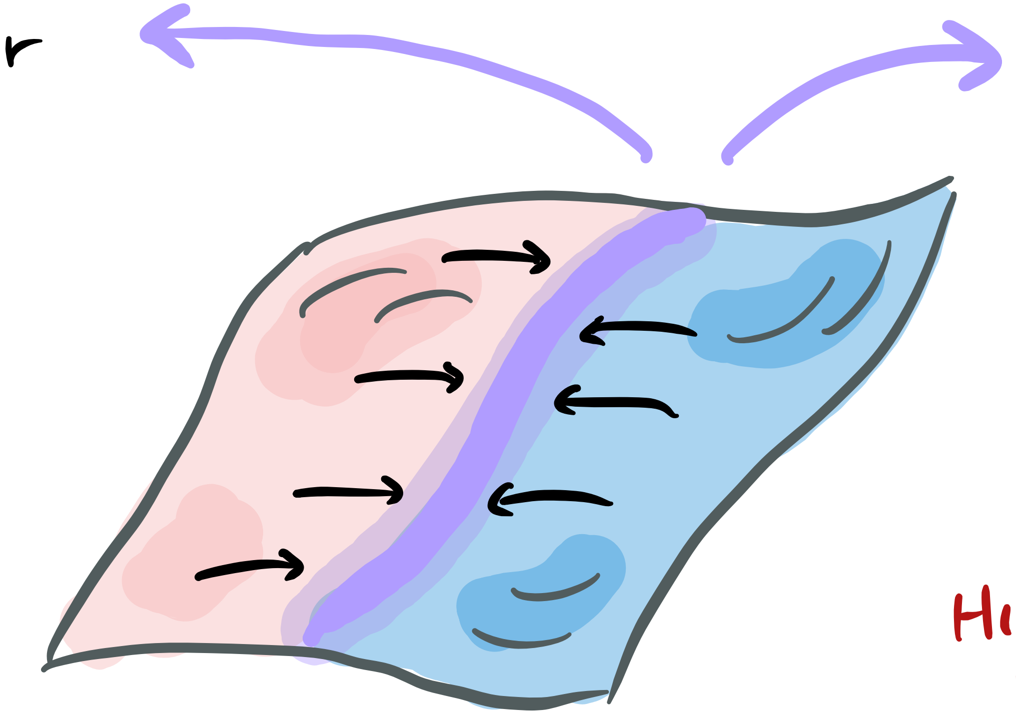

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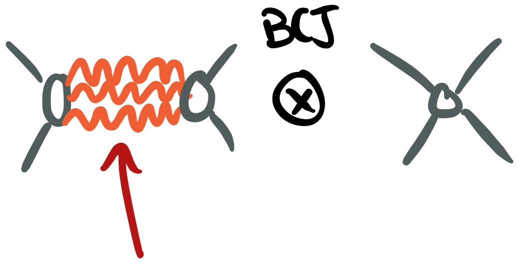
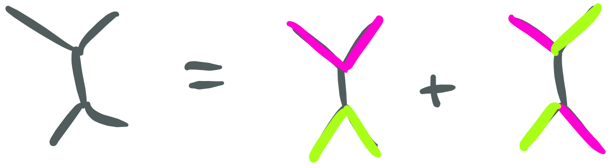


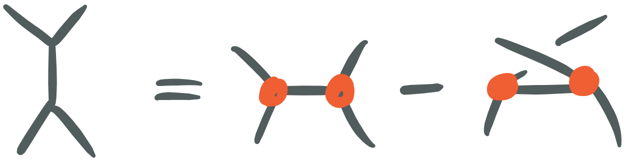
Higher-spin states in single-copies

Flavor Kinematics!



NHP [2210.12800]

Symmetric-structure



Carrasco, NHP [2211.04431]

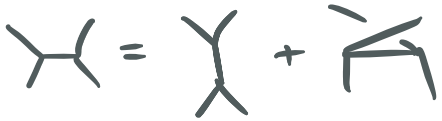
Higher-spin adjoint ⇔ local symmetric-structure

What underlies color jacobi-relations?

$$
\begin{aligned}
f^{abe}f^{ecd} = {} & \mathrm{Tr}[T^aT^bT^cT^d] - \mathrm{Tr}[T^aT^bT^dT^c] \\
& + \mathrm{Tr}[T^aT^dT^cT^b] - \mathrm{Tr}[T^aT^dT^bT^c]
\end{aligned}
$$

$\Rightarrow$

$$C_s^{ff} = C_t^{ff} + C_u^{ff}$$

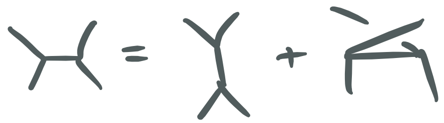


Higher-spin adjoint ⇔ local symmetric-structure

What underlies color jacobi-relations?

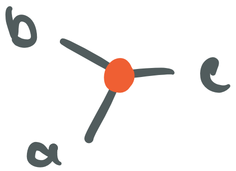
$$f^{abe} f^{ecd} = \text{Tr}[T^a T^b T^c T^d] - \text{Tr}[T^a T^b T^d T^c] + \text{Tr}[T^a T^d T^c T^b] - \text{Tr}[T^a T^d T^b T^c]$$

$$\Rightarrow \quad C_s^{ff} = C_t^{ff} + C_u^{ff}$$



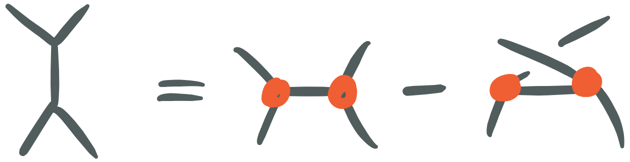
Could also introduce symmetric-structure

$$d^{abc} \equiv \text{Tr}[\{T^a, T^b\} T^c]$$



$$\Rightarrow \quad C_s^{ff} = C_t^{dd} - C_u^{dd}$$

$$f^{ade} f^{ecb} = d^{abe} d^{ecd} - d^{ace} d^{ebd} + \mathcal{O}(1/N_c)$$



Higher-spin adjoint ⇔ local symmetric-structure

Carrasco, NHP [2211.04431]

Applying this to NLSM

$$c_s^{ff} = c_t^{dd} - c_u^{dd}$$

$$\mathcal{A}^{NLSM} \sim \left(c_s^{ff} u + c_u^{ff} s\right) \to \left(c_s^{dd} s + c_t^{dd} t + c_u^{dd} u\right)$$

Higher-spin adjoint ⇔ local symmetric-structure

Carrasco, NHP [2211.04431]

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NLSM is also a symmetric Double-copy

$$\mathcal{A}^{\mathrm{NLSM}} = \sum_{g\in\Gamma^{(3)}} \frac{c_g^{\mathrm{dd}} n_g^{\pi,\mathrm{dd}}}{d_g} = \sum_{g\in\Gamma^{(3)}} \frac{c_g^{\mathrm{ff}} n_g^{\pi,\mathrm{ff}}}{d_g}$$

$$n_s^{ff} = t^2 - u^2$$

$$n_t^{dd} = t^2 \qquad n_u^{dd} = u^2$$

Higher-spin adjoint ⇔ local symmetric-structure

Carrasco, NHP [2211.04431]

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$$n_s^{ff} = t^2 - u^2$$

* verified through 6-point!

$$n_t^{dd} = t^2 \qquad n_u^{dd} = u^2$$

Virtue 1: Anomaly free BI constructed from local double copy

$$n_s^{\text{1-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\left(t^3 - u^3\right) \equiv n_t^{dd} - n_u^{dd} \implies n_s^{\text{dd}} \sim s^n \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

$$n_s^{\text{2-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\left(t^5 - u^5\right) \equiv n_t^{dd} - n_u^{dd}$$

Virtue 1: Anomaly free BI constructed from local double copy

$$n_s^{\text{1-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\left(t^3 - u^3\right) \equiv n_t^{dd} - n_u^{dd}$$

$$n_s^{\text{2-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\left(t^5 - u^5\right) \equiv n_t^{dd} - n_u^{dd}$$

$$n_s^{\text{dd}} \sim \underbrace{s^n \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}}_{\text{completely local}}$$

$$\frac{c_s^{dd}\, n_s^{dd}}{s} \sim$$

$$\mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim \left(s^6 + t^6 + u^6\right)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{A}^{\text{NLSM}} = \left(c_s^{\text{dd}} s + c_t^{\text{dd}} t + c_u^{\text{dd}} u\right)$$

Carrasco, NHP [2211.04431]

Virtue 1: Anomaly free BI constructed from local double copy

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$$\mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim (s^6 + t^6 + u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{A}^{\text{BI+CT}} \sim \left(s^{n+1} + t^{n+1} + u^{n+1}\right)\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

Carrasco, NHP [2211.04431]

Virtue 1: Anomaly free BI constructed from local double copy

$$n_s^{\text{1-loop CT}} = \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\,(t^3 - u^3) \equiv n_t^{dd} - n_u^{dd}$$

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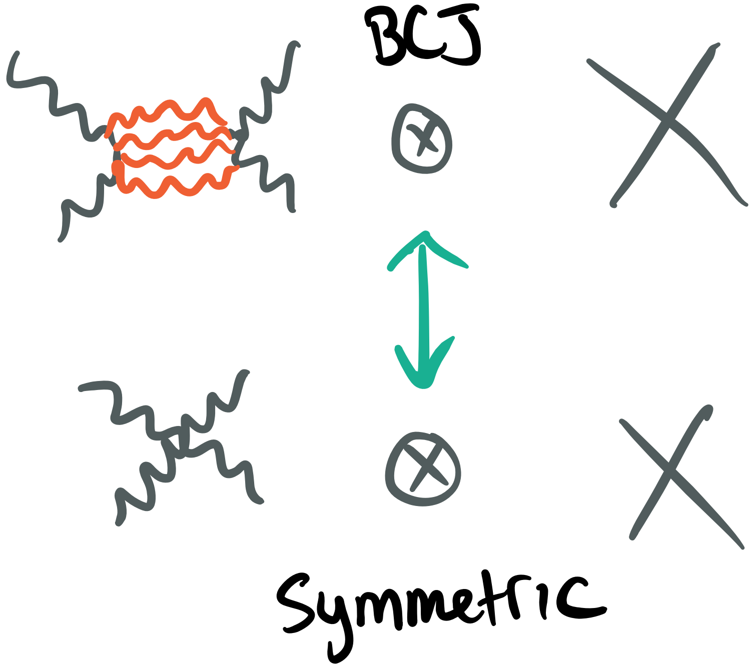
$$\underbrace{n_s^{\text{dd}} \sim s^n \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}}_{\text{completely local}}$$

$$\mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim (s^6 + t^6 + u^6)\,\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{A}^{\text{BI+CT}} \sim \left(s^{n+1} + t^{n+1} + u^{n+1}\right)\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

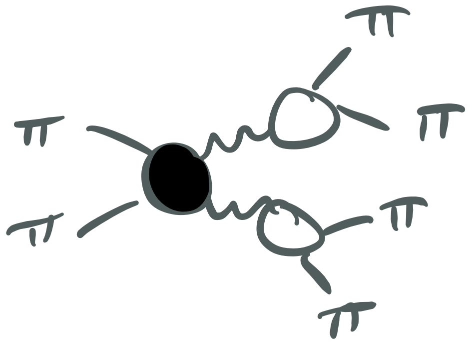
$\Rightarrow$



Carrasco, NHP [2211.04431]

Q : Can we avoid YM+NLSM tower?

Need to avoid $F^2 + F^3$!! $\Leftrightarrow$



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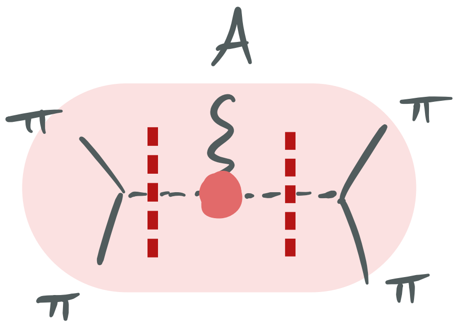

Need to avoid F^2+F^3!!

$$s_{1|2}A_5^{\text{cov}.\pi}(1_\pi,2_\pi,3_\pi,4_\pi,5_A)+s_{13|2}A_5^{\text{cov}.\pi}(1_\pi,3_\pi,2_\pi,4_\pi,5_A) + \underbrace{s_{134|2}A_5^{\text{cov}.\pi}(1_\pi,3_\pi,4_\pi,2_\pi,5_A)}_{\text{make this functionally distinct!}}\neq 0,$$

give pions a flavor structure

Q: Can we avoid YM+NLSM tower?

Need to avoid F^2+F^3 !!

$$s_{1|2}A_5^{\text{cov}.\pi}(1_\pi,2_\pi,3_\pi,4_\pi,5_A)+s_{13|2}A_5^{\text{cov}.\pi}(1_\pi,3_\pi,2_\pi,4_\pi,5_A)$$
$$+\,s_{134|2}A_5^{\text{cov}.\pi}(1_\pi,3_\pi,4_\pi,2_\pi,5_A)\neq 0,$$

- NLSM$\simeq$YM when $\overset{Y}{\epsilon}_{1,n}\to(0,1,0)$ $\overset{Z}{\epsilon}_{a}\to(k,0,ik)$

Cheung, Remmen, Shen, Wen
Cachazo, He, Yuan

Idea: Decompose Yan-Mills into color-dual flavor sectors

NHP [2210.12800]

$$A^{\text{vec}}_{(\sigma)}=\sum_{k=0}^{\lfloor|\sigma|/2\rfloor}\sum_{\rho\in S_\sigma^{2|k}}\epsilon_{(\rho)}\Delta^{(\rho)}_{(\sigma)}$$

$$\epsilon(\rho)\sim\{(\epsilon_1\epsilon_2),\ldots,(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4),\ldots\}$$

, each $\Delta^{(\rho)}_{(\sigma)}$ satisfies BCJ !!

Virtue 2: Flavorful color-dual pion-vector theory

$\epsilon_A \to (\epsilon^\mu, 0, \vec{0}) \qquad \epsilon_Y = (0,1,0) \qquad \epsilon_Z = (\vec{K}, 0, iK)$

Replace polarizations Y,Z modes

$$s_{1|2}A_5^{\text{YM}}(1_Y,2_Z,3_Z,4_Y,5_A)+s_{13|2}A_5^{\text{YM}}(1_Y,3_Z,2_Z,4_Y,5_A)$$
$$+\,s_{134|2}A_5^{\text{YM}}(1_Y,3_Z,4_Y,2_Z,5_A)=0.$$

Just dimensional reduction ⇒ satisfies BCJ

Carrasco, Lewandowski, NHP [2211.04441]

Virtue 2: Flavorful color-dual pion-vector theory

$\epsilon_A \to (\epsilon^{\mu}, 0, \vec{0})$ $\epsilon_Y = (0,1,0)$ $\epsilon_Z = (\vec{k}, 0, ik)$

$$s_{1|2} A_5^{\mathrm{YM}}(1_Y, 2_Z, 3_Z, 4_Y, 5_A) + s_{13|2} A_5^{\mathrm{YM}}(1_Y, 3_Z, 2_Z, 4_Y, 5_A) \\ + s_{134|2} A_5^{\mathrm{YM}}(1_Y, 3_Z, 4_Y, 2_Z, 5_A) = 0.$$

Just dimensional reduction ⇒ satisfies BCJ

$$A_5^{\mathrm{YM}}(1_Y, 2_Z, 3_Z, 4_Y, 5_A) = \frac{g}{f_\pi^2}\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}} - \frac{s_{13}\kappa_4^{(5)}}{s_{45}} + \kappa_{24}^{(5)}\right) \sim \boxed{s_{23}\Delta^{(14)(23)}_{(12345)},}$$

NHP [2210.12800]

$$A^{\mathrm{vec}}_{(\sigma)} = \sum_{k=0}^{\lfloor |\sigma|/2 \rfloor} \sum_{\rho \in S_\sigma^{2|k}} \epsilon_{(\rho)} \Delta^{(\rho)}_{(\sigma)}$$

Carrasco, Lewandowski, NHP [2211.04441]

Virtue 2: Flavorful color-dual pion-vector theory

$\epsilon_A \to (\epsilon^\mu, 0, \vec{0})$ $\quad \epsilon_Y = (0,1,0)$ $\quad \epsilon_Z = (\vec{K}, 0, iK)$

$$s_{1|2}A_5^{\mathrm{YM}}(1_Y,2_Z,3_Z,4_Y,5_A)+s_{13|2}A_5^{\mathrm{YM}}(1_Y,3_Z,2_Z,4_Y,5_A)$$
$$+s_{134|2}A_5^{\mathrm{YM}}(1_Y,3_Z,4_Y,2_Z,5_A)=0.$$

Just dimensional reduction ⇒ satisfies BCJ

$$A_5^{\mathrm{YM}}(1_Y,2_Z,3_Z,4_Y,5_A)=\underbrace{\frac{g}{f_\pi^2}\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}}-\frac{s_{13}\kappa_4^{(5)}}{s_{45}}+\kappa_{24}^{(5)}\right)}_{\text{factors to pions + YM}}\sim\boxed{s_{23}\Delta^{(14)(23)}_{(12345)},}$$

NHP [2210.12800]

No tower ⇒ No Adler zero violation

Virtue 3: Insight into the underlying structure of EM duality

Flavor decomposition teases out hidden structure

$$A^{\text{vec}}_{(\sigma)} = \sum_{k=0}^{\lfloor|\sigma|/2\rfloor} \sum_{\rho\in S_\sigma^{2|k}} \epsilon_{(\rho)} \Delta^{(\rho)}_{(\sigma)} \quad \Rightarrow \quad \mathcal{A}^{\text{NLSM}} \sim \sum s_{(\rho)} \mathcal{A}^{\text{YMS}}_{(\rho)}$$

NHP [2210.12800]

Virtue 3: Insight into the underlying structure of EM duality

Flavor decomposition teases out hidden structure

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$$\mathcal{L}^{\text{EMf}} \sim R - \frac{1}{4}\sum_I F_I^2$$

EM duality ⇒ Born-Infeld EM duality

$$\mathcal{M}^{\text{BI}} = \mathcal{A}^{\text{NLSM}} \otimes \mathcal{A}^{\text{YM}} \quad \Rightarrow \quad \mathcal{M}^{\text{BI}} \sim \sum s_{(\rho)} \mathcal{M}^{\text{EMf}}_{(\rho)}$$

NHP [2210.12800]

Future Directions

- Are there other HD towers from CK?
- Does $(DF^2+YM)\otimes(\mathcal{N}=4)$ cancel other higher-loop anomalies?
  - ↪ Does this cancel 4-loop UV divergence in $\mathcal{N}=4$ SG?
- Can symmetric structure give flexibility for EFT? Inflation? LSS?
- New partial amplitude relations?? $d^3f^3+d^3f^3+d^3f^3=0$
- Use flavor decomp for HD gravity operators?

Summary

- YM+ F^3 ⇒ (DF^2+YM+HD)
- YM+NLSM ⇒ (DF^2+YM+HD) | KK reduction

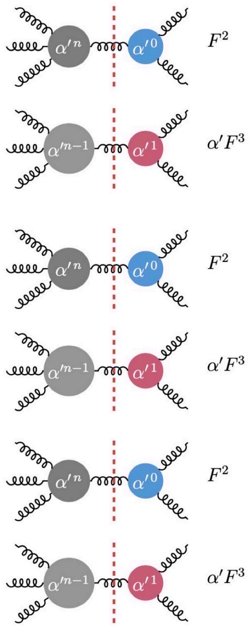
} UV tower @ 4-point

[2203.03592]

[2211.04441]

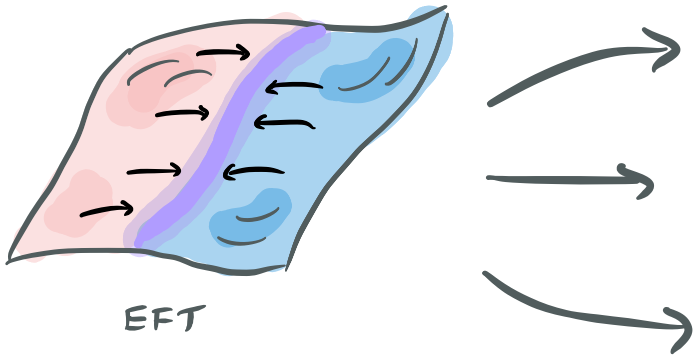
- BI counterterm from higher-spin⊗Adler zero [2211.04431]

  cancels 2-loop anomaly (loop-integrated) [2212.xxxxx]



- Flavor decomposition relaxes tower in YM+ pions [2210.12800]
- BI counterterms spanned by local D.C. w/ symmetric-structure [2211.04431]

Virtues!



$$\mathcal{A}^{\text{BI,2-loop}}_{(++++)} \sim (s^6+t^6+u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{M}^{\text{BI}} \sim \sum s_{(\rho)}\mathcal{M}^{\text{EMf}}_{(\rho)} \qquad \mathcal{A}^{\text{NLSM}} \sim \sum s_{(\rho)}\mathcal{A}^{\text{YMS}}_{(\rho)}$$

$$A_5^{\text{YM}}(1_Y,2_Z,3_Z,4_Y,5_A) = \frac{g}{f_\pi^2}\left(\frac{s_{24}\kappa_1^{(5)}}{s_{15}} - \frac{s_{13}\kappa_4^{(5)}}{s_{45}} + \kappa_{24}^{(5)}\right) \sim s_{23}\Delta^{(14)(23)}_{(12345)},$$