Double Copy – a review

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QCD Meets Gravity 2022

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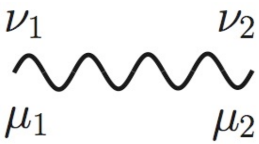
Some relevant refs:

Bern, Carrasco, Chiodaroli, HJ, Roiban (reviews) [1909.01358, 2203.13013];
Snowmass White Papers: [2203.09099, 2204.06547];
Brandhuber, Chen, HJ, Travaglini, Wen [2111.15649];
Ben-Shahar, HJ [2112.11452];
Chiodaroli, HJ, Pichini [2107.14779];
Edison, He, HJ, Schlotterer, Teng, Zhang [2211.00638]

Textbook perturbative gravity is complicated!

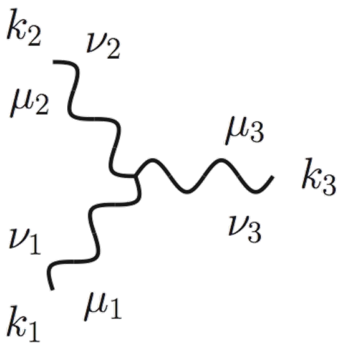
$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

DeWitt ('67)



$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2 + i\epsilon}$$

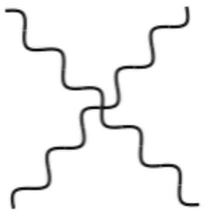
de Donder gauge



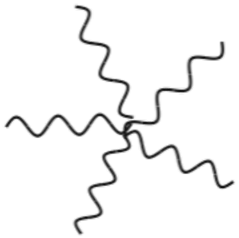
$$\begin{aligned} = \mathrm{sym}[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3}) \\ &+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3}) \\ &+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3}) \\ &+2P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]\end{aligned}$$

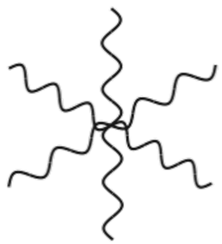
After symmetrization
$\sim$ 100 terms !

higher order vertices...



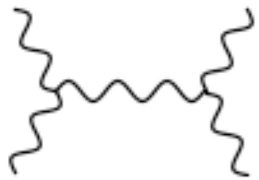
$\sim 10^3$ terms



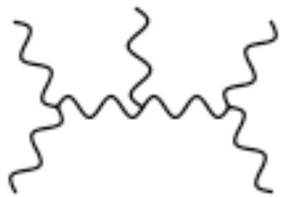


. . .

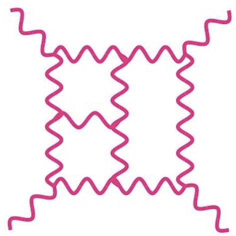
complicated diagrams:



$\sim 10^4$ terms



$\sim 10^7$ terms



$\sim 10^{21}$ terms

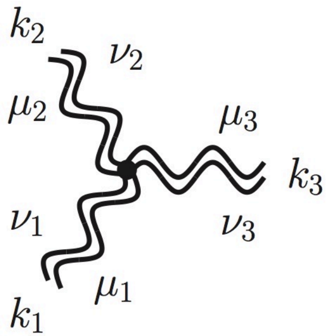
On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

$$|\text{spin } 2\rangle \sim |\text{spin } 1\rangle \otimes |\text{spin } 1\rangle$$

↑ Yang-Mills polarization

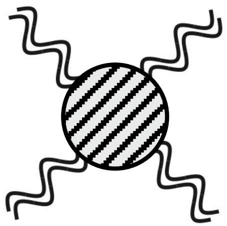
On-shell 3-graviton vertex:



$$= \Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

↑ Yang-Mills vertex

Gravity scattering amplitude:



Yang-Mills amplitude ↓

$$M^{\text{GR}}_{\text{tree}}(1,2,3,4) = \frac{st}{u}\Big[A^{\text{YM}}_{\text{tree}}(1,2,3,4)\Big]^2$$

Gravity processes = "squares" of gauge theory ones: KLT, BCJ, CHY

Kawai-Lewellen-Tye Relations ('86)

String theory
tree-level identity:

closed string ~ (left open string) × (right open string)

$$A_n \sim \int \frac{dx_1 \cdots dx_n}{\mathcal{V}_{abc}} \prod_{1 \le i < j \le n} |x_i - x_j|^{k_i \cdot k_j} \exp\left[\sum_{i<j}\left(\frac{\epsilon_i \cdot \epsilon_j}{(x_i - x_j)^2} + \frac{k_i \cdot \epsilon_j - k_j \cdot \epsilon_i}{(x_i - x_j)}\right)\right]\Bigg|_{\text{multi-linear}}$$

KLT relations emerge after nontrivial world-sheet integral identities

Field theory limit $\Rightarrow$ gravity theory ~ (YM theory) × (YM theory)

$$M_4^{\text{tree}}(1,2,3,4) = -i s_{12} A_4^{\text{tree}}(1,2,3,4)\, \widetilde{A}_4^{\text{tree}}(1,2,4,3)$$

$$M_5^{\text{tree}}(1,2,3,4,5) = i s_{12} s_{34} A_5^{\text{tree}}(1,2,3,4,5)\, \widetilde{A}_5^{\text{tree}}(2,1,4,3,5)$$
$$+ i s_{13} s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, \widetilde{A}_5^{\text{tree}}(3,1,4,2,5)$$

gravity states are products of YM states:

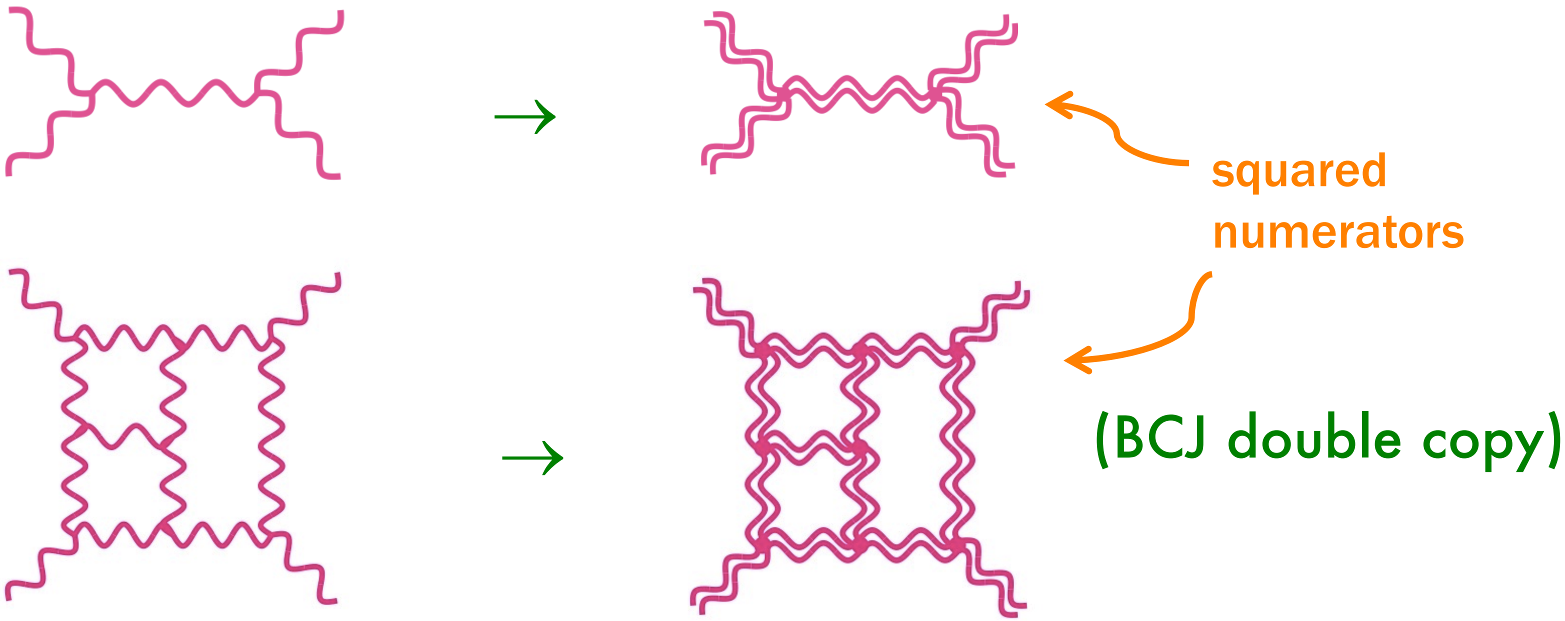
$$|2\rangle = |1\rangle \otimes |1\rangle$$
$$|3/2\rangle = |1\rangle \otimes |1/2\rangle$$

etc...

Squaring of YM theory – the double copy

Gravity processes = squares of gauge theory ones - entire S-matrix

Yang-Mills Gravity



E.g. pure Yang-Mills → Einstein gravity + dilaton + axion

4D YM + massless quarks → Pure 4D Einstein gravity

Example: axion-dilaton gravity

Consider double copy of *D*-dimensional pure YM:

States:

$$(\varepsilon^h)^{ij}_{\mu\nu} = \varepsilon_\mu^{((i}\varepsilon_\nu^{j))} \qquad \text{(graviton)}$$

$$(\varepsilon^B)^{ij}_{\mu\nu} = \varepsilon_\mu^{[i}\varepsilon_\nu^{j]} \qquad \text{(B-field)}$$

$$(\varepsilon^\phi)_{\mu\nu} = \frac{\varepsilon_\mu^i \varepsilon_\nu^j \delta_{ij}}{D-2} \qquad \text{(dilaton)}$$

Amplitudes consistent with the theory:

$$S = \int d^D x \sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2(D-2)}\partial^\mu\phi\partial_\mu\phi + \frac{1}{6}e^{-4\phi/(D-2)}H^{\lambda\mu\nu}H_{\lambda\mu\nu}\right]$$

In 4D this is axion-dilaton gravity:

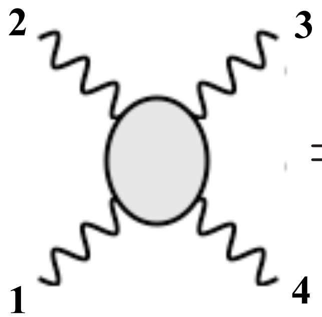
$$S = \int d^4 x \sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{4}\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}e^{-2\phi}\partial_\mu\chi\partial^\mu\chi\right]$$

Symmetry $\chi \to -\chi$, $\phi \to -\phi$ allows for consistent truncation of scalars

The (Square-)Root of Gravity

Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:



$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color factors: $c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic numerators:

$$n_s = \Big[(\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4) p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\big] \,,$$

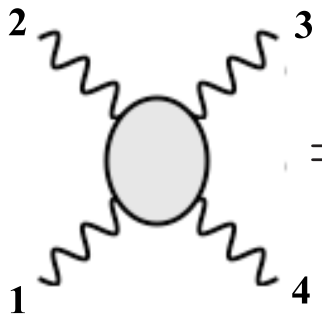
consider linearized gauge transformation $\delta A_\mu = \partial_\mu \xi$

$$n_s\Big|_{\varepsilon_4 \to p_4} = s\Big[(\varepsilon_1 \cdot \varepsilon_2)\big((\varepsilon_3 \cdot p_2) - (\varepsilon_3 \cdot p_1)\big) + \text{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon, p)$$

(individual diagrams not gauge inv.)

Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:



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$$+s\big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\big]\,,$$

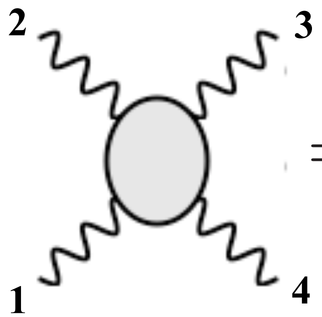
consider linearized gauge transformation $\delta A_\mu = \partial_\mu \xi$

$$\left.\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right|_{\varepsilon_4\to p_4} = \underbrace{(c_s + c_t + c_u)}_{=0}\,\alpha(\varepsilon, p)$$

= 0 Jacobi identity

Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:



$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color factors: $c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic numerators:

$$n_s = \Big[(\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4) p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\big]\,,$$

$c_s + c_t + c_u = 0$ Jacobi Id. (gauge invariance)

$\Leftrightarrow$

$n_s + n_t + n_u = 0$ kinematic Jacobi Id. (diffeomorphism inv.)

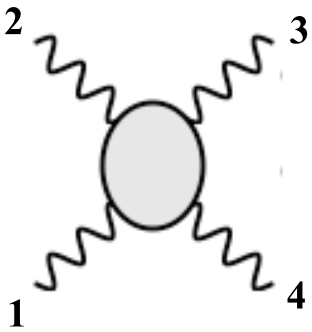
BCJ ('08)

Double copy

Color and kinematics are dual...

$$c_s + c_t + c_u = 0 \quad \Leftrightarrow \quad n_s + n_t + n_u = 0$$

...replace color by kinematics $c_i \to n_i$ BCJ double copy



$$= \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

← gravity ampl.

Properties of ampl:
- spin-2 scattering $\quad \varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$
- 2-derivative interactions $\quad \partial_\mu \to \partial_\mu \partial_\nu$
- diffeomorphism inv. $\quad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

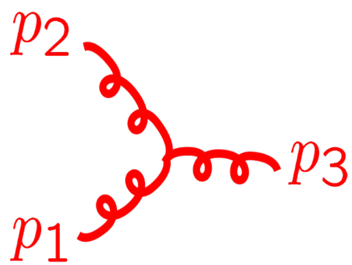
$$\left. \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u} \right|_{\varepsilon_4^{\mu\nu} \to p_4^\mu \varepsilon_4^\nu + p_4^\nu \varepsilon_4^\mu} = 2(n_s + n_t + n_u)\, \alpha(\varepsilon, p) = 0$$

What is the Kinematic Algebra ?

– YM numerators obey Jacobi Id. → a kinematic algebra should exist!
– Algebra *may* dramatically simplify GR calculations!

What is known?

Self dual YM in light-cone gauge: Monteiro, O'Connell ('11)



Generators of area-preserving diffeomorphisms:

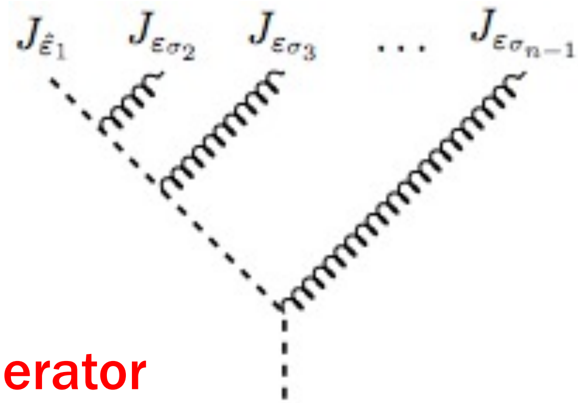
$$L_k = e^{-ik\cdot x}(-k_w\partial_u + k_u\partial_w)$$

Lie Algebra: $[L_{p_1}, L_{p_2}] = iX(p_1,p_2)L_{p_1+p_2} = iF_{p_1p_2}{}^{k}L_k$

YM vertex

Beyond the simplest helicity sectors (NMHV)

Chen, HJ, Teng, Wang [1906.10683, 2104.12726]



$$J_{\hat{\varepsilon}_1}(p) \star J_{\varepsilon_i}(p_i) = \varepsilon_i\cdot p\, J_{\hat{\varepsilon}_1}(p+p_i) - \frac{1}{2}J_{\hat{\varepsilon}_1\otimes\varepsilon_i\otimes(p+p_i)}(p+p_i)$$

vector generator

tensor generator

Cheung-Shen Lagrangian

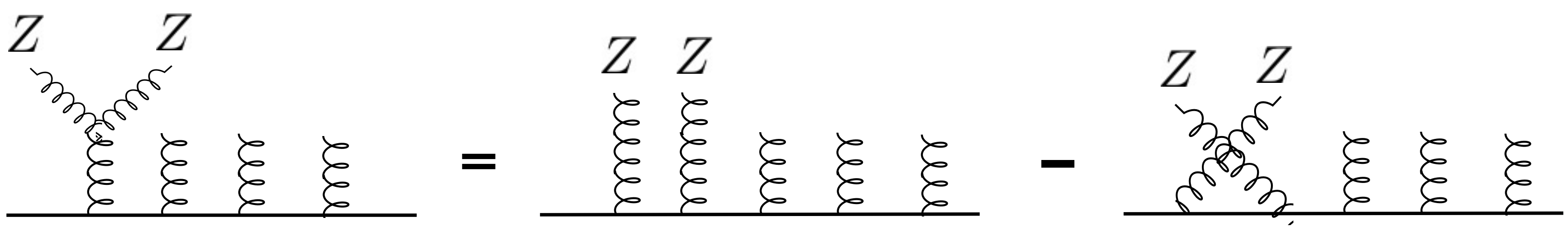
Cubic Lagrangian that manifests color-kinematics duality, gives:
→ NLSM pions at tree level
→ YM trees for MHV sector

Cheung, Shen ('16)

$$\mathcal{L}_{\rm CS} = Z^{a\mu}\Box X^a_\mu + \frac{1}{2}Y^a\Box Y^a - g f^{abc} Z^{a\mu}\left(Z^{b\nu}X^c_{\mu\nu} + Y^b\partial_\mu Y^c\right)$$

Jacobi Id. manifest:

$$X^a_{\mu\nu} = \partial_\mu X^a_\nu - \partial_\nu X^a_\mu\,;$$



NLSM pions: external states Y^a or $\partial_\mu Z^{a\mu}$

Gives all YM numerator terms of type: $n^{\rm YM} \sim \ (\varepsilon_1\cdot\varepsilon_n)\prod_{i,j}(\varepsilon_i\cdot p_j)$

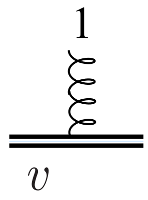
sufficient for MHV amplitude: Chen, HJ, Teng, Wang

Hopf algebra structure and heavy mass EFT

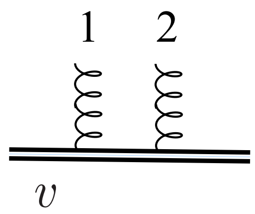
Gauge invariant BCJ numerators from heavy-quark limit

Brandhuber, Chen, HJ, Travaglini, Wen '21

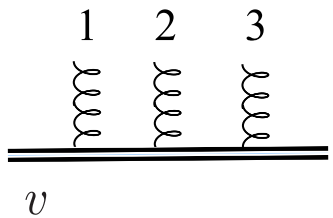
$$\mathcal{N}(1,v) = v\cdot\varepsilon_1\,,$$



$$\mathcal{N}(12,v) = -\frac{v\cdot F_1\cdot F_2\cdot v}{2v\cdot p_1}\,,$$



$$\mathcal{N}(123,v) = \frac{v\cdot F_1\cdot F_2\cdot F_3\cdot v}{3v\cdot p_1} - \frac{v\cdot F_1\cdot F_2\cdot V_{12}\cdot F_3\cdot v}{3v\cdot p_1 v\cdot p_{12}} - \frac{v\cdot F_1\cdot F_3\cdot V_1\cdot F_2\cdot v}{3v\cdot p_1 v\cdot p_{13}}$$



$$F_i^{\mu\nu} := p_i^\mu \varepsilon_i^\nu - \varepsilon_i^\mu p_i^\nu$$
$$V_\tau^{\mu\nu} := v^\mu \sum_{j\in\tau} p_j^\nu$$

Brandhuber, Brown, Chen, Gowdy, Travaglini '22

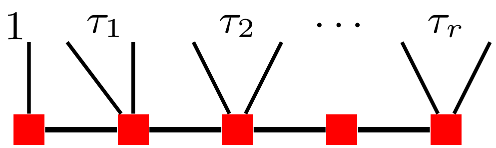
See talks → Chen, Brown

YM numerators at any multiplicity given by an associative Hopf algebra

$$\mathcal{N}(12\ldots n{-}2,v) := \langle T_{(1)} \star T_{(2)} \star \cdots \star T_{(n-2)}\rangle$$

Quasi-shuffle product: $T_{(12)} \star T_{(3)} = -T_{(123)} + T_{(12),(3)} + T_{(13),(2)}$

$$\langle T_{(1\tau_1),(\tau_2),\ldots,(\tau_r)}\rangle :=$$



$$= \frac{v\cdot F_{1\tau_1}\cdot V_{\Theta(\tau_2)}\cdot F_{\tau_2}\cdots V_{\Theta(\tau_r)}\cdot F_{\tau_r}\cdot v}{(n{-}2)v\cdot p_1 v\cdot p_{1\tau_1}\cdots v\cdot p_{1\tau_1\tau_2\cdots\tau_{r-1}}}$$

First complete Kinematic Algebra ?

Pure Chern-Simons: complete kinematic algebra at tree/loop level

Ben-Shahar, HJ

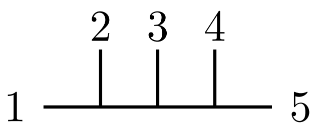
Generators $$L^\mu(p) = e^{ip\cdot x}\Delta^{\mu\nu}\partial_\nu$$

3D transversality "projector" $$\Delta^{\mu\nu}(p) = i\epsilon^{\rho\mu\nu}p_\rho$$

Infinite-dimensional kinematic Lie algebra $$[L^\mu(p_1), L^\nu(p_2)] = F^{\mu\nu}{}_\rho\, L^\rho(p_1+p_2)$$

Kinematic structure constants $$F^{\mu_1\mu_2}{}_\nu(p_1,p_2) = \Delta^{\rho\mu_1}(p_1)\epsilon_{\rho\nu\sigma}\Delta^{\sigma\mu_2}(p_2)$$

BCJ numerators



$$= \mathrm{tr}\Big([[[L^{\mu_1}(p_1), L^{\mu_2}(p_2)], L^{\mu_3}(p_3)], L^{\mu_4}(p_4)], L^{\mu_5}_{\mathrm{amp}}(p_5)\Big)$$
$$= F^{\mu_1\mu_2}{}_\nu F^{\nu\mu_3}{}_\rho F^{\rho\mu_4\mu_5}\delta^3(p_1+p_2+p_3+p_4+p_5)\ ,$$

Lie algebra of 3D volume-preserving diffeomorphisms!

See talk → Ben-Shahar

Double Copy Theories

Example: pure GR

Pure 4D Einstein gravity: $\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{g}R$

HJ, Ochirov

Does not match YM² spectrum: $\mathrm{YM} \otimes \mathrm{YM} = \mathrm{GR} + \phi + a$

Deform YM theories with massless fundamental quarks

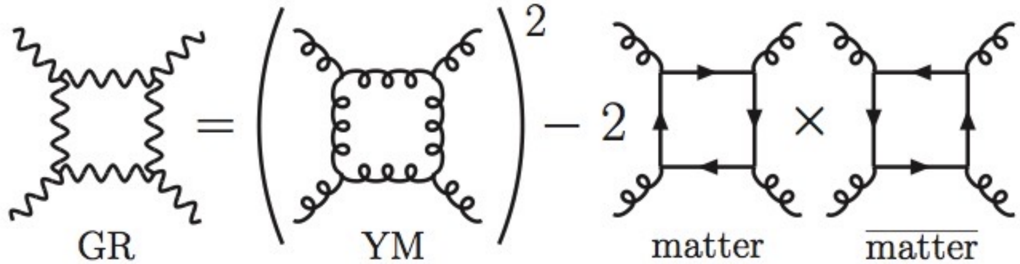
$$(\mathrm{YM} + \mathrm{quark}) \otimes (\mathrm{YM} + n_f\ \mathrm{quarks})$$
$$= \mathrm{GR} + 2(n_f + 1)\ \mathrm{scalars}$$

Anti-align the spins of the quarks → gives scalars in GR

e.g. $\phi = q \otimes \bar{q} + \bar{q} \otimes \bar{q}$

$a = q \otimes \bar{q} - \bar{q} \otimes \bar{q}$

become ghosts if $n_f = -1$



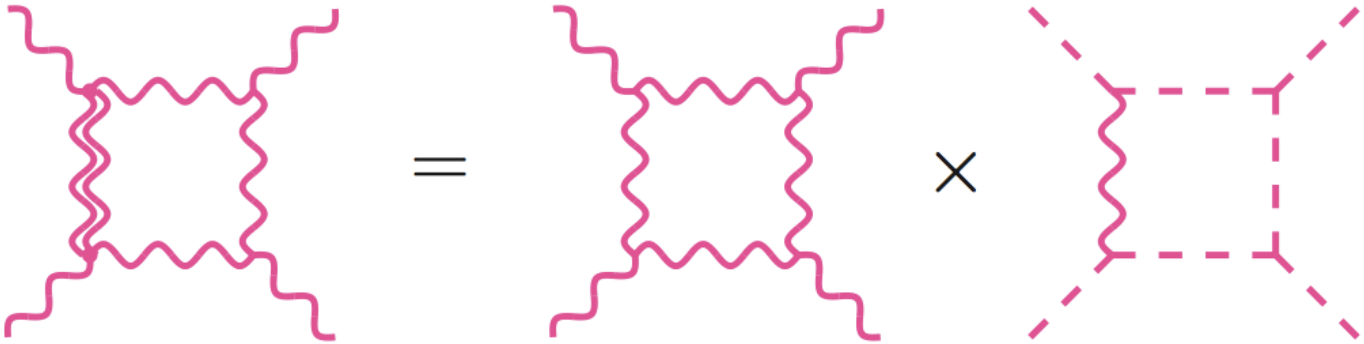
Example: YM-Einstein theory

GR+YM amplitudes are "heterotic" double copies

$$\mathrm{GR} + \mathrm{YM} = \mathrm{YM} \otimes (\mathrm{YM} + \phi^3)$$

Chiodaroli, Gunaydin, HJ, Roiban

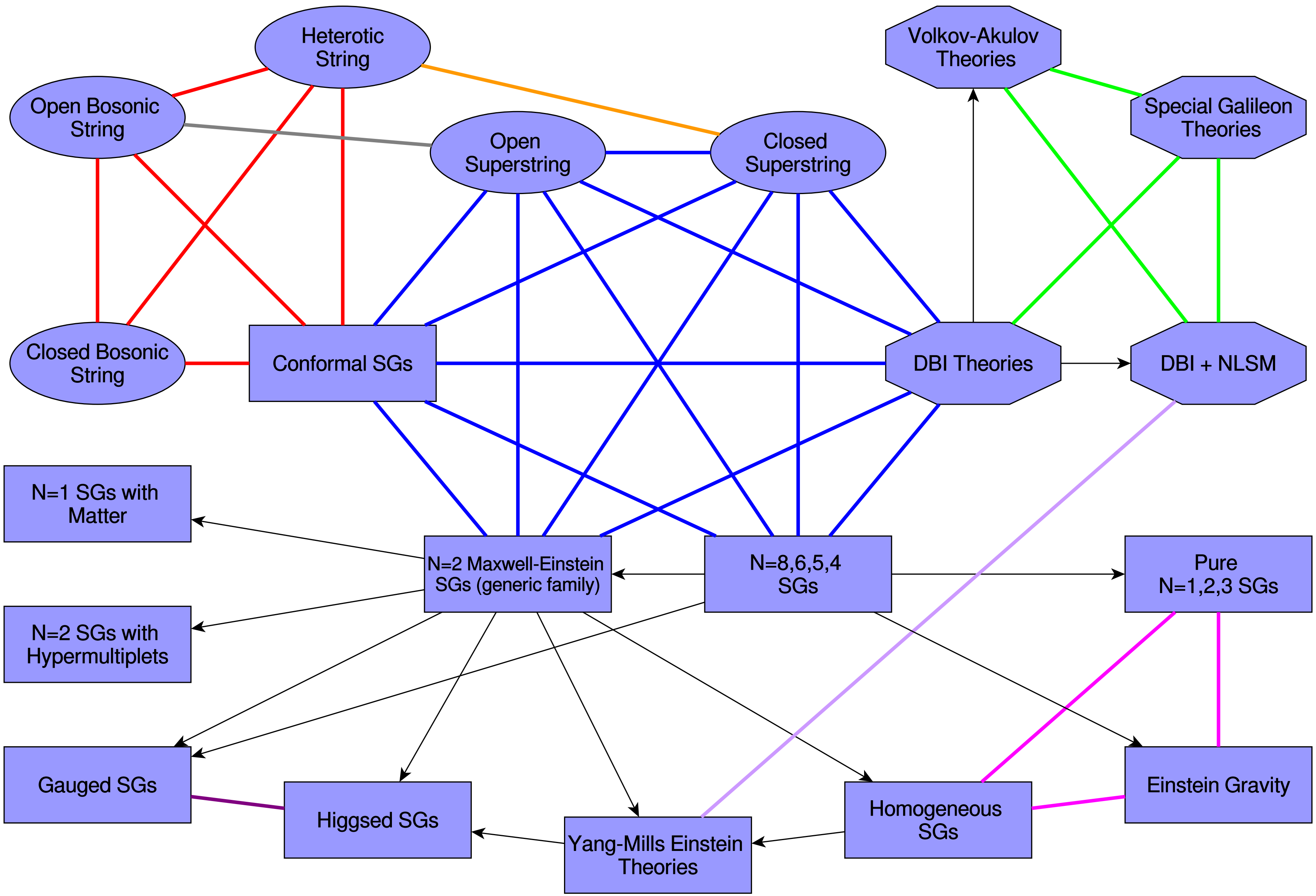
$$h^{\mu\nu} \quad \sim \quad A^\mu \otimes A^\nu$$
$$A^{\mu a} \quad \sim \quad A^\mu \otimes \phi^a$$



$\mathcal{N}$= 0,1,2,4 YM-E supergravity **$\mathcal{N}$= 0,1,2,4 SYM** **YM + ϕ^3**

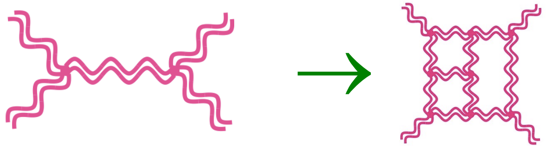
- $\mathcal{N}$=0,1,2 YM-E all have axion-dilaton states $\rightarrow$ g, θ parameters
- Construction extends to SSB (Coulomb branch) Chiodaroli, Gunaydin, HJ, Roiban ('15)

Web of double-copy constructible theories



See reviews [1909.01358], [2203.13013] – Bern, Carrasco, Chiodaroli, HJ, Roiban

Generalizations of C/K & double copy

Trees → loops:  Bern, Carrasco, HJ ('10)

→ Theories that are not truncations of $\mathcal{N}$=8 SG HJ, Ochirov; Chiodaroli, Gunaydin, Roiban,...

→ Theories with fundamental matter HJ, Ochirov; Chiodaroli, Gunaydin, Roiban,...

→ Spontaneously broken theories (gauge/susy) Chiodaroli, Gunaydin, HJ, Roiban

→ Form factors Boels, Kniehl, Tarasov, Yang & CFT correlators Farrow, Lipstein, McFadden

→ Gravity off-shell symmetries from YM Anastasiou, Borsten, Duff, Hughes, Nagy,...

→ Classical (black hole) solutions Luna, Monteiro, O'Connell, White; Ridgway, Wise; Goldberger,...

→ Gravitational radiation/potential Luna, Monteiro, Nicholson, O'Connell, White; Goldberger,.. Bern, Cheung, Roiban, Solon; Bjerrum-Bohr et al. ..

→ Amplitudes in curved background Adamo, Casali, Mason, Nekovar; Herderschee, Roiban, Teng

→ CHY scattering eqs, twistor strings Cachazo, He, Yuan, Skinner, Mason, Geyer, Adamo, Monteiro,..

→ Scalar EFTs: NLSM, DBI, Galileon Cachazo, He, Yuan; Du, Chen; Cheung, Shen; Elvang et al.

→ New double copies for string theory Mafra, Schlotterer, Stieberger, Taylor, Broedel, Carrasco... ... Azevedo, Marco Chiodaroli, HJ, Schlotterer

→ Conformal gravity HJ, Nohle; Mogull, Teng; Menezes

→ Celestial amplitudes Casali, Puhm; Sharma; Monteiro; Brown, Gowdy, Stieberger, Taylor

→ Non-perturbative DC Cheung, Mangan, Parra-Martinez, Shah; Armstrong-Williams, White, Wikeley, Stark-Muchão, ...

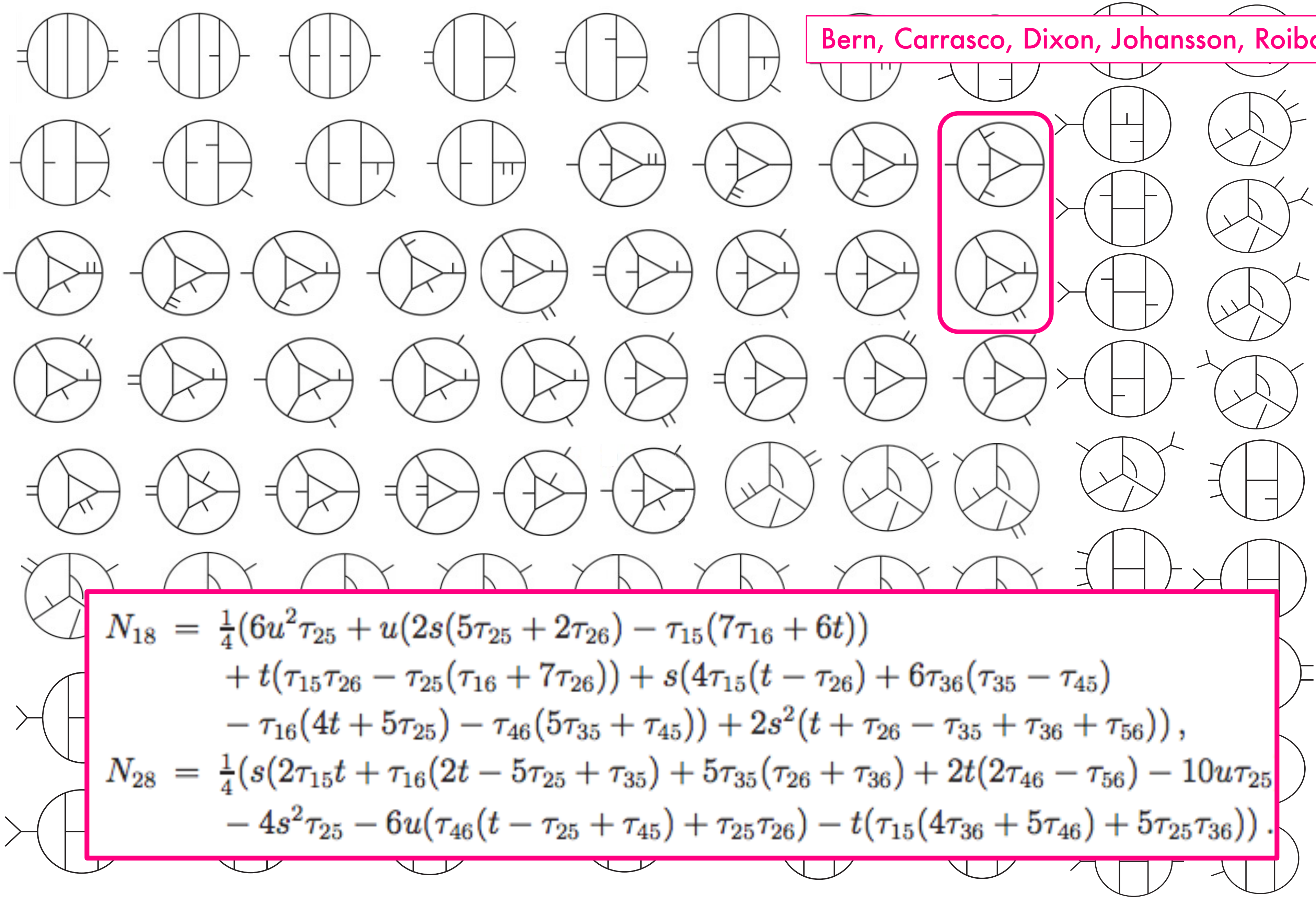
→ New massive DCs: Momeni, Rumbutis, Tolley; Engelbrecht, Jones, Paranjape; Carrillo González; Burger, Emond, Moynihan; Lust, Markou, Mazloumi, Stieberger ...

See talks → Mangan, Wikeley, Markou, Moynihan, Engelbrecht; Monteiro; Brown, Gowdy

Loop calculations – CK duality & double copy

$\mathcal{N}$=4 SYM @ 4-loops: 85 diagrams, 2 masters

Bern, Carrasco, Dixon, Johansson, Roiban



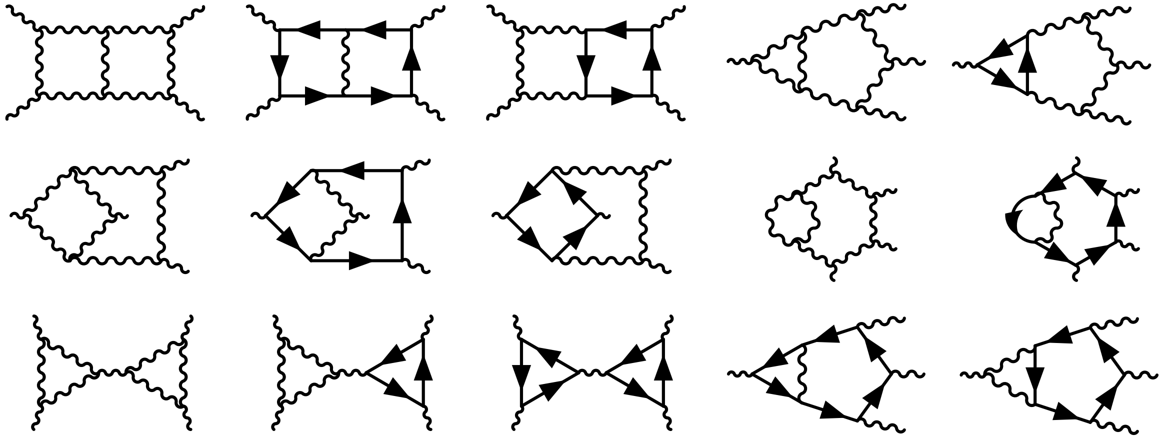
$$
\begin{aligned}
N_{18} = {} & \tfrac{1}{4}\big(6u^2\tau_{25} + u(2s(5\tau_{25} + 2\tau_{26}) - \tau_{15}(7\tau_{16} + 6t)) \\
& + t(\tau_{15}\tau_{26} - \tau_{25}(\tau_{16} + 7\tau_{26})) + s(4\tau_{15}(t - \tau_{26}) + 6\tau_{36}(\tau_{35} - \tau_{45}) \\
& - \tau_{16}(4t + 5\tau_{25}) - \tau_{46}(5\tau_{35} + \tau_{45})) + 2s^2(t + \tau_{26} - \tau_{35} + \tau_{36} + \tau_{56})\big)\,, \\
N_{28} = {} & \tfrac{1}{4}\big(s(2\tau_{15}t + \tau_{16}(2t - 5\tau_{25} + \tau_{35}) + 5\tau_{35}(\tau_{26} + \tau_{36}) + 2t(2\tau_{46} - \tau_{56}) - 10u\tau_{25} \\
& - 4s^2\tau_{25} - 6u(\tau_{46}(t - \tau_{25} + \tau_{45}) + \tau_{25}\tau_{26}) - t(\tau_{15}(4\tau_{36} + 5\tau_{46}) + 5\tau_{25}\tau_{36})\big)\,.
\end{aligned}
$$

Complete *N*=2 SQCD 2-loop calculation

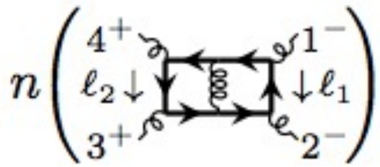
Integrand computed using color-kinematics duality and supersymmetric decomposition

HJ, Kälin, Mogull ('17)

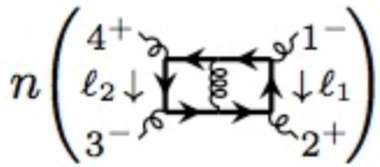
- two-loop SQCD amplitude
- color-kinematics manifest
- planar + non-planar
- N_f massless quarks
- integrand valid in $D \leq 6$



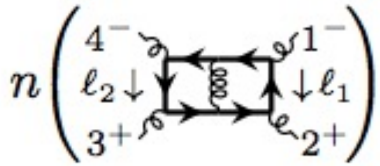
e.g. simple SQCD numerators



$$= -\kappa_{12}\mu_{12}\,,$$



$$= \frac{\kappa_{13}}{u^2}\,\mathrm{tr}_-(1\ell_1 24\ell_2 3)$$



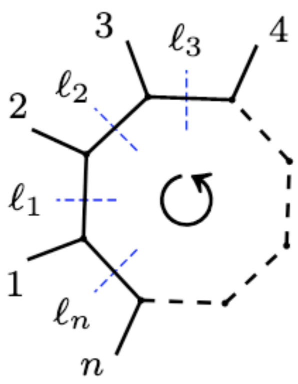
$$= \frac{\kappa_{14}}{t^2}\,\mathrm{tr}_-(1\ell_1 23\ell_2 4)$$

trace-rep. from 1811.09604 Kälin, Mogull, Ochirov

Perfecting one-loop BCJ numerators ?

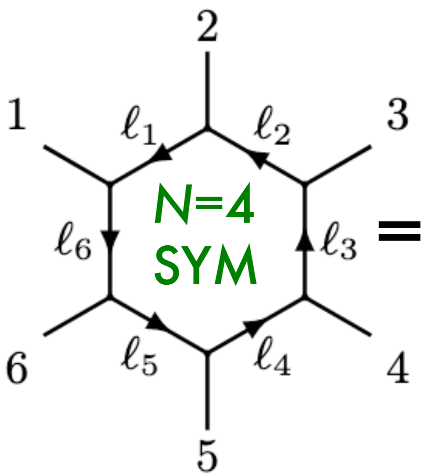
Edison, He, HJ, Schlotterer, Teng, Zhang

All-multiplicity numerators modulo contact terms:



$$= \left\{ \begin{array}{l} n_{\rm v}\,\mathrm{tr}\displaystyle\prod_{i=1}^{n}\Big((\epsilon_i\cdot\ell_i)\mathbf{1}_{\rm Vec}-f_i\Big)+(n_{\rm s}-2n_{\rm v})\prod_{i=1}^{n}(\epsilon_i\cdot\ell_i) \\ -\dfrac{n_{\rm l}+n_{\rm r}}{4}\mathrm{tr}\displaystyle\prod_{i=1}^{n}\Big((\epsilon_i\cdot\ell_i)\mathbf{1}_{\rm Dirac}-\not{f}_i\Big) \\ +\dfrac{n_{\rm l}-n_{\rm r}}{4}\mathrm{tr}\Big[\Gamma\not{\epsilon}_1\not{\ell}_1\displaystyle\prod_{i=2}^{n}\Big((\epsilon_i\cdot\ell_i)\mathbf{1}_{\rm Dirac}-\not{f}_i\Big)\Big], \end{array}\right.$$

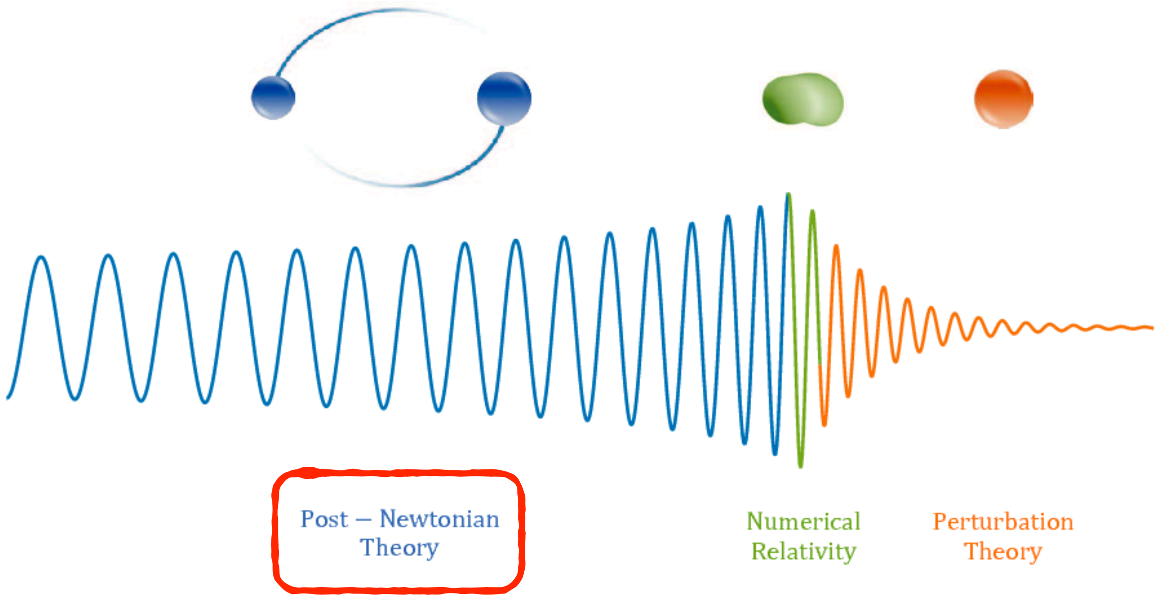
Explicit numerators for 6-7pt *N* = 4 SYM and 5pt *N* = 2 SYM in d-dim



$$= \left\{ \begin{array}{l} \Big[\epsilon_1\cdot\ell_1\epsilon_2\cdot\ell_2\mathrm{tr}_{(\max)}(f_3f_4f_5f_6)+(1,2|1,2,3,4,5,6)\Big] \\ -\Big[\epsilon_1\cdot\ell_1\mathrm{tr}_{(\max)}(f_2f_3f_4f_5f_6)+\mathrm{cyclic}(1,2,3,4,5,6)\Big]+\mathrm{tr}_{(\max)}(f_1f_2f_3f_4f_5f_6) \\ +\dfrac{1}{40}\Big[\epsilon_1\cdot\epsilon_2\,(3\ell_2^2-10\ell_1^2+3\ell_6^2)\mathrm{tr}_{(\max)}(f_3f_4f_5f_6) \\ \qquad +\epsilon_1\cdot\epsilon_3\,(\ell_3^2-3\ell_2^2-3\ell_1^2+\ell_6^2)\mathrm{tr}_{(\max)}(f_2f_4f_5f_6) \\ \qquad -\epsilon_1\cdot\epsilon_4\,(\ell_1^2+\ell_6^2)\mathrm{tr}_{(\max)}(f_2f_3f_5f_6)+\mathrm{cyclic}(1,2,3,4,5,6)\Big]. \end{array}\right.$$

Double copy and black hole amplitudes

Double copy and gravitational waves



Explicit PM calculations done using double copy:

Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove ('18)
Bern, Cheung, Roiban, Shen, Solon, Zeng ('19)+ Ruf, Parra-Martinez ('21)
Brandhuber, Chen, Travaglini, Wen (21)

Some methods developed for PM calc. using double copy:

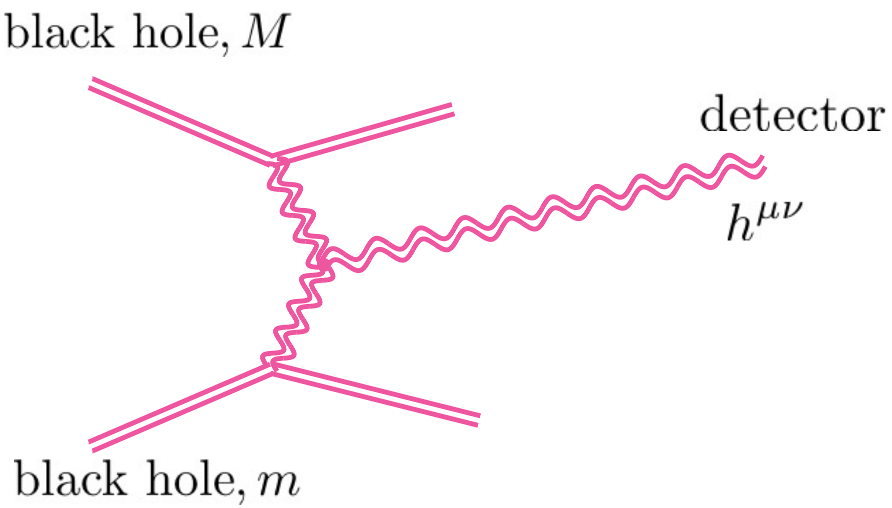
Bjerrum-Bohr, Cristofoli, Damgaard, Gomez+Brown;
Cristofoli, Gonzo, Kosower, O'Connell;
Maybee, O'Connell, Vines; Luna, Nicholson, O'Connell, White; …

See talks → Roiban, Alessio, Aoude, Pichini, Ochirov

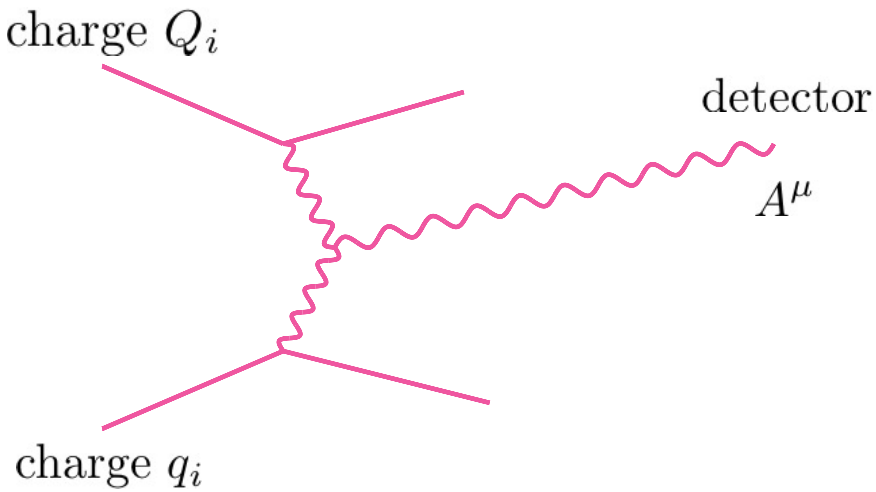
Gravitational radiation

LIGO/VIRGO observations → motivates high-order PN, PM calcs.

BH gravitational scattering



analog non-abelian
gauge-theory process



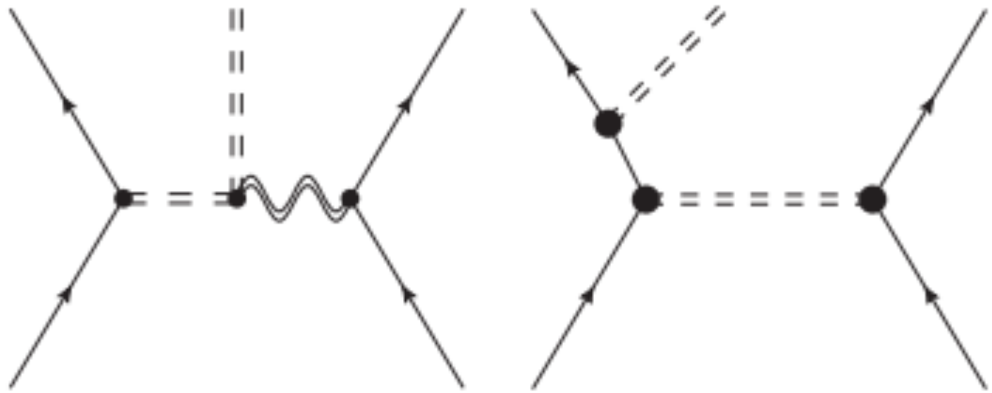
Using double copy
for GW, potentials,
observables :

Goldberger, Ridgway; Prabhu, Thompson; Li,
Luna, Monteiro, Nicholson, O'Connell, White; Shen;
Plefka, Steinhoff, Wormsbecher; Plefka, Shi, Steinhoff, Wang;
Maybee, O'Connell, Vines;
Bern, Cheung, Roiban, Shen, Solon, Zeng;
Bern, Kosmopoulos, Luna, Roiban, Teng;
Bern, Parra-Martinez, Roiban, Ruf, Shen; [...]

Removing the dilaton ?

For massive processes the dilaton couples to mass

$$\mathcal{L}_{\rm matter} \sim \sqrt{-g}(\partial_\mu \bar{\varphi} \partial^\mu \varphi - m^2 e^{-\phi} \bar{\varphi}\varphi) + \dots$$



see e.g.
Luna, Nicholson, O'Connell, White;
Plefka, Shi, Wang;
HJ, Ochirov

Can be removed by compensating diagrams Luna, Nicholson, O'Connell, White
or projectors applied to on-shell states Bern, Cheung, Roiban, Shen, Solon, Zeng; Carrasco, Vazquez-Holm

However, methods not completely satisfactory:
→ What it the most efficient approach?
→ Is removal complete for all physical processes?
→ General framework for different theories? (cf. HJ, Ochirov for pure GR)

AHH amplitudes ←→ Kerr BH?

Arkani-Hamed, Huang, Huang ('17) wrote down natural higher-spin ampl's:

Kerr 3pt:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}, \qquad M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Root-Kerr 3pt:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}, \qquad A(1\phi^s, 2\bar{\phi}^s, 3A^-) = \frac{m}{x}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Shown to reproduce Kerr by: Guevara, Ochirov, Vines ('18); Vines ('17)

Gravity Compton ampl.
via BCFW recursion ?

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle\mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}.$$

Problem: spurious pole for $s > 2$

EFTs for root-Kerr AHH amplitudes ?

Rewrite the 3pt AHH amplitudes on covariant form → identify theory

Chiodaroli, HJ, Pichini;
HJ, Ochirov

1) introduce generating series, e.g.

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A^+) = \frac{mx}{1-\frac{\langle\mathbf{12}\rangle^2}{m^2}}$$

2) rewrite covariantly (both helicity sectors):

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\varepsilon_1\cdot\varepsilon_2)^2 A_{\phi\phi A}}{(1+\varepsilon_1\cdot\varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_1}$$

$$A_{\phi\phi A} \equiv i\sqrt{2}\,\varepsilon_3\cdot p_1\,, \quad A_{WWA} \equiv i\sqrt{2}\,(\varepsilon_1\cdot\varepsilon_2\,\varepsilon_3\cdot p_2 + \varepsilon_2\cdot\varepsilon_3\,\varepsilon_1\cdot p_3 + \varepsilon_3\cdot\varepsilon_1\,\varepsilon_2\cdot p_1)$$

$s=0$ & $s=1/2$ minimally coupled scalar & fermion

$s=1$ W-boson $s=3/2$ charged/massive gravitino

EFTs for Kerr AHH amplitudes?

Related to the root-Kerr via double copy

$$M(1\phi^s, 2\bar{\phi}^s, 3h^{\pm}) = iA(1\phi^{s_{\rm L}}, 2\bar{\phi}^{s_{\rm L}}, 3A^{\pm})A(1\phi^{s_{\rm R}}, 2\bar{\phi}^{s_{\rm R}}, 3A^{\pm})$$

3pt works for any decomposition: $s = s_{\rm L} + s_{\rm R}$

Preferred decomposition $s = 1 + (s-1)$ give fewest derivatives :

$$\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA}\Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1 \cdot \varepsilon_2)^2 A_{0\oplus 1/2}}{(1 + \varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1 \cdot p_2\, \varepsilon_2 \cdot p_1}\Big)$$

From double-copy structure:

$s = 0,\ s = 1/2,\ s = 1,\ s = 3/2$ min-coupled matter (Proca, Rarita-Schwinger)

$s = 2$ Kaluza-Klein graviton

Also works for Compton and beyond (Lagrangians known)

Summary of EFTs

EFTs	$s = {}^{1}/_{2}$	$s = 1$	$s = {}^{3}/_{2}$	$s = 2$	$s = {}^{5}/_{2}$	$s \geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	[26]	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	-	HS

The $s \leq 2$ Kerr ampl's for admit double copies to any multiplicity

$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{scalar}) = (\text{GR} + \text{scalar})$$
$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{fermion}) = (\text{GR} + \text{fermion})$$
$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{W-boson}) = (\text{GR} + \text{Proca})$$
$$(\text{YM} + \text{W-boson}) \otimes (\text{YM} + \text{fermion}) = (\text{GR} + \text{massive gravitino})$$
$$(\text{YM} + \text{W-boson}) \otimes (\text{YM} + \text{W-boson}) = (\text{GR} + \text{massive KK graviton})$$

Lagrangians unique: have no non-minimal terms beyond cubic order in fields

Can be used for $(S^\mu)^{\leq 4}$ PM/PN calculations.

Compton $(S^\mu)^4$ to be confirmed via other methods (BHPT, worldline).

See: Bautista, Guevara, Kavanagh, Vines; Aoude, Haddad, Helset;
Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

→ Pichini

Summary & Outlook

- Color-kinematics duality lies at the root of gravity:
  → makes perturbative GR more manageable!
  → allows for simpler classification of gravity theories
  → kinematic algebra is a well-hidden gem of YM (and GR)
  → useful for PM calculations

- Explored amplitudes for massive spinning matter → Kerr BH ?
  → Double copy works well up to spin-2 (KK graviton)
  → Paolo Pichini can give more details on higher-spin resultss

- Not discussed: string theories exhibit novel double copy structures.

$$\text{string tree ampl} = \text{String} \otimes \text{QFT}$$

Azevedo, Chiodaroli, HJ, Schlotterer ('18)

- Not discussed: C/K duality in AdS space (Herderschee, Roiban, Teng; [...])
- Not discussed: classical double copies of BH solutions (O'Connell et al. [...])
- Not discussed: new massive DC, celestial DC, non-perturbative DC.....

The topic of double copy & CK duality has grown significantly in the last few years, you will hear more about it at QCD Meets Gravity Zurich !

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Organizers:

Daniel Baumann, Zvi Bern, Alessandra Buonanno,
John Joseph Carrasco, Paolo Di Vecchia, Henrik Johansson,
Andrea Phum, Oliver Schlotterer

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July 24 – 28, 2023

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- multiloop integration
- gravitational waves and classical GR
- EFT methods
- celestial amplitudes
- cosmology, inflation,
- curved space amplitudes